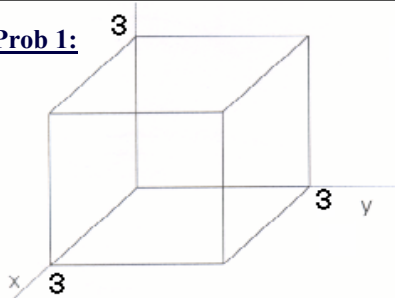


EE 340 Electromagnetics Lab

Problem Session #2

Prob 1:



$$\text{a) } \int_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{s} = \int_{\text{left}} + \int_{\text{right}} + \int_{\text{top}} + \int_{\text{bottom}} + \int_{\text{front}} + \int_{\text{back}}$$

since $\mathbf{F} = a_y \mathbf{y}$, I have only y component in \mathbf{F} , then there is a value for the surfaces in the direction of +ve y and -ve y

$$\begin{aligned} \int_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{s} &= \int_{\text{left}} + \int_{\text{right}} = 5 \int_{z=0}^3 \int_{x=0}^3 dx dz - 5 \int_{z=0}^3 \int_{x=0}^3 dx dz = \\ &= 5 \int_{z=0}^3 [x dz]_0^3 - 5 \int_{z=0}^3 [x dz]_0^3 = 45 - 45 = 0 = 0 \end{aligned}$$

b) $\mathbf{F} = x^2 y^2 \mathbf{a}_x$

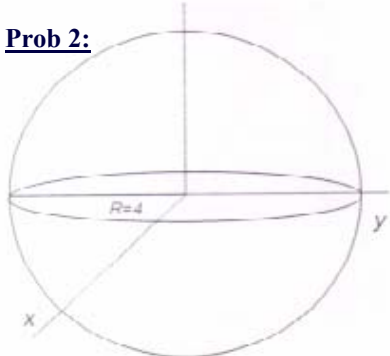
since I have only x component in \mathbf{F} , then there is a value for the surfaces in the direction of +ve x and -ve x

$$\int_{\text{front}} \mathbf{F} \cdot d\mathbf{s} = \int_{z=0}^3 \int_{y=0}^3 x^2 y^2 dy dz \Big|_{x=3} = 9 \int_{z=0}^3 \int_{y=0}^3 y^2 dy dz = 9 \int_{z=0}^3 \left[\frac{y^3}{3} \right]_0^3 dz = 81 \int_{z=0}^3 dz = 81[z]_0^3 = 243$$

$$\int_{\text{back}} \mathbf{F} \cdot d\mathbf{s} = \int_{z=0}^3 \int_{y=0}^3 x^2 y^2 dy dz \Big|_{x=0} = 0$$

$$\int_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{s} = \int_{\text{front}} + \int_{\text{back}} = 243 + 0 = 243$$

Prob 2:



$\bar{ds} = r^2 \sin \theta d\theta d\phi \bar{a}_r$

a) $F = \frac{\mathbf{a}_r}{r^2}$

$$\oint F \cdot d\mathbf{s} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin \theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin \theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin \theta d\theta d\phi = - \int_{\phi=0}^{2\pi} [\cos \theta]_{\theta=0}^{\pi} d\phi = - \int_{\phi=0}^{2\pi} [-1 - 1] d\phi = 2[\phi]_{\phi=0}^{2\pi} = 4\pi$$

b) $F = \frac{\sin^2 \phi}{r^2} \bar{a}_r + \cos \phi \bar{a}_\phi \Rightarrow F \cdot d\mathbf{s} = \frac{\sin^2 \phi}{r^2}$

$$\oint F \cdot d\mathbf{s} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} 4 \sin \theta \sin^2 \phi d\theta d\phi = 8\pi$$

