KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
Electrical Engineering Department

EE 380 - CONTROL ENGINEERING **Experiment # 10**

CASCADE COMPENSATION VIA FREQUENCY DOMAIN TECHNIQUES

OBJECTIVES

To develop an understanding of the lag and lead compensators and to use the frequency domain methods in designing such compensators.

INTRODUCTION

Control systems are designed to perform specific tasks. The requirements imposed upon the control system are usually spelled out as performance specifications. They generally relate to accuracy, relative stability, and speed of response.

Setting the gain is the first step in adjusting the system for satisfactory performance. In many practical cases, however, the adjustment of the gain alone may not provide sufficient alteration of the system behavior to meet the performance specifications. In general, gain adjustment can improve the steady-state behavior, but the system will tend to be oscillatory or even unstable. It is then necessary to insert additional components in order to alter the overall behavior so that the system will behave as desired. An additional device inserted into the system for such purpose is called a **COMPENSATOR**. This device compensates for deficient performance of the original system. The four commonly employed methods of compensation are (a) cascade or series compensation, (b) feedback compensation, (c) output or load compensation, and (d) input compensation. These schemes are illustrated in Fig. 1.

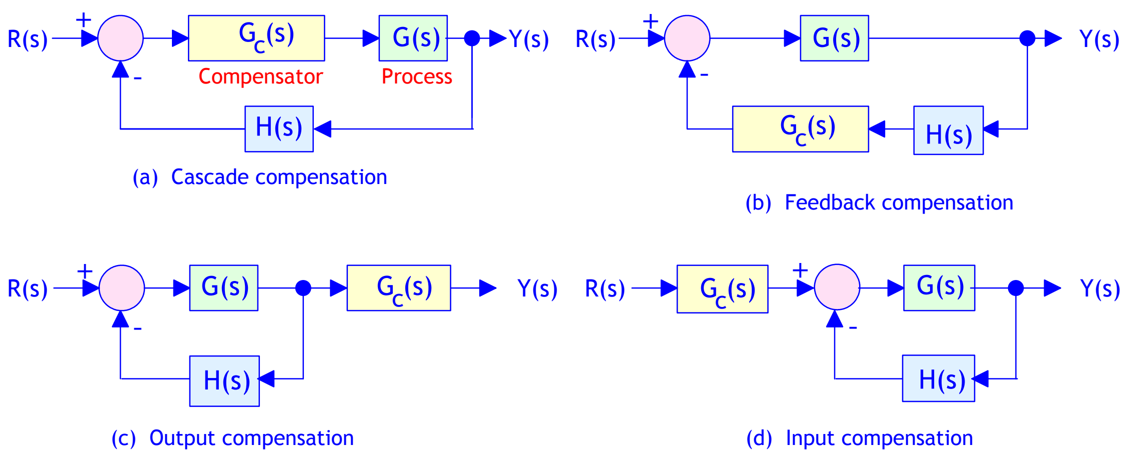

Fig. 1 Types of compensation

As indicated in Fig. 1a, a cascaded compensation consists of placing elements in series with the forward loop transfer function. such compensation may be classified into the following categories:

(1) **Phase-lag** compensation
(2) **Phase-lead** compensation
(3) Lag-lead compensation.
(4) Compensation by cancellation.

In this experiment, the design of the phase-lag and the phase-lead compensators is considered.

THEORY:

Part I : Phase-lag compensation

When the output of an element lags the input in phase and the magnitude decreases as a function of frequency, the element is called a phase-lag element. Consider the phase-lag network of Fig. 2.

The transfer function is

$$G_c(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1+\tau s}{1+\alpha\tau s} = \frac{1}{\alpha}\frac{(s+z)}{(s+p)}$$

$$\tau = R_2 C;\ \alpha = \frac{R_1+R_2}{R2} \quad (>\mathbf{1})$$

$$z = \frac{1}{\tau},\ p = \frac{1}{\alpha\tau}$$

The bode diagram and the pole-zero diagram of the phase-lag network are shown in Figs. 3 and 4 Respectively.

