KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
Electrical Engineering Department

EE 380 Control Engineering **Experiment # 7**

Investigation of Error Performance Indexes

OBJECTIVE:

To investigate, by way of simulation, the error performance indexes that are used in the design of control systems.

APPARATUS:

IBM -compatible Personal Computers, MATLAB/SIMULINK software, and printers.

INTRODUCTION:

In the design of a control system, it is important that the system meets given performance specifications. Since control systems are dynamic, the performance specifications may be given in terms of the transient response behavior to specific inputs, such as step inputs, ramp inputs, etc. or the specifications may be given in terms of a performance index.

- Performance Indexes

A performance index is a number which indicates the "goodness" of system performance. A control system is considered optimal if the values of the parameters are chosen so that the selected performance index is minimum or maximum. The optimal values of the parameters depend directly upon the performance index selected.

- Requirements of Performance Indexes

A performance index must offer selectivity; that is, an optimal adjustment of the parameters must clearly distinguish nonoptimal adjustments of the parameters. In addition, a performance index must yield a single positive number or zero, the latter being obtained if and only if the measure of the deviation is identically zero. To be useful, a performance index must be a function of the parameters of the system, and it must exhibit a maximum or a minimum. Finally, to be practical, a performance index must be easily computed, analytically or experimentally.

- Error Performance Indexes

In control systems, consideration is given to several error criteria in which the corresponding performance indexes are integrals of some function or weighted

function of the deviation of the actual system output from the desired output. Since the values of the integrals can be obtained as functions of the system parameters, once a performance index is specified, the optimal system can be designed by adjusting the parameters to yield, say, the smallest value of the integral.
Various error performance indexes have been proposed in the literature. We shall consider the following four indexes in our simulation study.

1	**ISE**	$J_1 = \int_0^\infty e^2\,dt$	Integral square-error criterion
2	**ITSE**	$J_2 = \int_0^\infty te^2\,dt$	Integral-of-time-multiplied square-error criterion
3	**IAE**	$J_3 = \int_0^\infty \lvert e\rvert\,dt$	Integral absolute-error criterion
4	**ITAE**	$J_4 = \int_0^\infty t\lvert e\rvert\,dt$	Integral-of-time-multiplied absolute-error criterion

- Characteristics of Error Performance Indexes

Integral square-error criterion (J_1)

- Easy to compute both analytically and experimentally.
- Weighs large errors heavily and small errors lightly.
- Not very selective.
- A system designed by this criterion is oscillatory and has poor relative stability.
- Has practical significance because the minimization of the performance index results in the minimization of power consumption for some systems, such as spacecraft systems.
- Fig. 1 shows desired output $x(t)$, actual output $y(t)$, error $e(t)$, square error $e^2(t)$, and $\int e^2(t)dt$ as a function of t.

Integral-of-time-multiplied square-error criterion J_2

- Large initial error is weighed lightly, while errors occurring late in the transient response are penalized heavily.
- Better selectivity than the integral square-error criterion.

Integral absolute-error criterion J_3

- Results in a system which has reasonable damping and a satisfactory transient-response
- Selectivity is not too good
- Cannot be easily evaluated analytically
- Minimization of the integral absolute error is directly related to the minimization of fuel consumption of spacecraft systems.
- Fig. 2 shows the error $e(t)$, and the absolute error $|e(t)|$, versus t.

