Int. J. Elect. Enging Educ., Vol. 16, pp. 355–362. Manchester U.P., 1979. Printed in Great Britain

A FEW LIAPUNOV FUNCTIONS FOR A SYNCHRONOUS GENERATOR INFINITE BUS POWER SYSTEM PROBLEM

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INTRODUCTION

The use of Liapunov's direct method for determination of power system stability has been reported in the literature over a number of years.[1,2] Because of the higher orders of power system equations, explicit expressions of Liapunov functions are not usually reported. It is felt that a few worked-out examples will help final-year undergraduates, who are sufficiently familiar with control theory, in the use of Liapunov functions for the determination of power system stability. Since the standard texts consider examples of only up to third-order or so, these examples will also serve as illustrations of higher-order systems for an introductory course on modern control for postgraduate students.

A synchronous generator having a fast response, high gain exciter and feeding an infinite bus through a double circuit transmission line has been considered for this study. Such generators on load result in slowly growing oscillations necessitating additional stabilizing signals for satisfactory performance[3]. The stability of the system has been examined with and without auxiliary stabilizing signals through the direct method of Liapunov considering both linear and nonlinear system equations. The variable gradient method[4,6], with some modifications, has been used for the nonlinear model.

THE DIRECT METHOD OF LIAPUNOV

Consider a system which can be represented by the following differential equation

$$\dot{X} = f(X), \quad f(0) = 0 \tag{1}$$

where X is a state vector and f is a continuous function of X. The direct method of Liapunov states that equilibrium states of equation (1) are stable if it is possible to determine a scalar $V(X)$ such that $V(0) = 0$ and the derivative of the scalar function, $\dot{V}(X)$, is of sign opposite to $V(X)$ or vanishes identically. The function $V(X)$ is called a Liapunov function[5].

Expanding $f(X)$ in a Taylor series about the equilibrium state X_e and retaining only the first-order derivatives, the linearized equation can be expressed as

$$\dot{X} = AX \tag{2}$$

where X now represents the variation of the states from the equilibrium values and A is the Jacobian which depends on X_e. Consider A to be a real and nonsingular matrix.

A necessary and sufficient condition for $X = 0$ to be an assymptotically stable solution of equation (2), in the neighborhood of the equilibrium state where the linearization is valid, is that there exists a real symmetric matrix P satisfying the equation[4]

$$A^T P + PA = -I \tag{3}$$

where I is the identity matrix. The scalar function $V(X) = X^T PX$ is a Liapunov function for the system represented by equation (2). Differentiating the expression for V and substituting equation (2), it can be easily shown that $\dot{V}(X) = -X^T X$, which is of opposite sign to $V(X)$.

For large perturbations from equilibrium states, the first-order approximation is not valid. To obtain representative functions in such cases, the nonlinear dynamics should be taken into consideration. One of the methods which provide a systematic approach for generating a Liapunov function is a variable gradient method of Schultz and Gibson.[4,6] The method is based on the fact that if a particular Liapunov function (L.F.) exists indicating stability of the equilibrium state, then its gradient also exists. Assume V to be an explicit function of X. Then its derivative can be expressed as

$$\dot{V} = \frac{\partial V}{\partial x_1}\dot{x}_1 + \frac{\partial V}{\partial x_2}\dot{x}_2 + \ldots\ldots\ldots + \frac{\partial V}{\partial x_n}\dot{x}_n = (\nabla V)^T \dot{X} \tag{4}$$

where the gradient of V is assumed to be a combination of various states and is given as

$$\nabla V = \begin{bmatrix} \frac{\partial V}{\partial x_1} \\ \frac{\partial V}{\partial x_2} \\ \vdots \\ \frac{\partial V}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \nabla V_1 \\ \nabla V_2 \\ \vdots \\ \nabla V_n \end{bmatrix} = \begin{bmatrix} c_{11}x_1 + c_{12}x_2 + \ldots\ldots + c_{1n}x_n \\ c_{21}x_1 + c_{22}x_2 + \ldots\ldots + c_{2n}x_n \\ \vdots \quad\quad \vdots \quad\quad\quad \vdots \\ c_{n1}x_1 + c_{n2}x_2 + \ldots\ldots + c_{nn}x_n \end{bmatrix} \tag{5}$$

c_{ij}'s are completely unknown quantities.
V is obtained as a line integral of ∇V as

$$V = \int_0^X (\nabla V)^T \dot{X} \tag{6}$$

The integral can be made independent of the path of integration and is written in the expanded form as

$$V = \int_0^{x_1 (x_2 = x_3 = \ldots = x_n = 0)} \nabla V_1 dx_1 + \int_0^{x_2 (x_1 = x_1, x_3 = x_4 = \ldots = x_n = 0)} \nabla V_2 dx_2$$

$$\ldots, + \int_0^{x_n (x_1 = x_1, x_2 = x_2, \ldots\ldots x_{n-1} = x_{n-1})} \nabla V_n dx_n \tag{7}$$

