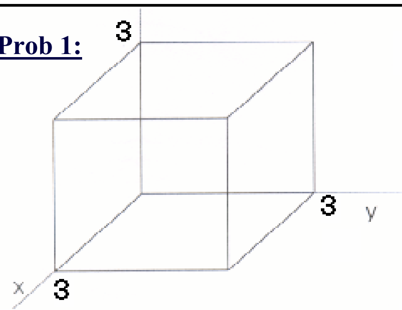
EE 340 Electromagnetics Lab

Problem Session #2

Prob 1:



a) $\int_S F = \int_{left} + \int_{right} + \int_{top} + \int_{bottom} + \int_{front} + \int_{back}$

since $\mathbf{F}=\mathbf{a}_y$, I have only y component in $\mathbf{F}$, then there is a value for the surfaces in the direction of +ve y and –ve y

$$\int_S F = \int_{left} + \int_{right} = 5\int_{z=0}^{3}\int_{x=0}^{3} dx\,dz - 5\int_{z=0}^{3}\int_{x=0}^{3} dx\,dz =$$

$$= 5\int_{z=0}^{3}\left[x\,dz\right]_0^3 - 5\int_{z=0}^{3}\left[x\,dz\right]_0^3 = 45 - 45 = 0 = 0$$

b) $\mathbf{F}=x^2y^2\,\mathbf{a}_x$

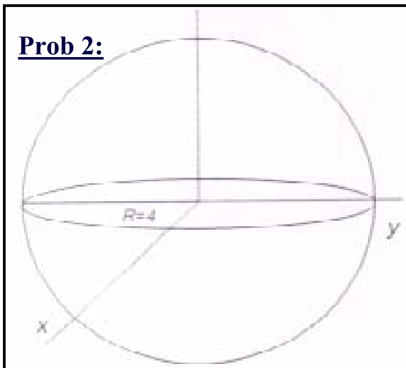
since I have only x component in $\mathbf{F}$, then there is a value for the surfaces in the direction of +ve x and –ve x

$$\int_{front} F.ds = \int_{z=0}^{3}\int_{y=0}^{3} x^2y^2\,dy\,dz \,\Big|_{x=3} = 9\int_{z=0}^{3}\int_{y=0}^{3} y^2\,dy\,dz = 9\int_{z=0}^{3}\left[\frac{y^3}{3}\right]_0^3 dz = 81\int_{z=0}^{3} dz = 81\left[z\right]_0^3 = 243$$

$$\int_{back} F.ds = \int_{z=0}^{3}\int_{y=0}^{3} x^2y^2\,dy\,dz \,\Big|_{x=0} = 0$$

$$\int_S F.ds = \int_{front} + \int_{back} = 243 + 0 = 243$$

Prob 2:



$\bar{ds} = r^2 \sin\theta d\theta d\phi\, \bar{\mathbf{ar}}$

a) $F = \frac{\mathbf{a}r}{\mathbf{r}^2}$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi =$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} \sin\theta d\theta d\phi = -\int_{\phi=0}^{2\pi}[\cos\theta]_{\theta=0}^{\Pi} d\phi = -\int_{\phi=0}^{2\Pi}[-1-1]d\phi = 2[\phi]_{\phi=0}^{2\Pi} = 4\Pi$$

(b) $\mathbf{F} = \frac{\sin^2\phi}{r^2}\bar{ar} + \cos\phi \bar{a}_\theta \Rightarrow F.ds = \frac{\sin^2\phi}{r^2}$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} r\sin\theta\sin^2\phi d\theta d\phi = 8\pi$$

$r = 4$

$\int_0^{\pi} \sin\theta\, d\theta = 2$

$\int_0^{2\pi} \sin^2\Phi\, d\Phi = \pi$

because of integration of sin function over one period

Prob 2: (c) $\mathbf{F} = \mathbf{a}x$

$\bar{F} = \sin\theta\cos\phi\bar{ar} + \cos\theta\cos\phi\bar{a}_\theta - \sin\phi\bar{a\phi}$

$F.ds = r^2 \sin^2 \cos\phi d\theta d\phi$

$$\oint F.ds = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\Pi} r^2\sin^2\theta\cos\phi d\theta d\phi = 0$$

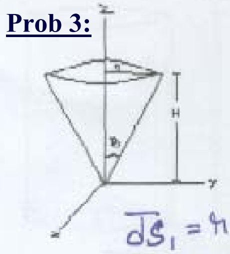
$r^2 = 16$

$\int_0^{\pi} \sin^2\theta\, d\theta = \frac{\pi}{2}$

$\int_0^{2\pi} \cos\phi\, d\phi = 0$

because of integration of sin function over one period

Prob 3:



$\overline{ds}_1 = r\sin\theta\, dr\, d\phi\ \overline{a_\theta}$ (sp.)

Surface 1:

a) $\overline{F} = ra_r = r\overline{a_r}$

$\quad F.ds = 0$

b) $\S F.ds = \iint r^2 \sin\theta\, dr\, d\phi = \frac{2\pi}{3} a(h^2 + a^2)$

$\overline{a_r} = \sin\theta\cos\phi\,\overline{a_x} + \sin\theta\sin\phi\,\overline{a_y} + \cos\theta\,\overline{a_z}$

$\overline{a_r} = \underline{\sin\theta\cos\phi(\cos\phi\,\overline{a_\rho})} - \sin\theta\cos\phi\cdot(\sin\phi\, a_\phi) + \underline{\sin\theta\sin\phi(\sin\phi\,\overline{a_\rho})} + \sin\theta\cdot\sin\phi(\cos\phi\, a_\phi) + \cos\theta\, a_z$

