King Fahd University of Petroleum & Minerals
Electrical Engineering Department

EE570 Stochastic Processes
Assignment I

September 24, 2007 Due Date: October 20, 2007

Question 1

A non symmetrical binary communication channel is shown in Figure 1.1. Assume the inputs are equiprobable.
(a) Find the probability that the output is 0.
(b) Find the probability that the input was 0 given that the output was 1. Find the probability that the input is 1 given that output is a 1. What input is more probable?

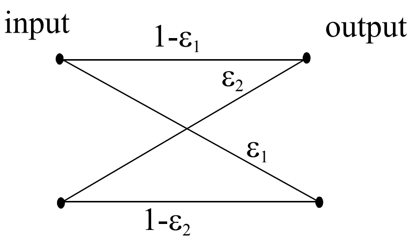

Figure 1.1 Binary unsymmetrical channel

Question 2

A transmitter outputs three symbols *A*, *B*, *C*. In Table 1, probabilities of sending symbol *j* and receiving symbol *k* making a pair (*j,k*), are listed:

Table 1: Probabilities of transmission and reception

j	*k*		
	A	*B*	*C*
A	0.1	0.06	0.12
B	0.07	0.15	0.05
C	0.10	0.15	0.2

(i) Sketch the channel model giving symbol probabilities at the input and the output of the channel.
(ii) Calculate the probability that the symbol *k* was sent, given that symbol *k* is received, for *k* = *A*, *B*, *C*.
(iii) Calculate the probability of error incurred in using this system. An error is defined as the reception of a symbol other than the one transmitted.
(iv) What is the probability of receiving symbol *C*?

Question 3

A random variable x has *pdf*

$$f_x(x) = \begin{cases} cx(1-x) & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases} \qquad (1)$$

(a) Find c.

(b) Find $P\left[\frac{1}{2} \le x \le \frac{3}{4}\right]$.

(c) Find $F_{\mathbf{x}}(x)$.

Question 4

The amount of time cars are parked in a parking lot follows an exponential probability law with parameter 1. The charge for parking in the lot is 2 Rials for each half hour or less.
(a) Find the probability that a car pays k rials.
(b) Suppose that there is a maximum charge of 10 Rials. Find the probability that a car pays k Rials.

Question 5

The number of orders waiting to be processed is given by a Poisson random variable with parameter $\alpha = \frac{\lambda}{n\mu}$ where λ is the average number of orders that arrive in a day, μ is the number of orders that can be processed by an employee per day, and n is the number of employees. Let $\lambda = 3$ and $\mu = 1$. Find the number of employees required so that the probability that more than 4 orders are waiting in less than 90%. What is the probability that there are no orders waiting?

Question 6

The duration of long distance telephone calls is generally modeled as a random phenomenon. It was found that the long distance calls made from Jeddah to Dammam fits a probability distribution function $F(.)$, given by

$$\begin{aligned} F_{\mathbf{x}}(x) &= 0 \qquad \text{for } x < 0 \\ &= 1 - \frac{1}{2}e^{-\left(\frac{x}{3}\right)} - \frac{1}{2}e^{-\left\lfloor\frac{x}{3}\right\rfloor} \qquad x \ge 0 \end{aligned} \tag{2}$$

where $\lfloor y \rfloor$ is being a real number and largest integer $y \ge 0$ less than or equal to y.

1. What type of random variable the distribution function defines? Find the probability density function of the random variable.
2. What is the probability that a long distance telephone call is (i) more than three minutes, (ii) equal to three minutes, (iii) more than four minutes (iv) equal to six minutes?
3. (b) What is the conditional probability that the duration in minutes of a long distance telephone call is (i) less than eight minutes, given that it is more than three minutes?

Question 7

A communication receiver filters and amplifies the voltage across an antenna terminal. The receiver output sampled at a certain time t is a random variable $\mathbf{x}$. The probability density function of $\mathbf{x}$, when only the background noise (no signal) is present at the antenna input (hypothesis H_0), is given by,

$$f_{\mathbf{x}}(x|H_0) = (2\pi\sigma^2)^{-1/2}\exp\left(-\frac{x^2}{2\sigma^2}\right) \tag{3}$$

The probability density function of $\mathbf{x}$, when a signal (presence of noise is implicit) is also present at the antenna input (hypothesis H_1), is

$$f_{\mathbf{x}}(x|H_1) = (2\pi\sigma^2)^{-1/2}\exp\left(-\frac{(x-m)^2}{2\sigma^2}\right), \qquad m > 0 \tag{4}$$

It is given that the signal is transmitted with a probability 1/2.

1. Given that a particular value x of this random variable $\mathbf{x}$ has been observed at the output of the receiver, what is the posterior (conditional) probability that a signal is present? That is, calculate $P_r(\text{signal present} \,|\mathbf{x} = x)$.
2. If we wish to minimize the probability of error, P_e, in deciding whether or not the signal is present, over what range of values of x should we decide "signal present" and over what range should we decide "signal absent"?