King Fahd University of Petroleum and Minerals

Electrical Engineering Department

PROBLEM SESSION # 3

3.15 Determine the gradient of the following scalar fields:

(a) $U = 4xz^2 + 3yz$.

(b) $V = e^{(2x+3y)} \cos 5z$.

(c) $W = 2\rho(z^2 + 1) \cos \varphi$.

(d) $T = 5\rho\, e^{-2z} \sin \varphi$.

(e) $H = r^2 \cos\theta \cos\varphi$.

(f) $Q = (\sin\theta \sin\varphi) / r^3$.

3.18 Find the divergence and curl of the following vector fields:

(a) $\mathbf{A} = e^{xy}\, \mathbf{a_x} + \sin xy\, \mathbf{a_y} + \cos^2 xz\, \mathbf{a_z}$

(b) $\mathbf{B} = \rho z^2 \cos\varphi\, \mathbf{a_\rho} + z \sin^2 \varphi\, \mathbf{a_z}$

(c) $\mathbf{C} = \boldsymbol{r} \cos\theta\, \boldsymbol{a}_r - \frac{1}{\boldsymbol{r}} \sin\theta\, \boldsymbol{a}_\theta + 2\boldsymbol{r}^2 \sin\theta\, \boldsymbol{a}_\phi$

3.30 Given that $\boldsymbol{E} = \frac{1}{\boldsymbol{r}^4} \sin^2 \phi\, \boldsymbol{a}_r$, evaluate

(a) $\oint_S E \cdot dS$

(b) $\int_V (\nabla \cdot E)\, dv$

over the region between the spherical surfaces $r = 2$ and $r = 4$.

3.33 Calculate the total outward flux of vector

$\boldsymbol{F} = \rho^2 \sin\phi\, \boldsymbol{a}_\rho + \boldsymbol{z} \cos\phi\, \boldsymbol{a}_\phi + \rho\, \boldsymbol{z}\, \boldsymbol{a}_z$

through the hollow cylinder defined by $2 \le \rho \le 3$, $0 \le z \le 5$.

3.39 Given the vector field

$R = (2x^2 y + yz) a_x + (xy^2 - xz^3) a_y + (c\,xyz - 2x^2 y^2) a_z$

determine the value of c for **R** to be solenoidal.

3.40 If the vector field

$T = (\alpha xy + \beta z^3) a_x + (3x^2 - \gamma z) a_y + (3xz^2 - y) a_z$

is irrotational, determine α, β, and γ. Find $\nabla \cdot \mathbf{T}$ at (2, -1, 0).

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