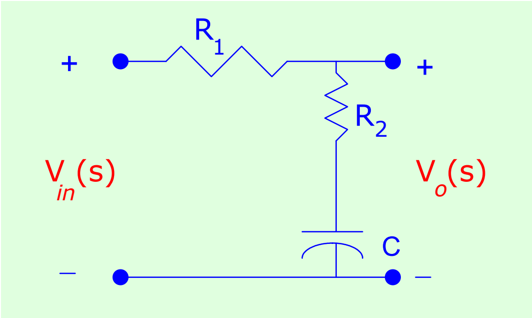

Fig. 2 Phase-lag network

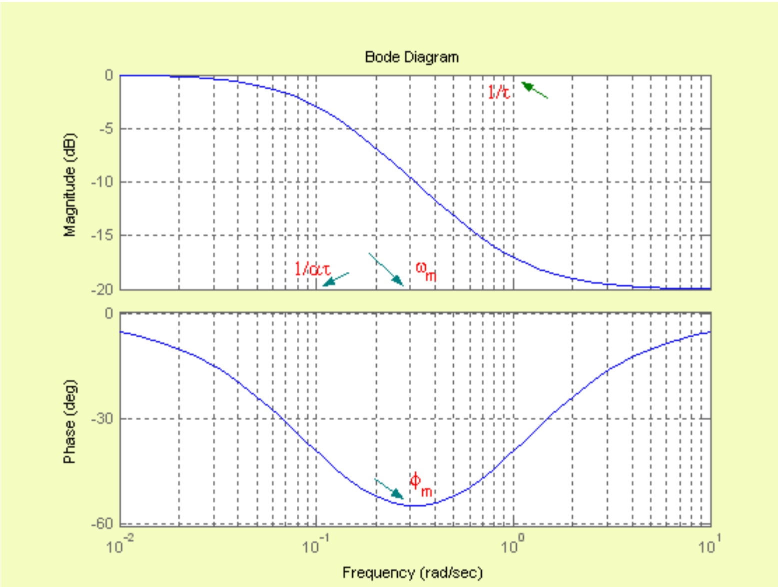

Fig. 3 Bode diagram of the Phase-lag network

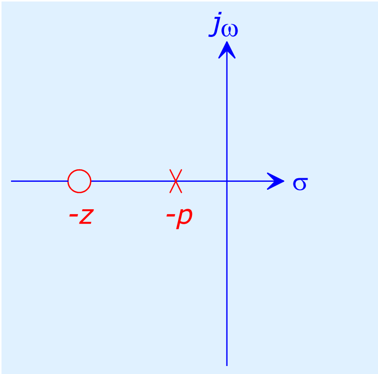

Fig. 4 Pole-zero diagram of phase-lag network.

- The lag compensator has **unity** low frequency gain and an attenuation of 'a' at high frequencies.
- By placing the compensator in cascade with the plant, the low frequency performance will be unaffected but the overall gain of the system will be reduced in the gain-crossover region to improve the stability margins. The amount of attenuation required to meet the design requirement determines the value of 'a' and the only other decision that has to be made is the actual value of the pole and zero of the compensator (i.e. τ).
- The lag compensator introduces a small phase lag at very low frequencies and at very high frequencies, and a much larger phase lag between the two corner frequencies. The obvious effect is that the **new** gain-crossover frequency is **lower** than the **original** gain-crossover frequency.
- By locating the corner frequencies of the compensator much lower than the gain cross-over frequency, the amount of phase lag introduced, **where attenuation is needed**, is small. A typical design figure is for the compensator to introduce less than 5° phase lag at the design point. This figure translates approximately into placing the upper corner frequency , $\frac{1}{a\tau}$, of the compensator at **one tenth** of the new gain cross-over frequency .
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Design steps:

1) Determine the open-loop gain such that the requirement on the particular error coefficient is satisfied.