Handwritten notes for part b: $\int_0^\pi \sin \theta d\theta = 2$, $\int_0^{2\pi} \sin^2 \phi d\phi = \pi$

because of integration of sin function over one period

Prob 2: c) $F = -a_x$

$$\bar{F} = \sin \theta \cos \phi \bar{a}_r + \cos \theta \cos \phi \bar{a}_\theta - \sin \phi \bar{a}_\phi$$

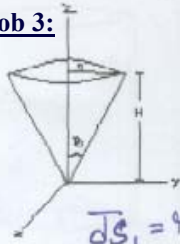
$$F \cdot d\mathbf{s} = r^2 \sin^2 \theta \cos \phi d\theta d\phi$$

$$\oint F \cdot d\mathbf{s} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin^2 \theta \cos \phi d\theta d\phi = 0$$

Handwritten notes for part c: $\int_0^\pi \sin^2 \theta d\theta = \frac{\pi}{2}$, $\int_0^{2\pi} \cos \phi d\phi = 0$

because of integration of sin function over one period

Prob 3:



$$d\vec{S}_1 = \rho \sin\theta d\theta d\phi \vec{a}_\theta \quad (\text{sp.})$$

Surface 1:

a) $\vec{F} = r\vec{a}_r = \rho \vec{a}_\rho$
 $\oint \vec{F} \cdot d\vec{S} = 0$

b) $\oint \vec{F} \cdot d\vec{S} = \int \int r^2 \sin\theta dr d\theta = \frac{2\pi}{3} a(h^2 + a^2)$

c) $F = \cos\theta \vec{a}_\theta + r \vec{a}_\theta$

Surface 2:

a) $\vec{F} = r\vec{a}_r = r \sin\theta \vec{a}_\rho + r \cos\theta \vec{a}_z$
 $= \rho \vec{a}_\rho + z \vec{a}_z$

$$\vec{a}_n = \sin\theta \cos\phi \vec{a}_x + \sin\theta \sin\phi \vec{a}_y + \cos\theta \vec{a}_z$$

$$\vec{a}_n = \sin\theta \cos\phi (\cos\phi \vec{a}_\rho) - \sin\theta \cos\phi (\sin\phi \vec{a}_\phi) + \sin\theta \sin\phi (\cos\phi \vec{a}_\rho) + \cos\theta \vec{a}_z$$

$$\vec{a}_n = \sin\theta (\cos^2\phi + \sin^2\phi) \vec{a}_\rho + \cos\theta \vec{a}_z$$

$$\vec{a}_n = \sin\theta \vec{a}_\rho + \cos\theta \vec{a}_z$$

∴ $\vec{F} = \rho \vec{a}_\rho$
 $= \rho \sin\theta \vec{a}_\rho + \rho \cos\theta \vec{a}_z$
 $= \rho \vec{a}_\rho + z \vec{a}_z$

Prob 3:

$$\oint \vec{F} \cdot d\vec{S} = \int \int z \rho d\rho d\theta = z \int \rho^2 d\theta = \pi h a^2$$

b) $\vec{F} = r\vec{a}_z = r \cos\theta \vec{a}_\rho - r \sin\theta \vec{a}_z = z \vec{a}_\rho - \rho \vec{a}_z$
 $\oint \vec{F} \cdot d\vec{S} = -\int \int \rho^2 d\rho d\theta = -\frac{2\pi a^3}{3}$

c) $\vec{F} = \cos\theta \vec{a}_\theta + r\vec{a}_\theta = r \cos\theta \vec{a}_\rho + \cos\theta \vec{a}_z - r \sin\theta \vec{a}_z$
 $= z \vec{a}_\rho + \cos\theta \vec{a}_z - \rho \vec{a}_z$
 $\oint \vec{F} \cdot d\vec{S} = -\int \int \rho^2 d\rho d\theta = -\frac{\pi a^3}{3}$

$$d\vec{S} = \rho d\phi d\rho \vec{a}_z \quad (\text{cy})$$

$$\oint \vec{F} \cdot d\vec{S} = \int \int (\rho \vec{a}_\rho + z \vec{a}_z) \cdot (\rho d\phi d\rho \vec{a}_z)$$