In order to obtain the scalar V from the line integral of the vector function ∇V uniquely, the following F matrix formed by $\frac{\partial \nabla V_i}{\partial x_j}$

$$F = \begin{bmatrix} \frac{\partial \nabla V_1}{\partial x_1} & \frac{\partial \nabla V_1}{\partial x_2} & \cdots\cdots & \frac{\partial \nabla V_1}{\partial x_n} \\ \frac{\partial \nabla V_2}{\partial x_1} & \frac{\partial \nabla V_2}{\partial x_2} & \cdots\cdots & \frac{\partial \nabla V_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial \nabla V_n}{\partial x_1} & \frac{\partial \nabla V_n}{\partial x_2} & \cdots\cdots & \frac{\partial \nabla V_n}{\partial x_n} \end{bmatrix} \tag{8}$$

must be symmetric.

SYSTEM REPRESENTATION

Fig. 1 gives a block diagram of the synchronous generator-infinite bus system considered.

The differential equations for a 46 MW synchronous generator as taken from reference [7] are

$$\begin{aligned} pi_{fd} &= -.40013\, i_{fd} - 22.233\, i_d + 369.7(1+n) i_q - 298.44 \sin\delta + 0.711\, E_{fd} \\ pi_d &= -.15191\, i_{fd} - 27.376\, i_d + 455.22(1+n) i_q - 368 \sin\delta + 0.271\, E_{fd} \\ pi_q &= 171.3775(1+n) i_{fd} - 451.406(1+n) i_d - 22.672\, i_q - 305 \cos\delta \\ pn &= -.0936\, i_{fd}\, i_q + 0.0406\, i_d i_q + K \\ p\delta &= 377\, n \end{aligned} \tag{9}$$

where i_{fd}, i_d, i_q are the field current, direct and quadrature axis armature currents respectively. δ, n and E_{fd} are the torque angle, per unit speed deviation of rotor and

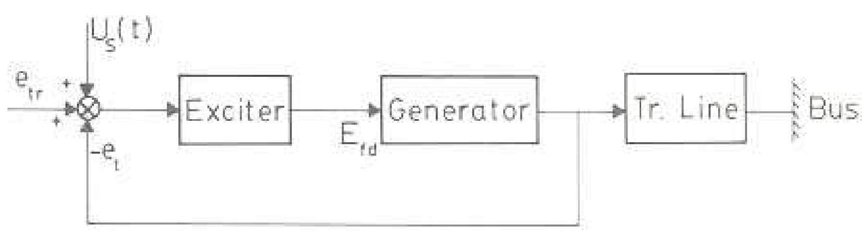

FIG. 1

normalized field voltage. The first three equations are obtained by combining Park's equation for the synchronous generator with the transmission line equations. The last two are obtained by breaking the second-order electromechanical equation (swing equation) of the generator. The excitation system is assumed to have a single time constant T_r, the value of which is taken to be 0.0125 sec. For a gain K_r, the exciter dynamics can be expressed by the equation

$$p E_{fd} = -80 K_r e_t + 80 K_r U_s(t) + 80(E_{fdo} + K_r e_{to}) - 80 E_{fd} \tag{10}$$

e_t is the terminal voltage of the generator and $U_s(t)$ is the additional stabilizing signal (see Fig. 1). Linearization of equations (9) and (10) about the equilibrium state $[i_{fdo}\ i_{do}\ i_{qo}\ n\ \delta_o\ E_{fdo}] = [1.78\ \ 0.3\ \ 0.718\ \ 0\ \ 60^\circ\ \ 1]$ yields the following set of first-order differential equations