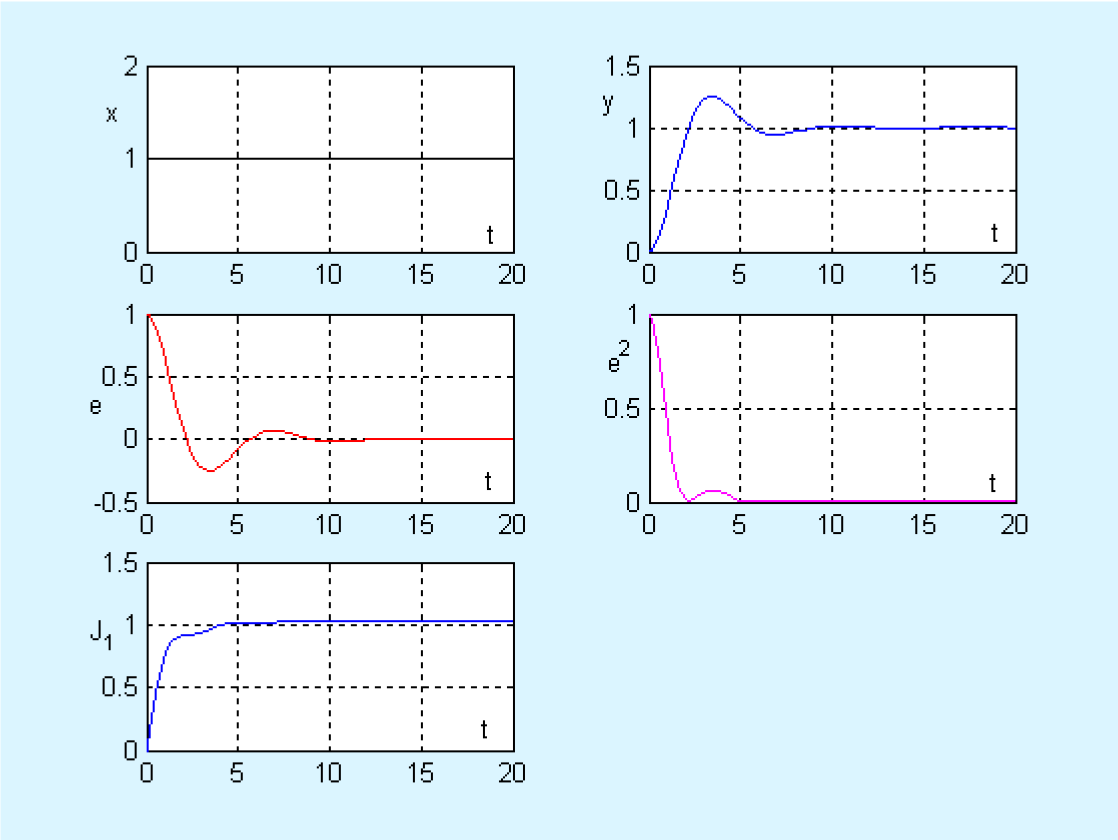

Fig. 1 Plots showing desired output $x(t)$, actual output $y(t)$, error $e(t)$, square error $e^2(t)$, and $\int e^2(t)dt$ as a function of t.

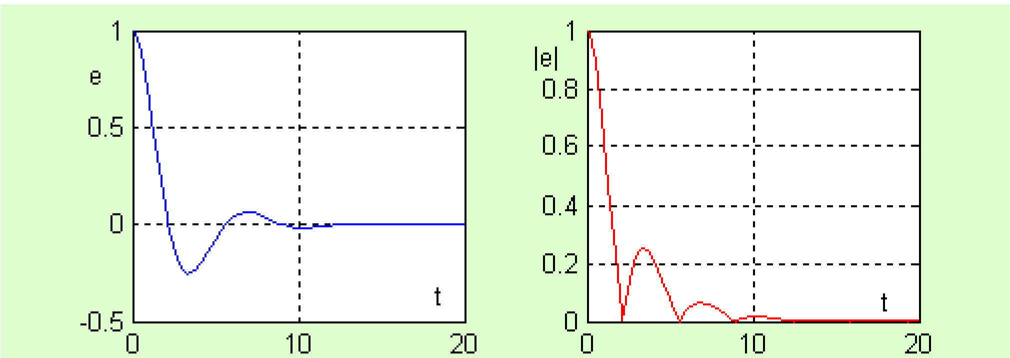

Fig. 2 Plots showing error $e(t)$, and absolute error $|e(t)|$ versus t.

Integral-of-time-multiplied absolute-error criterion J_4

- Large initial error is weighed lightly, while errors occurring late in the transient response are penalized heavily.
- Results in a system which has small overshoot and well damped oscillations.

- Has good selectivity and is an improvement over IAE.
- Very difficult to evaluate analytically.

SIMULATION

In this laboratory exercise, we shall solve a simple optimization problem in which we minimize the performance indexes discussed earlier. The system to be studied is shown in Fig. 1.

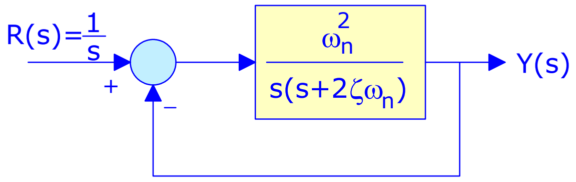

Fig. 1 Control system

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \frac{E(s)}{R(s)} = \frac{s^2 + 2\zeta\omega_n s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

1. Suppose that the input to the system, $R(s)$, is a unit-step applied at $t = 0$. Assume that the system is initially at rest.
2. Assume that $0.1 \le \zeta \le 2.0$, and that $\omega_n = 1.0$.
3. Using SIMULINK , develop a program to evaluate the performance indexes $J_1 \ldots J_4$ for the specified range of ζ , in appropriate steps., and the optimum value for ζ in each case.
4. Plot the variations of each of the performance indexes against the system damping ratio ζ. (preferably on a single graph).

DISCUSSION:

1. Comments on all your results. Is the optimal value of ζ the same for all performance indexes? Explain your answer.
2. Give a qualitative comparison of the selectivity of the various performance indexes considered.
3. Obtain , analytically, the optimal value of ζ that will minimize the performance index J_1 and the minimum value of J_1 [see 'Hint']. Compare these values with the corresponding values obtained from plotting your simulation results.
4. Repeat part (3) for the performance index J_2.
5. Addition of the square of error rate to the integrand of J_1 results in the performance index $J_5 = \int_0^\infty (e^2 + \dot{e}^2)\,dt$. Plot the variations of the performance index J_5 against the system damping ratio ζ. . Comments on the optimal value of ζ and the selectivity of J_5.

6. Addition of the absolute error rate to the integrand of J_4 results in the performance index $J_6 = \int_0^\infty t(|e| + |\dot{e}|)\,dt$. Plot the variations of the performance index J_6 against the system damping ratio ζ. . Comments on the optimal value of ζ and the selectivity of J_6.

HINT

$$J_1 = \int_0^\infty e^2 dt = \lim_{t\to\infty} \int_0^t e^2 dt = \lim_{s\to 0} s\{\frac{1}{s}\mathcal{L}[e^2(t)]\}$$

$$E(s) = \frac{s^2 + 2\zeta s}{s(s^2 + 2\zeta s + 1)} = \frac{s + \zeta}{s^2 + 2\zeta s + 1} + \frac{\zeta}{s^2 + 2\zeta s + 1}$$

$e(t) = \dots$

$e^2(t) = \dots$

$\mathcal{L}[e^2(t)]\} = \dots$

⋮

⋮