DYNAMIC BRAKING RESISTOR-REACTOR SWITCHING STRATEGIES THROUGH A NOVEL LINEAR TRANSFORMATION TECHNIQUE

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ABSTRACT:

Optimum switching strategies for dynamic braking resistor and shunt reactor is proposed for transient stability of a single machine infinite bus power system. The strategy is derived through a novel method of transforming the nonlinear dynamic model of the system to linear one. The simple optimum strategies derived from the linear model was observed to be very effective in stabilization.

1. INTRODUCTION

Power system stability problems can be broadly divided into two types the transient stability associated with large disturbances or changes in the network, and steady state and so called dynamic stability when small changes are involved. A number of measures for enhancing the perturbed systems are reported in the literature. Amongst these, the dynamic braking resistor is a widely known tool for transient stability improvement [1,2]. Application of braking resistor devices through a novel control technique is addressed in this article.

The braking resistor can be considered as a fast load injection to absorb excess transient energy of an area when the machines accelerate following a disturbance. The resistors are switched off when there is a deceleration. A modification was suggested by Aliyu [3] who reported significant improvement in transient by switching in a reactor thus reducing the electrical power output when the machines were generally decelerating. Further investigations on switching of the resistors and reactors for transient control also have been reported [4].

One problem associated with the brakes is determination of the instant of switching in and out of these devices so that transients are controlled most effectively. Because transient stability analysis requires nonlinear modeling of the power system, a closed loop optimal switching strategy normally cannot be arrived at. In this article a procedure is proposed which transforms the nonlinear system dynamics to a linear one. The optimal switching strategy for the linear system is then obtained in a more straightforward manner. The minimum time switching strategies are obtained for a simple single machine problem, considering only the nonlinearities of the electromechanical swing equation of the generator.

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3. DYNAMIC MODEL OF A SINGLE MACHINE POWER SYSTEM

A synchronous generator feeding an infinite bus power system over a double circuit transmission line is shown in Figure 2. The system is assumed to be equipped with dynamic braking resistor and switchable reactor at high voltage side of the generator transformer.

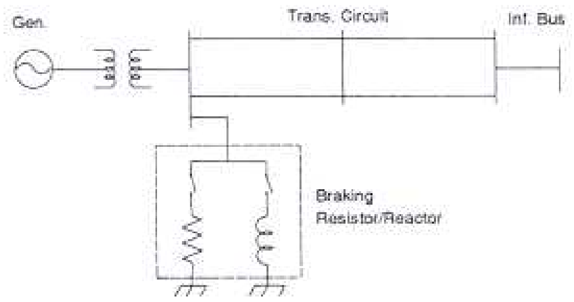

FIGURE 2: Single Machine Infinite Bus System

In transient stability studies the time of interest is very small. As first approximation, stability information can be obtained from consideration of the system dynamics through the simple electromechanical swing equation

$$M \;\; d^2\delta/dt^2 \; + \; D \;\; d\delta/dt \; = \; P_m \; - \; P_e \; - \; P_b \tag{3}$$

where P_m, and P_e are the mechanical input power, and electrical output power of the generator (without the brakes) respectively. δ is the rotor angular position; M and D are the generator inertia and damping coefficient respectively. P_b is the power absorbed by the switchable brake.

The output power P_e can be expressed approximately as

$$P_e \; = \; \frac{EV}{X} \sin\delta \tag{4}$$

where E, V, and X are the internal voltage of the generator, bus voltage, and the reactance between them. Equation (3) then can be broken up as

$$\dot{\delta} \; = \; \Delta\omega$$

$$\Delta\dot{\omega} \; = \; \frac{P_m}{M} \; - \; \frac{EV}{XM}\sin\delta \; - \; \frac{1}{M}P_b \; - \; \frac{D\;\;\Delta\omega}{M} \tag{5}$$

$\Delta\omega$ is the change in frequency.
By assuming $u = P_m - P_b$ then equation (5) is of the form (1) with

$$f \; = \; \begin{bmatrix} \Delta\omega \\ \frac{EV}{XM}\sin\delta \; - \; \frac{D\Delta\omega}{M} \end{bmatrix} \; ; \qquad g \; = \; \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}. \tag{6}$$

2. THEORY

In this section, the method and underlying ideas for the construction of a linear transformation for a special class of nonlinear systems will be discussed. Consider the class of nonlinear systems

$$\dot{x} = f(x) + g(x)u \tag{1}$$

where u is a scalar input, x is an n-dimensional vector, f and g are smooth vector fields, and f(0) = 0. Recently there have been considerable amount of research [5-9] deal with the problem of finding a transformation, T, that will transform the system of form (1) into a particular linear system, that is, a series of integrators (Fig. 1). Expressed in state space form it is

$$\dot{z} = Az + bv \tag{2}$$

where z = T(x), A and b are matrices of dimensions n x n and n x 1 respectively. They are of the form

$$A = \begin{pmatrix} 0 & 1 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1 \\ 0 & 0 & 0 & . & . & . & 0 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 \\ 0 \\ . \\ 0 \\ 1 \end{pmatrix}$$

Figure 1 gives a block diagram showing the transformations involved. Linear system theory can be used to design the controller shown.

