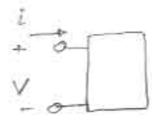
10.1

a) $\bar{V} = 100\angle 50^\circ V$, $\bar{I} = 10\angle 15^\circ A$



$P = \frac{1}{2} IV\cos\theta = \frac{1}{2}(10)(100)\cos(50^\circ - 15^\circ) = 409.58W$

$Q = \frac{1}{2} IV\sin\theta = \frac{1}{2}(10)(100)\sin(50^\circ - 15^\circ) = 286.79\ VAR$

The circuit absorbs 409.58W and absorbs 286.79 VAR.

b) $\bar{V} = 40\angle -15^\circ V$, $\bar{I} = 20\angle 60^\circ A \Rightarrow \theta = -15-60 = -75^\circ$

$P = \frac{1}{2}(20)(40)\cos(-75^\circ) = 103.53W$ (abs.)

$Q = \frac{1}{2}(20)(40)\sin(-75^\circ) = -386.37VAR$ (del.).

c) $\bar{V} = 400\angle 30^\circ V$, $\bar{I} = 10\angle 150^\circ A$

$\therefore \theta = -120^\circ$

$P = \frac{1}{2}(10)(400)\cos(-120^\circ) = -1000W$ (del.)

$Q = \frac{1}{2}(10)(400)\sin(-120^\circ) = -1732.05VAR$ (del.)

d) $\bar{V} = 200\angle 160^\circ V$, $\bar{I} = 5\angle 40^\circ A$

$\therefore \theta = 120^\circ$

$P = \frac{1}{2}(5)(200)\cos(120^\circ) = -250W$ (del.)

$Q = \frac{1}{2}(5)(200)\sin(120^\circ) = 433.01\ VAR$ (abs)

10.7

$$i_{rms} = \sqrt{\frac{\text{Area under the square}}{\text{Period}}} = \sqrt{\frac{\int_{t_0}^{t_0+T} i^2(t)\,dt}{T}}$$

$T = 0.1$ s

$$i(t) = \begin{cases} 200t & , 0 \le t \le 0.075 \text{ sec} \\ 60 - 600t & , 0.075 \le t \le 0.1 \text{ sec} \end{cases}$$

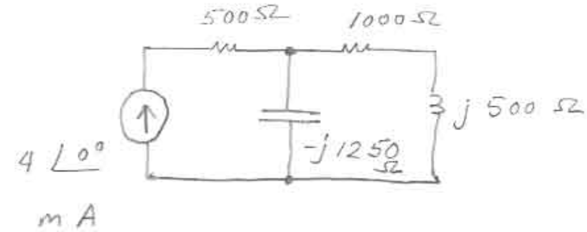
Let $t_0 = 0$.

$$\therefore \int_0^{0.1} i^2(t)\,dt = \int_0^{0.075} (200t)^2\,dt + \int_{0.075}^{0.1} (60-600t)^2\,dt$$

$$= 5.625 + 1.875 = 7.5$$

$$\therefore i_{rms} = \sqrt{\frac{7.5}{0.1}} = \sqrt{75} = 8.66 \text{ A(rms)}$$

10.10



$$\bar{Z}_{eq} = 500 + (-j1250) // (1000 + j500)$$

$$= 500 + \frac{(-j1250)(1000 + j500)}{-j1250 + (1000 + j500)}$$

$$= 500 + \frac{(1250 \angle -90^\circ)(1118.04 \angle 26.57^\circ)}{1250 \angle -36.87^\circ}$$

$$= 500 + 1118.04 \angle -26.56^\circ = 500 + (1000 - j500)$$

$$= 1500 - j500 \ \Omega$$

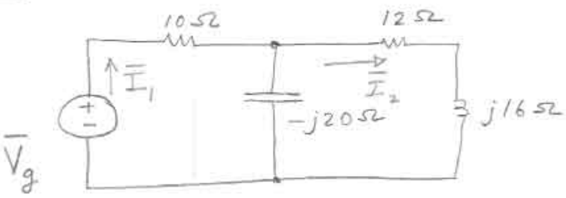
Since only the resistive part of the impedance absorbs average power, then the average power absorbed by the load:

$$P_L = P_R = \frac{1}{2} I^2 R = \frac{1}{2} (0.004)^2 (1500) = 12 \text{ mW}$$

∴ The source delivers 12 mW to the load.

10.15

a)



$\bar{V}_g = 170\angle 0^\circ$ V(rms).

KVL $\Rightarrow$ $-170 + 10\bar{I}_1 + (-j20)(\bar{I}_1 - \bar{I}_2) = 0 \Rightarrow$

$(10 - j20)\bar{I}_1 + j20\bar{I}_2 = 170$ ①

KVL $\Rightarrow$ $-j20(\bar{I}_2 - \bar{I}_1) + (12 + j16)\bar{I}_2 = 0$

$j20 I_1 + (12 - j4)\bar{I}_2 = 0$ ②

solving ① & ② $\Rightarrow$ $\bar{I}_1 = (4 + j)$ A

$\bar{I}_2 = (3.5 - j3.5)$A

$\bar{S}_g = -\bar{V}_g \bar{I}_1^* = (-170\angle 0^\circ)(4 - j)$

$= -680 + j170$ VA

b) $P_g = -680$ W (del.)

c) $Q_g = 170$ VAR (abs.)

d) $P_{10\Omega} = I_1^2 10 = 10(17) = 170$ W.