2) Using the gain thus determined, draw the Bode diagram of the uncompensated system and determine the phase and gain margin. [MATLAB is very handy here].

3) If the specifications are not satisfied, then find the frequency point where the phase angle of the open-loop transfer function is $-180^\circ +$ phase margin $+5^\circ$. Choose this frequency as the **new gain cross-over frequency.**

4) Determine the attenuation of the system at the new cross-over frequency . This attenuation is equal to $--20\log_{10}a$. Calculate a.

5) Choose the corner frequency $\frac{1}{\tau}$ (corresponding to the zero of the lag network) one decade below the new gain cross-over frequency. Thus the other corner frequency $\frac{1}{a\tau}$ (corresponding to the pole of the lag network) is determined.

Part II : Phase-lead compensation

When the output of an element leads the input in phase and the magnitude increases as a function of frequency, the element is called a phase-lead element. Consider the phase-lead network of Fig. 5. The transfer function is:

$$G_c(s) = \frac{1}{a}\frac{1+a\tau s}{1+\tau s} = \frac{(s+z)}{(s+p)},\ \tau = \frac{R_1 R_2}{R_1 + R_2}C;\ a = \frac{R_1+R_2}{R_2};\ (>\mathbf{1})$$

Note that the gain of forward loop transfer function is increased to offset the effect of a.

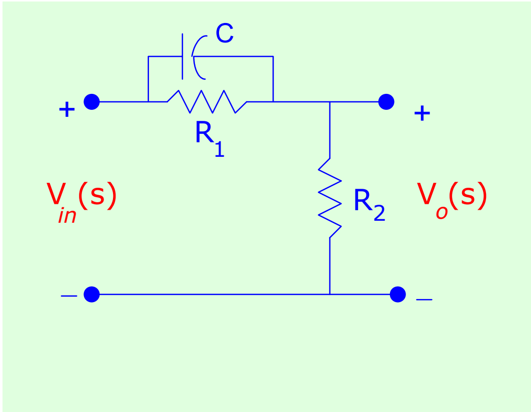
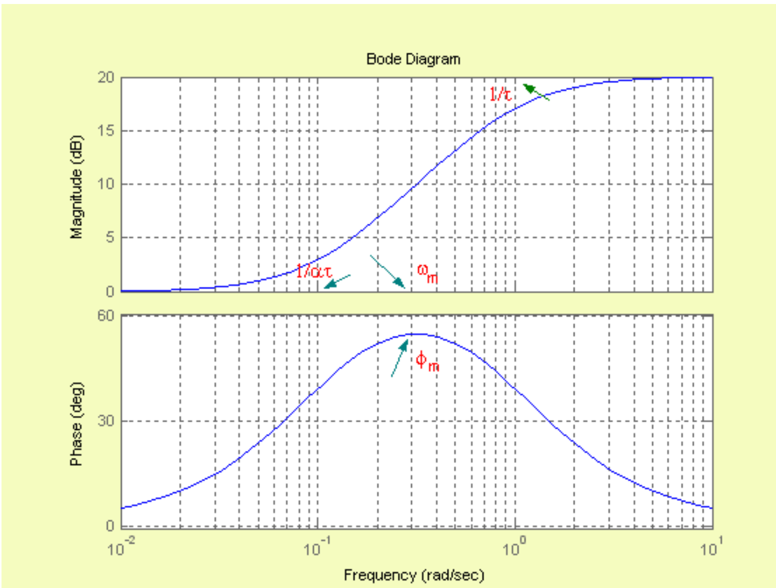


Fig. 5 Phase-lead network

The bode diagram and the pole-zero diagram of the phase-lead network are shown in Figs. 6 and 7 Respectively.