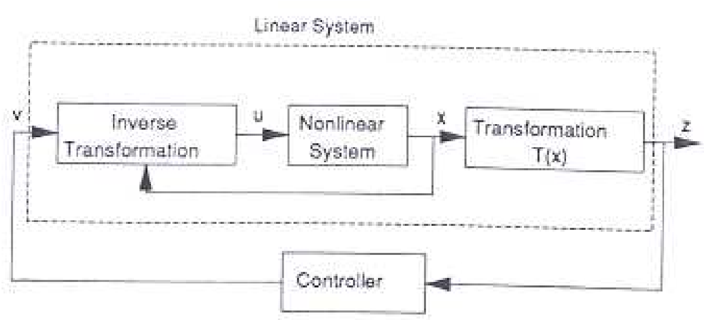

FIGURE 1: Transformation into Linear System

The mathematical theory behind this transformation, as given in reference [6] is presented in Appendix A. Here, in this paper an application of the above simplified procedure is considered for a simple power system problem.

4. THE TRANSFORMED SYSTEM MODEL

When a large disturbance appears in a power system, it may even be first swing unstable if proper control action is not taken fast enough. The optimum control problem for transient stability enhancement can be formulated so as to minimize the transient duration, or minimization of

$$J = \int_{t_0}^{t_f} dt \tag{7}$$

The control problem can be stated as: Given the system of equations (5) find admissible control u which is limited by some upper and lower bounds

$$u_{\min} \leq u \leq u_{\max} \tag{8}$$

So that the system has a transition from the initial to some target state minimizing the functional (7).

Through the control problem can be solved through open loop iterative procedure, the optimal u cannot be determined as a function of the states even for this second order nonlinear system. The following analysis shows how a linear model for this system is arrived at and the optimum control is then derived from the linear system. Notice that this is not a small signal linearization (or quairlinearization) process as is often reported for nonliner systems.

The first step of the design procedure requires us to establish the linearizability of the single machine system. The lie bracket (refer to Appendix A) is given as

$$\begin{aligned}[f,g] &= \frac{\partial g}{\partial x}f - \frac{\partial f}{\partial x}g \\ &= \begin{bmatrix} -\dfrac{1}{M} \\ \dfrac{D}{M^2} \end{bmatrix}. \end{aligned} \tag{9}$$

Next, check the following two conditions of the linearization theorem:

Condition (i). A simple calculation shows that the set $\{ g , [f , g] \}$ is linearly independent set in R^2 provided that $M \neq 0$.

Condition (ii). The second set of vectors which actually consist of g only is obviously involutive.

Since conditions (i) and (ii) are satisfied, therefore system (5) is transformable into linear system of form (2) with the matrices A, and b are 2 x 2 and 2 x 1 respectively.

The transformation $T=(T_1,T_2,T_3)$ which carries system (5) into a linear system of form (2) is constructed as follows [7]. From equation (A.7) in Appendix A we write,

$$\begin{bmatrix} \frac{d\delta}{ds} \\ \frac{d\omega}{ds} \end{bmatrix} = [f, g] = \begin{bmatrix} \frac{-1}{M} \\ \frac{D}{M^2} \end{bmatrix} \tag{10}$$

with $\delta(0) = 0$, and $\omega(0) = 0$ to obtain $\delta = -\frac{1}{M}s$ and $\omega = \frac{D}{M^2}s$. From (A.8) we can arrive at the following

$$\begin{bmatrix} \frac{d\delta}{dt_1} \\ \frac{d\Delta\omega}{dt_1} \end{bmatrix} = g = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} \tag{11}$$

with $\delta(s,0) = \delta(s)$ and $\Delta\omega(s,0) = \Delta\omega(s)$ to obtain $\delta = -\frac{1}{M}s$ and $\Delta\omega = \frac{1}{M}t + \frac{D}{M^2}s$.

Finally, by setting $T_1 = s$ and $\delta = -\frac{1}{M}s$ from (A.4) we have the transformation

$$T_1 = -M\delta,$$

$$T_2 = -M\Delta\omega,$$

$$\text{and} \quad T_3 = \frac{EV}{X}\sin\delta + D\Delta\omega - u. \tag{12}$$

To demonstrate that such a transformation does indeed transform system (5) into a system of the form (2), let $z_1 = T_1$, $z_2 = T_2$, and $v = T_3$. Then

$$\frac{dz_1}{dt} = \frac{\partial T_1}{\partial \delta}\frac{\partial \delta}{\partial t} = -M\Delta\omega = T_2 = z_2 \tag{13.a}$$

and

$$\frac{dz_2}{dt} = \frac{\partial T_2}{\partial \Delta\omega}\frac{\partial \Delta\omega}{\partial t} = -M\left(\frac{-\frac{EV}{X}\sin\delta - D\Delta\omega + u}{M}\right) = T_3 = v \tag{13.b}$$

Thus the transformed system is linear system.