$Q_{10\Omega} = 0$

$P_{12\Omega} = 12\ I_2^{\ 2} = 12\left(3.5^2 + 5.5^2\right) = 510\,W$

$Q_{12\Omega} = 0$

$P_{-j20\Omega} = 0 \quad , \quad Q_{-j20\Omega} = -20\ \left|\bar{I}_1 - \bar{I}_2\right|^2 = -20\left(0.5^2 + 6.5^2\right)$

$= -850\ VAR$

$P_{j16\Omega} = 0 \quad , \quad Q_{j16\Omega} = 16\ I_2^{\ 2} = 16\left(3.5^2 + 5.5^2\right) = 680\ VAR$

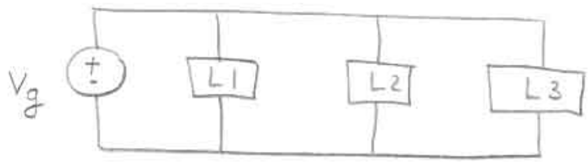
e) $\sum P_{del.} = 680\,W$

$\sum P_{abs} = 170 + 510 = 680\,W$

f) $\sum Q_{del.} = 850\ VAR$

$\sum Q_{abs} = 170 + 680 = 850\ VAR$

10.17 a)



$\bar{Z}_1 = 240 + j70\ \Omega = 250 \angle 16.26^\circ$

$\therefore\ Pf_1 = \cos\theta_1 = \cos 16.26^\circ = 0.96$ (lagging)

$\therefore\ rf_1 = \sin\theta_1 = \sin 16.26^\circ = +0.28$

$\bar{Z}_2 = 160 - j120\ \Omega = 200 \angle -36.87^\circ$

$Pf_2 = \cos(-36.87^\circ) = 0.8$ (leading)

$rf_2 = \sin(-36.87^\circ) = -0.6$

$\bar{Z}_3 = 30 - j40\ \Omega = 50 \angle -53.13^\circ\ \Omega$

$Pf_3 = \cos(-53.13^\circ) = 0.6$ (leading)

$rf_3 = \sin(-53.13^\circ) = -0.8$

b) $\bar{Z}_{eq} = \dfrac{1}{\bar{Y}_{eq}}$, $\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$

$$= \frac{1}{250 \angle 16.26^\circ} + \frac{1}{200 \angle -36.87^\circ} + \frac{1}{50 \angle -53.13^\circ}$$

$= 0.004\angle{-16.26^\circ} + 0.005\angle{36.87^\circ} + 0.02\angle{53.13^\circ}$

$= 0.01984 + j0.01788 \quad (S)$

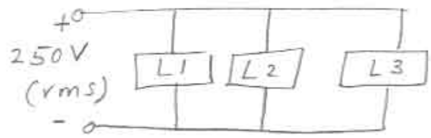
$\therefore \bar{Z}_{eq} = \dfrac{1}{0.01984 + j0.01788} = \dfrac{1}{0.02671\angle{42.025^\circ}}$

$= 37.44\angle{-42.025^\circ} \ \Omega$

$\therefore Pf = \cos 42.025^\circ = 0.743$ (leading).

$\therefore rf = \sin(-42.025^\circ) = -0.669$

<u>10.18</u> a)



$\bar{S}_1 = (16 + j18)$ KVA

$\theta_2 = -\cos^{-1}(0.6) = -53.13^\circ$

$S_2 = |\bar{S}_2| = 10$ KVAR

$\dfrac{P_2}{S_2} = \cos\theta_2 \implies P_2 = S_2\cos(-53.13^\circ) = S_2(Pf_2)$

$= 10(0.6) = 6$ KW

$\dfrac{Q_2}{S_2} = \sin\theta_2 \implies Q_2 = S_2\sin(-53.13^\circ) = S_2(rf_2)$

$= 10(-0.8) = -8$ KVAR

$\therefore \bar{S}_2 = (6 - j8)$ KVA

$\bar{S}_3 = 8$ KW, since $Pf_3 = 1 \implies Q_3 = 0$.

$$\bar{S}_{tot} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = (16 + j18) + (6 - j8) + 8$$

$$= (30 + j10)\ kVA$$

$$\bar{S}_{tot} = \bar{V}\,\bar{I}^* \Rightarrow (30 + j10)k = 250\ \bar{I}^*$$

$$\bar{I}^* = \frac{30000 + j10000}{250} = (120 + j40)\ A\ (rms)$$

$$\therefore \bar{I} = (120 - j40)\ A\ (rms)$$

$$\therefore \bar{Z}_{eq} = \frac{\bar{V}}{\bar{I}} = \frac{250\angle 0^\circ\ V(rms)}{(120 - j40)\ A(rms)}$$

$$= 1.98\angle 18.43^\circ\ \Omega$$

b) $pf = \cos(18.43^\circ) = 0.949$ (lagging).