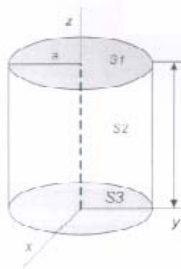
$$\theta_1 = \int \int z \rho d\phi d\rho$$

$$\theta_1 = z \int_0^{2\pi} \int_0^a \rho d\rho d\phi =$$

$$\theta_1 = (h) \int_0^{2\pi} \left[\frac{\rho^2}{2} \right]_0^a d\phi$$

$$\theta_1 = \frac{h a^2}{2} \cdot 2\pi = \pi h a^2$$

Prob 4a:



$ds1 = \rho d\phi d\rho \bar{a}_z$
 $ds2 = \rho d\phi dz \bar{a}_\rho$
 $ds3 = -\rho d\phi d\rho \bar{a}_z$
 $ds3 = \text{"-" due to direction}$

a) $F = \rho^2 \bar{a}_\rho + \rho \sin \phi \bar{a}_\phi + \rho^7 \sin \phi \bar{a}_z$

$\int_{S1} = \int_{\phi=0}^{2\pi} \int_{\rho=0}^a \rho^3 \sin \phi d\phi d\rho = 0$ (due to $\int \sin \phi d\phi = 0$)

$\int_0^a \rho^3 d\rho = \left[\frac{\rho^4}{4} \right]_0^a = \frac{a^4}{4}$
 $\int_0^{2\pi} \sin \phi d\phi = -\cos \phi \Big|_0^{2\pi} = -(1-1) = 0$

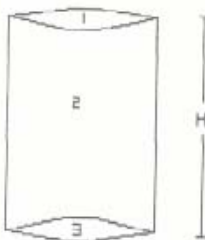
integration of sin over one period

$\int_{S2} = \int_{z=0}^h \int_{\phi=0}^{2\pi} \rho^3 d\phi dz = [a^3][2\pi][h] = 2\pi h a^3$
 $\int_{S3} = -\int_{\phi=0}^{2\pi} \int_{\rho=0}^a \rho^3 \sin \phi d\rho d\phi = 0$

integration of sin over one period

$\int F = \int_{S1} + \int_{S2} + \int_{S3} = 2\pi h a^3$

Prob 4b:



$b) F = x\bar{a}_x + z\bar{a}_z$
 $= x \cos \theta \bar{a}_\rho - x \sin \theta \bar{a}_\phi + z\bar{a}_z$
 $= \rho \cos^2 \theta \bar{a}_\rho - \rho \cos \theta \sin \theta \bar{a}_\phi + z\bar{a}_z$

$a) F = \rho^2 \bar{a}_\rho + \rho \sin \theta \bar{a}_\theta + \rho^2 \sin \theta \bar{a}_z$
 $\int F \cdot ds1 = \int \rho^3 d\theta d\phi dz = 2\pi a^3 h$

$\int F \cdot ds1 = \int \int \rho^2 \cos^2 \theta d\theta d\phi dz \quad \left\{ \begin{matrix} z=0 \rightarrow h \\ \phi=0 \rightarrow 2\pi \end{matrix} \right\}$
 $= \pi a^2 h$

$\int F \cdot ds2 = \int \int \rho^3 \sin \theta d\theta d\phi$
 $= \int -\cos \theta | \rho^3 d\phi = 0$

$\int F \cdot ds2 = \int \int \rho z d\theta d\rho dz \quad \left\{ \begin{matrix} z=0 \rightarrow a \\ \phi=0 \rightarrow 2\pi \end{matrix} \right\}$
 $= h \int \int \rho d\rho d\phi = \pi a^2 h$

$\int F \cdot ds3 = 0$ (as $z=0$)

$F \cdot ds3 = 0$

$\Rightarrow F \cdot ds = 2\pi a^3 h$