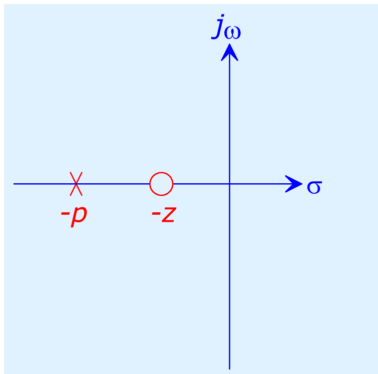


Fig. 7 Pole-zero diagram of phase-lead network

Fig. 6 Bode diagram of the Phase-lead network

The maximum phase lead ϕ_m occurs at ω_m. It may be shown that :

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{a}} \; ; \quad \sin(\varphi_m) = \frac{a-1}{a+1} \; ; \quad [\text{ Also } \; a = \frac{1+\sin(\varphi_m)}{1-\sin(\varphi_m)} \;]$$

Note that the amount of modification in the magnitude curve at $\omega_m = \frac{1}{\sqrt{a}\,\tau}$ is $\sqrt{a}$.

In a lag compensator, the **amount of attenuation** it offers is of primary interest and the phase lag it produces is discounted by centering the compensator well away from the design point. With a lead compensator, however, its **phase characteristic** is what the designer is after and its **magnitude characteristic** is a nuisance that he cannot wish away.

Design steps:

1. Determine the open-loop gain such that the requirement on the particular error coefficient is satisfied.
2. Using the gain thus determined, draw the Bode diagram of the uncompensated system and determine the phase margin.
3. Determine the necessary phase lead angle ϕ to be added to the system. Allow for a small amount of safety. Then determine φ_m.
4. Determine the value of $\alpha = \frac{1+\sin(\phi_m)}{1-\sin(\phi_m)}$.
5. Determine the frequency where the magnitude of the uncompensated system is equal to $20\log_{10}\sqrt{\alpha}$. Select this frequency as the new gain cross-over frequency. This frequency corresponds to $\omega_m = \frac{1}{\sqrt{\alpha}\,\tau}$ and the maximum phase shift φ_m occurs at this frequency.
6. Determine the corner frequencies $\frac{1}{\alpha\tau}$ (corresponding to the pole of the lead compensator) and $\frac{1}{\tau}$ (corresponding to the zero of the lead compensator).
7. Finally , insert an amplifier with gain equal to α .

DESIGN DRILL:

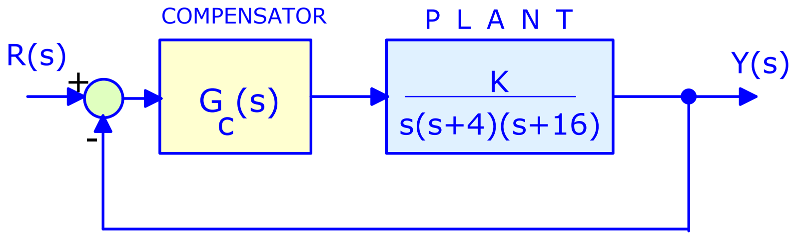
Consider a position servo whose transfer function is :

$$G_p(s) = \frac{K}{s(s+4)(s+16)}$$

Specifications:

- Velocity error coefficient $K_v = 10\,\text{sec}^{-1}$.
- Phase margin of 45°.

1. Design a **lag compensator** to satisfy the specifications.
2. Design a **lead compensator** to satisfy the specifications.



REPORT

1. Show all the design steps for the phase-lag compensator.
2. Show and comment on the bode diagram of the uncompensated system, the compensator, and the compensated systems.
3. Design a circuit to implement the phase-lag compensator.
4. Draw the root locus of the uncompensated and the compensated systems. Discuss the results.
5. Calculate the bandwidth of both the uncompensated and the compensated systems. What do you observe?
6. Provide the step-response for both the uncompensated and the compensated systems. Comment on your results.
7. Repeat steps 1 to 6 for the phase-lead compensator.