$$\begin{bmatrix} p\Delta i_{fd} \\ p\Delta i_d \\ p\Delta i_q \\ pn \\ p\Delta\delta \\ p\Delta E_{fd} \end{bmatrix} = \begin{bmatrix} -0.40013 & -22.233 & 369.7 & 0.0 & -144.52 & 0.711 \\ -0.15191 & -27.376 & 455.22 & 0.0 & -184.155 & 0.27 \\ 171.3775 & -451.406 & -22.672 & 0.0 & 263.8 & 0.0 \\ -0.0687 & 0.02988 & -0.211 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 377 & 0.0 & 0.0 \\ -27.79 K_r & 12.257 K_r & -2.52 K_r & a_{64} & 22.53 K_r & (.0003 K_r - 80) \end{bmatrix} \begin{bmatrix} \Delta i_{fd} \\ \Delta i_d \\ \Delta i_q \\ n \\ \Delta\delta \\ \Delta E_{fd} \end{bmatrix} \tag{11}$$

where $a_{64} = 80 K_r U_s(t)/n$. $U_s(t)$ is zero in the absence of a stabilizing signal or equals 20 times the per unit speed deviation signal.

L.F. FOR THE LINEARIZED MODEL

For a given A, the matrix equation

$$A^T P + PA = -I \tag{12}$$

was solved for P by the Gauss Jordan method with normalization. Several cases were considered with and without automatic voltage regulator (a.v.r.) action and stabilizing signal $U_s(t)$. The P matrix obtained in each case was tested for positive definiteness by Sylvester's criterion.[4]

(i) *Without a.v.r. action:* In the absence of voltage regulator action, the linearized representation reduces to five first-order differential equations, the last row and columns of the matrix in equation (11) being zero. The positive definite symmetric matrix P corresponding to this 5×5 A matrix is

$$P = \begin{bmatrix} 5.696 & -4.6468 & -0.00015 & -15.998 & 3.859 \\ -4.647 & 3.7176 & -0.00019 & 20.417 & -3.155 \\ -0.00015 & -0.00019 & 0.03445 & -2.0127 & 0.017 \\ -15.9978 & 20.417 & -2.0127 & 16204.867 & -0.00132 \\ 3.859 & -3.155 & -0.01708 & -0.00132 & 5.0364 \end{bmatrix} \tag{13}$$

(ii) *With a.v.r. action:* The A matrix considering voltage regulator action is given in equation (11). The corresponding positive definite P matrix for $K_r = -2$ and stabilizing signal $U_s(t) = 0$ is

$$P = \begin{bmatrix} 8.366 & -6.838 & -0.0014 & -66.788 & 8.616 & .0456 \\ -6.838 & 5.621 & -0.0003 & 70.937 & -7.0194 & -0.037 \\ -0.0014 & -0.0003 & 0.0348 & -4.465 & -0.0276 & -0.0005 \\ -66.7 & 70.93 & -4.465 & 36453.87 & -0.0013 & -0.094 \\ 8.616 & -7.019 & -0.0276 & -0.0013 & 11.298 & 0.0552 \\ 0.0454 & -0.037 & -0.0005 & -0.094 & 0.0552 & 0.0065 \end{bmatrix} \quad (14)$$

(iii) *With a.v.r. and stabilizing signal:* For $K_r = -100$ and $U_s(t) = 20$ times rotor speed deviation, the positive definite P matrix is

$$P = \begin{bmatrix} 56.52 & -47.31 & 0.087 & 2630.44 & -3.06 & 0.065 \\ -47.31 & 39.72 & -0.0805 & 2231.53 & 2.90 & -0.0554 \\ 0.0878 & -0.0805 & 0.2415 & -16.129 & -0.555 & 0.0229 \\ -2630.44 & 2231.53 & -16.12 & 19142.12 & 908.07 & -2.139 \\ -3.0607 & 2.9039 & -0.555 & 908.07 & 28.85 & 0.099 \\ 0.065 & -0.0554 & -0.0229 & -2.1396 & 0.099 & 0.0066 \end{bmatrix} \quad (15)$$

It can be seen that in all three cases the respective Liapunov functions $V = X^T pX$ are positive definite, their derivatives $\dot{V} = -X^T X$ being negative definite. With a.v.r. action, the problem was solved for a large number of exciter gains and it was observed that in the absence of stabilizing signal the matrix P obtained from solution of equation (12) is not positive for magnitude of gain greater than 3.8. As a check, the eigenvalues of the linearized system were tested for unstable cases according to the first method of Liapunov. [The first method of Liapunov states that if the real parts of the eigenvalues of the A matrix obtained by linearization of the original nonlinear equation are nonpositive, then the equilibrium state is stable. But this applies only in the neighborhood of the equilibrium state.] However, with an additional feedback signal proportional to shaft velocity, the equilibrium states are stable even for quite large values of exciter gain.

