King Fahd University of Petroleum & Minerals
Electrical Engineering Department

EE570 Stochastic Processes
Assignment III

November 25, 2007 — Due Date: December 2, 2007

Question 1

Show that the mean and variance of the Weibull random variable are

$$E(T) = \alpha^{-1/\beta}\Gamma\left(1+\frac{1}{\beta}\right)$$
$$\mathrm{VAR}[T] = \alpha^{-2/\beta}\left\{\Gamma\left(1+\frac{2}{\beta}\right)-\left[\Gamma\left(1+\frac{1}{\beta}\right)\right]^2\right\} \tag{1}$$

The Weibull distribution is given by

$$f_{\mathbf{T}}(t) = \alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}} \tag{2}$$

Question 2

The number $\mathbf{x}$ of electrons counted by a receiver in an optical communication system is Poisson random variable with rate λ_1 when a signal is present and with rate $\lambda_o < \lambda_1$ when the signal is absent. Suppose that a signal is present with probability p.

(a) Find $P[\text{signal present}|\mathbf{x}=k]$ and $P[\text{signal absent}|\mathbf{x}=k]$.

(b) The receiver uses the following decision rule:

If $P[\text{signal present}|\mathbf{x}=k] > P[\text{signal absent}|\ \mathbf{x}=k]$, decide signal present; otherwise, decide signal is absent.

Show that this decision rule leas to the following threshold rule:

If $\mathbf{x} > T$, decide signal present; otherwise decide signal absent.

(c) What is the probability of error for the above decision rule?

Question 3

Let $\mathbf{x}$ be a the input to a communication channel. $\mathbf{x}$ takes on the values ±1 with equal probability. Suppose that the output of the channel is $\mathbf{y}=\mathbf{x}+\mathbf{N}$, where $\mathbf{N}$ is a Laplacian random variable with pdf

$$f_{\mathbf{N}}(z) = \frac{1}{2}\alpha e^{-\alpha|z|} \qquad -\infty \le z \le \infty \tag{3}$$

(a) Find $P[\mathbf{x} = k, \mathbf{y} \le y]$ for $k = \pm 1$.

(b) Find the marginal pdf of $\mathbf{y}$.

(c) Suppose we are given that $\mathbf{y} > 0$. Which is more likely, $\mathbf{x}=1$ or $\mathbf{x} = -1$?

Question 4

A message requires N time units to be transmitted, where N is a geometric random variable with pmf $p_j = (1-a)a^{j-1}$, $j = 1, 2,...$ A single new message arrives during a time unit with proba-

bility p, and no message arrive with probability 1-p. Let K be the number of new messages that arrive during the transmission of a single message.
(a) Find the pmf of K.

Hint: $(1-\beta)^{-(k+1)} = \sum_{n=k}^{\infty} \binom{n}{k} \beta^{n-k}$

(b) Find $E[K]$ and VAR[K] using conditional expectation.

Question 5

The lifetime **x** of a device is an exponential random variable with mean 1/**R**. Suppose that die to irregularities in the production process, the parameter **R** is random and has gamma distribution.
(a) Find joint probability density function of **x** and **R**
(b) Find probability density function of **x.**
(c) Find the mean and the variance of **x**.