

**PROBLEM SESSION III**

**KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS**  
**Electrical Engineering Department**  
**EE 340: Introduction to Electromagnetics**

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**3.15** Determine the gradient of the following scalar fields:

- (a)  $U = 4xz^2 + 3yz.$   
 (b)  $V = e^{(2x+3y)} \cos 5z.$   
 (c)  $W = 2\rho(z^2 + 1) \cos \phi.$   
 (d)  $T = 5\rho e^{-2z} \sin \phi.$   
 (e)  $H = r^2 \cos \theta \cos \phi.$   
 (f)  $Q = (\sin \theta \sin \phi) / r^3.$

**3.18** Find the divergence and curl of the following vectors:

- (a)  $\mathbf{A} = e^{xy} \mathbf{a}_x + \sin xy \mathbf{a}_y + \cos^2 xz \mathbf{a}_z$   
 (b)  $\mathbf{B} = \rho z^2 \cos \phi \mathbf{a}_\rho + z \sin^2 \phi \mathbf{a}_z$   
 (c)  $\mathbf{C} = r \cos \theta \mathbf{a}_r - \frac{1}{r} \sin \theta \mathbf{a}_\theta + 2r^2 \sin \theta \mathbf{a}_\phi$

**3.30** Given that  $\mathbf{E} = \frac{1}{r^4} \sin^2 \phi \mathbf{a}_r$ , evaluate

- (a)  $\oint_S \mathbf{E} \cdot d\mathbf{S}$   
 (b)  $\int_V (\nabla \cdot \mathbf{E}) dv$

over the region between the spherical surfaces  $r = 2$  and  $r = 4$ .

**3.33** Calculate the total outward flux of vector

$$\mathbf{F} = \rho^2 \sin \phi \mathbf{a}_\rho + z \cos \phi \mathbf{a}_\phi + \rho z \mathbf{a}_z$$

through the hollow cylinder defined by  $2 \leq \rho \leq 3, 0 \leq z \leq 5$ .

**3.39** Given the vector field

$$\mathbf{R} = (2x^2y + yz)\mathbf{a}_x + (xy^2 - xz^3)\mathbf{a}_y + (cxyz - 2x^2y^2)\mathbf{a}_z$$

determine the value of  $c$  for  $\mathbf{R}$  to be solenoidal.

**3.40** If the vector field

$$\mathbf{T} = (\alpha xy + \beta z^3)\mathbf{a}_x + (3x^2 - \gamma z)\mathbf{a}_y + (3xz^2 - y)\mathbf{a}_z$$

is irrotational, determine  $\alpha$ ,  $\beta$ , and  $\gamma$ . Find  $\nabla \cdot \mathbf{T}$  at  $(2, -1, 0)$ .