L.F. FOR THE NONLINEAR MODEL

In order to generate a Liapunov function for the nonlinear model by the variable gradient method, the following procedure was followed.

1. ∇V, the gradient of V, is assumed to be a combination of the different states as given in equation (5).
2. Some of the coefficients in the expression for ∇V are determined by constraining

$$\dot{V} = \nabla V_1 \, p i_{fd} + \nabla V_2 \, p i_d + \ldots\ldots + \nabla V_6 \, p E_{fd} \quad (16)$$

to be negative definite or at least negative semi-definite.

3. The remaining unknown coefficients in ∇V are obtained from the 15 curl equations (for the sixth order system) from the symmetry of the F matrix given in expression (8).
4. $\dot{V}$ is rechecked as the addition of terms required in step 3 might alter it. V is then obtained by performing the line integration shown in equation (7). This V function must be positive for assymptotic stability.

(i) *Without a.v.r. action:* To satisfy the curl requirement on the F matrix, the following assumptions on the c_{ij}'s were made to find the L.F. for the nonlinear machine equations without a.v.r. as given in equation (9).

$$\begin{aligned}&\text{(a)}\ c_{ij} = c_{ji} \qquad i, j = 1, 2, \ldots 5\\ &\text{(b)}\ c_{ii} > 0 \qquad i = 1, 2, \ldots 4\\ &\text{(c)}\ c_{11} > (c_{2j}, j = 1, 2) > (c_{3j}, j = 1, 2, 3) >\\ &\quad c_{4j}, j = 1, 2, 4) \geqslant (c_{5j}, j = 1, 2, 5)\end{aligned} \tag{17}$$

Assumptions (b) and (c) were made to make V positive definite. Substituting the values of c_{ij}, the expression for $\dot{V}$ given by

$$\dot{V} = \nabla V_1\, pi_{fd} + \nabla V_2\, pi_d + \nabla V_3\, pi_q + \nabla V_4\, p_n + \nabla V_5\, p\delta \tag{18}$$

is tested for negative definiteness. The values of c_{ij} for the equilibrium state [2.473 0.564 0.734 0 60°] are:

$$\begin{bmatrix} 5.872 & -4.025 & -1.28 & -1.0 & -1.0\\ -4.025 & 4.025 & 1.28 & 1.0 & 1.0\\ -1.28 & 1.28 & 1.28 & 1.0 & 1.0\\ -1.0 & 1.0 & 1.0 & 1.0 & 1.0\\ -1.0 & 1.0 & 1.0 & 1.0 & -0.282 \end{bmatrix} \tag{19}$$

The Liapunov function obtained is

$$\begin{aligned}V &= 0.9325\, i_{fd}^2 + 1.375\,(i_{fd} + i_d)^2 + 0.14\,(-i_{fd} + i_d + i_q)^2\\ &+ 0.5\,(-i_{fd} + i_d + i_q + n - 0.202\,\delta)^2\end{aligned} \tag{20}$$

which is positive definite. The $\dot{V}$ function is negative definite only in the neighbourhood of the equilibrium state. The limiting values of states for which $\dot{V} \leq 0$ were obtained by a computer programme searching a particular axis while others were kept at their equilibrium values. The boundaries obtained were

$i_{fd} = 5.0024, i_d = 1.6143, i_q = 1.003, n = 0.047, \delta = 2.012$ rad.

(ii) *With a.v.r. action:* Considering voltage regulator action but $U_s(t) = 0$, the c_{ij}'s for the synchronous machine exciter system obtained by the same procedure are (for $K_r = -2$):

$$\begin{bmatrix} 16.4399 & -11.72 & -3.02 & -2.92 & -1.0 & -1.0\\ -11.72 & 11.72 & 3.02 & 2.92 & 1.0 & 1.0\\ -3.02 & 3.02 & 3.02 & 2.92 & 1.0 & 1.0\\ -2.92 & 2.92 & 2.92 & 2.92 & 1.0 & 1.0\\ -1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0\\ -1.0 & 1.0 & 1.0 & 1.0 & 1.0 & -5.66 \end{bmatrix} \tag{21}$$

The Liapunov function is

$$\begin{aligned}V &= 2.3599\, i_{fd}^2 + 4.35\,(-i_{fd} + i_d)^2 + .05\,(-i_{fd} + i_d + i_q)^2\\ &+ 0.96\,(-i_{fd} + i_d + i_q + n)^2 + 0.5\,(-i_{fd} + i_d + i_q + n + \delta - 5.66\, E_{fd})^2\end{aligned} \tag{22}$$