$\overline{a_r} = \sin\theta(\cos^2\phi + \sin^2\phi)\overline{a_\rho} + \cos\theta\, a_z$

$\overline{a_r} = \boxed{\sin\theta\ \overline{a_\rho} + \cos\theta\,\overline{a_z}}$

$\therefore\ \overline{F} = r\overline{a_r}$

$\quad = r\sin\theta\,\overline{a_\rho} + r\cos\theta\,\overline{a_z}$

$\quad = \rho\,\overline{a_\rho} + z\,\overline{a_z}$

c) $F = \cos\phi\, a_\phi + r\, a_\theta$

Surface 2:

a) $F = r\overline{a_r} = r\sin\theta\,\overline{a_\rho} + r\cos\theta\,\overline{a_z}$

$\quad = \rho a_\rho + z a_z$

Prob 3:

$\S F.ds = \iint z\rho\, d\rho\, d\phi$

$\quad = z\int \frac{\rho^2}{2}\Big|\, d\phi = \pi h a^2$

b) $\overline{F} = r\overline{a_\theta} = r\cos\theta\,\overline{a_\rho} - r\sin\theta\,\overline{a_z} = z\,\overline{a_\rho} - \rho\,\overline{a_z}$

$\quad \S F.ds = -\iint \rho^2 d\rho\, d\phi = -\frac{2\pi a^3}{3}$

c) $\overline{F} = \cos\phi\, a_\phi + r a_\theta = r\cos\theta\, a_\rho + \cos\phi\, a_\phi - r\sin\theta\,\overline{a_z}$

$\quad = z a_\rho + \cos\phi\, a_\phi - \rho\, a_z$

$\quad \S F.ds = -\iint \rho^2 d\rho\, d\phi = -\frac{\pi a^3 2}{3}$

$\boxed{\overline{dS} = \rho\, d\phi\, d\rho\ \overline{a_z}}$ (cyl.)

$\oint \overline{F}\cdot\overline{ds} = \iint (\rho\,\overline{a_\rho} + z\,\overline{a_z})\cdot(\rho\, d\phi\, d\rho\,\overline{a_z})$

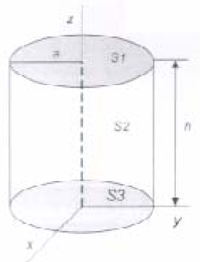
$\quad = \iint z\rho\, d\phi\, d\rho$

$\quad = z\int_0^{2\pi}\int_0^a \rho\, d\rho\, d\phi$

$\quad = (h)\int_0^{2\pi} \frac{\rho^2}{2}\Big]_0^a d\phi$

$\quad = \frac{ha^2}{2}\cdot 2\pi = \boxed{\pi h a^2}$

Prob 4a:



$ds1 = \rho d\phi d\rho \overline{a_z}$

$ds2 = \rho d\phi dz \overline{a_\rho}$

$ds3 = -\rho d\phi d\rho \overline{a_z}$

$ds3 =$ "$-$" due to direction

a) $F = \rho^2 a_\rho + \rho \sin\phi a_\phi + \rho^2 \sin\phi \overline{a_z}$

$$\int_{S1} = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \rho^3 \sin\phi d\phi d\rho = 0 \rightarrow (\overline{a_z})$$

$$\int_0^a \rho^3 d\rho = \frac{\rho^4}{4}\Big]_0^a = \frac{a^4}{4}$$

$$\int_0^{2\pi} \sin\phi d\phi = -\cos\phi\Big]_0^{2\pi} = -(1-1) = 0$$

integration of sin over one period

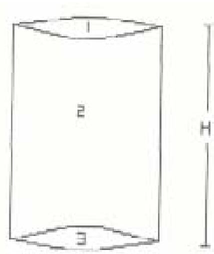
$$\int_{S2} = \int_{z=0}^{h} \int_{\phi=0}^{2\pi} \rho^3 d\phi dz = [a^3][2\pi][h] = 2\pi h a^3$$

$$\int_{S3} = -\int_{\phi=0}^{2\pi} \int_{\rho=0}^{a} \rho^3 \sin\phi d\phi d\rho = 0$$

integration of sin over one period

$$\int F = \int_{S1} + \int_{S2} + \int_{S3} = 2\pi h a^3$$

Prob 4b:



a) $F = \rho^2 a_\rho + \rho\sin\phi a_\phi + \rho^2 \sin\phi a_z$

$\oint F.ds1 = \iint \rho^3 d\phi dz$

$= 2\pi a^3 h$

$\oint F.ds2 = \iint \rho^3 \sin\phi d\phi d\rho$

$= \int -\cos\phi | \rho^3 d\rho = 0$

$F.ds3 = 0$

$\Rightarrow F.ds = 2\pi a^3 h$

b) $F = x\overline{a_x} + z\overline{a_z}$

$= x\cos\phi \overline{a_\rho} - x\sin\phi \overline{a_\phi} + z\overline{a_z}$

$= \rho\cos^2\phi \overline{a_\rho} - \rho\cos\phi\sin\phi \overline{a_\phi} + z\overline{a_z}$

$\oint F.ds1 = \iint \rho^2 \cos^2\phi d\phi dz$ $\left\{ \begin{matrix} z = 0 \rightarrow h \\ \phi = 0 \rightarrow 2\pi \end{matrix} \right\}$

$= \pi a^2 h$

$\oint F.ds2 = \iint \rho z d\phi d\rho a_z$ $\left\{ \begin{matrix} \rho = 0 \rightarrow a \\ \phi = 0 \rightarrow 2\pi \end{matrix} \right\}$

$= h\iint \rho d\rho d\phi$

$= \pi a^2 h$

$\oint F.ds3 = 0$ (as $z = 0$)