King Fahd University of Petroleum and Minerals
Electrical Engineering Department

EE570 Stochastic Processes

Student Name:	Student Number:

Final Examination - 071	Time allowed: 3 Hours	Date: January 23, 2007

PART I - SHORT QUESTIONS (18 MARKS)

Question 1

A urn contains six balls, numbered {1, 2, 3, 4, 5, 6}.
(a) A ball is drawn and is not replaced. A second draw follows. What is the probability that the two balls drawn have the same number?

Answer:

(b) The first drawn ball is replaced before a second ball is drawn. What is the probability the second ball drawn is the same as the first one?

Answer:

(c) Suppose the ball numbered 2 is drawn. What is the joint probability of drawing same numbered ball is not drawn i.e. $P[2,j]$, j =1, 3, 4, 5, 6 in the experiment?

Question 2

If $A = \{1 \le x \le 4\}$ and $B = \{3 \le x \le 5\}$, find $A \cup B, AB,$ and $(A \cup B)(\overline{AB})$.

Question 3

A Probability Distribution Function is defined as

$$F_{\mathbf{x}}(x) = 0; \qquad x < 0$$
$$= 1 - \frac{1}{2}e^{-(x/3)} - \frac{1}{2}e^{-\lfloor x/3 \rfloor}; \qquad x \geq 0 \tag{1}$$

$\lfloor . \rfloor$ denote integer value. Identify the type of distribution:

(a) discrete (b) continuous (c) mixed (d) none of the given choices?

(b) What is the probability that $x = 3$?

Question 4

The probability of a bit error in a communications line is 10^{-4}. Find the probability that a block of 10000 bits has three or more errors.

Question 5

(a) What measure defines the degree of linear dependence between random variables?

(b) Write an expression for the parameter identified in part (a).

Question 6

A random variable, **y** is given by $\boldsymbol{y} = \boldsymbol{x}_1 + \boldsymbol{x}_2 + \boldsymbol{x}_3 + \boldsymbol{x}_4$. The *pdf*'s of random variables, $\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3,$ and $\boldsymbol{x}_4$ are given to be $f_{X_1}(x_1), f_{X_2}(x_2), f_{X_3}(x_3),$ and $f_{X_4}(x_4)$ respectively. Our interest is to find the *pdf* of **y**. Suggest methods to find $p_{\mathrm{y}}(\mathbf{y})$ and write simplified expressions for the suggested methods.

Question 7

Define stationary, wide-sense stationary, and cyclostationary random process illustrating the differences between them.

Question 8

(a) Write the relationship between the power spectral density and auto-correlation function of a random variable.

(b) Give the salient properties of power spectral density of a random variable.

Question 9

(a) Define with the help of an equation a Markov process.

(b) What is the difference between a Markov process and a Markov chain?

PART II LONG QUESTIONS (42 MARKS)

Question 1

The sample **X** of speech signal has a pdf of $p_X(x) = \frac{1}{2}\exp(-|x|), -\infty \le x \le \infty$, which is a Laplacian random variable with parameter $\alpha = 1$. Suppose that X is quantized by a 4-level non-uniform quantizer with intervals: $(-\infty, -a]$, $(-a, 0]$, $(0, a]$, and (a, ∞).

(a) Find the value of a so that X is equally likely to fall in each of four intervals.

(b) Find the representation point $x_1 = q(X)$ for X in $(0, a]$ that minimizes the mean squared error, that is

$$\int_0^a (x - x_1)^2 f_X(x)dx \qquad (2)$$

is minimized.

(c) Evaluate the mean-squared error of the quantizer: $E[(\boldsymbol{X} - q(\boldsymbol{X}))^2]$.

Question 2

A point (X, Y, Z) is selected at random inside the unit sphere ($x^2 + y^2 + z^2 = r^2$, where *x, y, z,* are the coordinates and $r = 1$ is the radius of the sphere). If $f_{X, Y, Z}(x, y, z)$ is the probability density function of randomly selected point *x, y, z* then:

(a) Find the marginal point *pdf* of X and Y.

(b) Find the marginal *pdf* of X.

(c) Find the conditional joint *pdf* of X and Y given Z.

(d) Are X, Y, and Z independent random variables?

Question 3

Let X and Y be samples of a random signal at two time instants. Suppose that X and Y are independent zero mean Gaussian random variables with the same variance. When signal “0” is present the variance is σ_o^2, and when signal “1” is present the variance is $\sigma_1^2 > \sigma_o^2$. Suppose signals “0” and “1” occur with probabilities *p* and 1-*p*. respectively. Let $R^2 = X^2 + Y^2$ be the total energy of the two observations.

(a) Find *pdf* of R^2 when signal “0” is present: when signal “1” is present. Find the *pdf* of R^2.

(b) Suppose we use the following "signal detection" rule: If $R^2 > T$, then we decide signal "1" is present: otherwise, we decide signal "0" is present. Find an expression for the probability of error in terms of *T*.

(c) Find the value of *T* that minimizes the error.

Question 4

Messages arrive at a computer from two telephone lines according to independent Poisson process of rates λ_1 and λ_2, respectively.

(a) Find the probability that a message arrives first on line 1.

(b) Find the pdf for the time until a message arrives on the other line.

(c) Find probability mass function (*pmf*) for $N(t)$, the total number of messages that arrive in an interval of length t.

(d) Generalize the result of part (c) for the "merging" of k independent Poisson processes of rates $\lambda_1, \lambda_2, \ldots, \lambda_k$, respectively i.e. $N(t) = N_1(t) + N_2(t) + \ldots + N_k(t)$

Question 5

A certain part of a machine can be in two states: working (state 0) or undergoing repair (state 1). A working part fails during the course of the day with probability a. A part undergoing repair is put into working order during the course of the day with a probability b. Let X_n be the state of the part.

(a) Show that X_n is a two state Markov chain and give its one step transition probability matrix P.

(b) Find the n-step transition matrix P^n.

(d) Find the steady state probability of each of the two states.

Useful Formulas

Poisson distribution - $P[k] = \frac{\alpha^k}{k!}e^{-\alpha} \qquad k = 0, 1, \ldots$ and $\alpha > 0$

Gaussian distribution - $f_X(x) = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sqrt{2\pi}\sigma}$ $-\infty < x < \infty$ and $\sigma > 0$

Laplacian distribution - $f_X(x) = \frac{e^{-\alpha|x|}}{2}$ $-\infty < x < \infty$ and $\alpha > 0$

Volume of a sphere with radius $r = \frac{4}{3}\pi r^3$

A useful integral $\int_0^{\alpha} \sqrt{a^2 - t^2}\, dt = \frac{1}{4}\pi\alpha^2$

To find the eignvalues and eigenvectors - $|\boldsymbol{P} - \lambda \boldsymbol{I}| = 0$

$\boldsymbol{PV} = \lambda \boldsymbol{V}$ $\boldsymbol{V}$ is the right hand eigenvector.

Raising to the power n, $\boldsymbol{P}^n = E^{-1}\begin{bmatrix} \lambda_1^n & 0 \\ 0 & \lambda_2^n \end{bmatrix} E$