The boundary states are: $i_{fd} = 5.1213, i_d = 1.6102, i_q = 1.0321, n = 0.048, \delta = 2.14$ rad, $E_{fd} = 3.22$

(iii) *With a.v.r. and stabilizing signal:* With a.v.r. action, $K_r = -20$ and $U_s(t) = 20 \times n$, the c_{ij}'s are

$$\begin{bmatrix} 6.721 & -4.729 & -3.02 & -2.82 & -1.0 & -1.0 \\ -4.729 & 4.729 & 3.02 & 2.82 & 1.0 & 1.0 \\ -3.02 & 3.02 & 3.02 & 2.82 & 1.0 & 1.0 \\ -2.82 & 2.82 & 2.82 & 2.82 & 1.0 & 1.0 \\ -1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ -1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 0.029 \end{bmatrix} \tag{23}$$

The Liapunov function

$$V = 0.995\, i_{fd}^2 + 0.854\,(-i_{fd} + i_d)^2 + 0.096\,(-i_{fd} + i_d + i_q)^2 + 0.914$$
$$(-i_{fd} + i_d + i_q + n)^2 + 0.5\,(-i_{fd} + i_d + i_q + n + \delta + 0.029\, E_{fd})^2 \tag{24}$$

is positive definite for all states. The boundaries for $\dot{V} \leq 0$ as determined by the procedure discussed are:

$i_{fd} = 5.7321, i_d = 1.6281, i_q = 1.0021, n = 0.042, \delta = 2.028$ rad, $E_{fd} = 4.21$

The stability of the equilibrium states with a nonlinear system model was also investigated for a number of exciter gains in the range 0 to 1000. For magnitude of gain greater than 4, it was not possible to generate a positive definite Liapunov function in the absence of a stabilizing signal. As in the linearized case, addition of signal proportional to velocity of the shaft resulted in convergence of the algorithm. For large gains the region in which $\dot{V} \leq 0$ narrows down and the iteration required to obtain a V function increased significantly.

CONCLUSIONS:

The stability of a few equilibrium points of a synchronous generator infinite bus power system problem has been examined through the direct method of Liapunov. Considering both linear and non-linear models, it was observed that the system can be operated with a relatively large exciter gain if a signal proportional to speed variation of the machine is used in addition to normal voltage regulator action.

To obtain a Liapunov function for the linearized model is relatively simple. But the results obtained by this first approximation of the nonlinear system apply only in the neighbourhood of the equilibrium state. The Liapunov function obtained for a nonlinear model is again not unique; different functions may exhibit different regions of stability. The complexity of the problem increases with higher-order systems. A good first guess of the unknown coefficients from physical considerations of the system and an efficient algorithm of updating these coefficients help to reduce the number of iterations required to generate a satisfactory function.

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ABSTRACTS–ENGLISH, FRENCH, GERMAN, SPANISH

A few Liapunov functions for a synchronous generator infinite bus power system problem
The transient stability of a single machine infinite bus power system problem was examined through the direct method of Liapunov. It was observed, through the direct method, that the system can be operated at a larger exciter gain if an additional feedback signal proportional to shaft speed deviation is used.

Quelques fonctions de Lyapounov pour le problème du générateur synchrone raccordé à un réseau infini
La stabilité transitoire d'un seul générateur raccordé à un réseau de puissance infinie est étudiée par la méthode directe de Lyapounov. Grâce à cette méthode directe, on peut observer que le système peut opérer avec un gain plus élevé de l'excitatrice si un signal de contrôle supplémentaire, proportionnel à la variation de vitesse, est utilisé.

Einige Liapunov-Funktionen für ein Inselbetriebproblem eines Synchrongenerators
Die transiente Stabilität bei einem Inselbetriebproblem einer Einzelmaschine wird mittels der direkten Methode von Liapunov untersucht. Durch diese direkte Methode wurde gefunden, dass das System mit einer höheren Erregerverstärkung betrieben werden kann, wenn ein zusätzliches, der Wellengeschwindigkeitsabweichung proportionales Rückführungssignal benutzt wird.

Algunas funciones de Liapunov correspondientes al problema de un generador síncrono conectado a una barra de potencia infinita
Se estudia la estabilidad transitoria del problema clásico de una máquina conectada a una barra de potencia infinita, aplicando el método directo de Liapunov. Se observa, mediante el método directo, que el sistema puede operar con una mayor ganancia en la excitación si se utiliza una señal adicional retroalimentada proporcional a la desviación de velocidad del eje.