5. MINIMUM TIME DYNAMIC BRAKING CONTROL

Having transform the power system model (5) into a linear system we are in a position to apply any linear design technique to stabilize the transformed system which in turn will stabilize the original system.

A power system is said to be stable if, following a disturbance, it returns to a state of equilibrium. Although, in theory, the system can take an infinite amount of time to return to equilibrium, it is necessary that this happens in a finite interval of time in practice. By an equilibrium state we mean a condition where the angular velocity and acceleration are zero. Also, the steady-state torque angle of each generator connected to the system must be less than 90° with respect to the chosen frame of reference.

In transient stability problems, the objective is to restore the power system near its equilibrium state in the shortest possible time. One way of achieving this objective is to switch the braking resistor/ractor optimally and the problem then can be defined as follows:

Given the system described by equation (13), and the target set S_α defined for $\alpha > 0$ by the relation:

$$S_\alpha = \{(z_1, z_2):\ z_2 = 0,\ -.025 \le z_1 \le 0\} \tag{14}$$

Find the control $v(t)$ which minimizes the cost function (7) and transfers the system from any initial state to S_α at the same time satisfying the inequality constraint.

$$v_{min} \le v(t) \le v_{max} \tag{15}$$

[Note that the range in equation (14) corresponds to $0, \delta(t_2) < \frac{\pi}{2}$ and $\Delta\omega(t_f) = 0$]

The control $v(t)$ is related to $u(t)$ by the transformation

$$v(t) = \frac{EV}{X}\sin\delta + D\Delta\omega - u \tag{16}$$

$u = P_m - P_b$ is limited by an upper and lower bound from the limits of P_b given as

$$0 \le P_b \le 1 \tag{17}$$

The bounds on u, in turn, decide the maximum and minimum allowable values of $v(t)$.

This is the minimum time problem of the so-called double integral plant [11] for which the switching curves are given by

$$\Sigma_1 = z_1 - \frac{1}{2v_{max}} z_2^2 \qquad (z_2 < 0)$$

$$\Sigma_2 = z_1 - \frac{1}{2v_{min}} z_2^2 + 0.025 \quad (z_2 > 0) \tag{18}$$

and $\Sigma = \Sigma_1 \cup S_\alpha \cup \Sigma_2$ is the switch curve. The control is given by

$$v = v_{max} \quad \forall \quad (z_1, z_2) \in R_1 \cup \Sigma_1$$

$$v = v_{min} \quad \forall \quad (z_1, z_2) \in R_2 \cup \Sigma_2 \tag{19}$$

where R_1 and R_2 are regions divided by the switch curve Σ as shown in Figure 3.

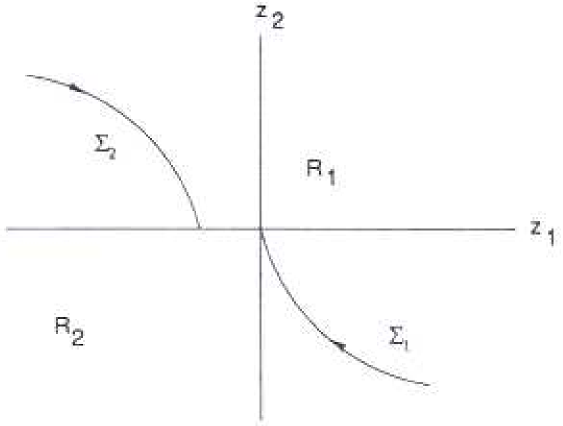

FIGURE 3. Switch curves in $z_1\, z_2$ plane.

The steps involved in realization of the controller would be

1. Measure states x_1 and x_2.
2. Find z_1 and z_2 in terms of x_1 and x_2 .
3. Determine the location of states in the z_1 - z_2 plane (R_1, R_2 or on the switch curves).
4. Depending whether v is v or v determine if P_b is 1 or 0.
5. If $P_b = 1$ switch in the braking resistor (reactor off).
6. If $P_b = 0$ switch in the reactor (resistor off).

   Not that when the reactor is switched in, the machine output power will be virtually zero. v_{min} is calculated on the basis of this assumption.

6. RESULTS

The proposed time optimal braking resistor control strategy through the linear model was tested on the single machine infinite bus system given in Fig.2. The system data and the operating points considered are in Appendix B. The control was tested for 3-ϕ faults of different durations on the high voltage bus of the transmission link. The faults were cleared by opening breakers on the transmission line, isolating a section of it. Results for only two sample cases are presented here.

The rotor angle variation of the generator for a fault duration of 0.22 sec is given in Figure 4 while Figure 5 shows the transient frequency variation following the fault. The fault duration was chosen such that the system is just unstable for this particular operating condition. The optimum brake controls stabilize the transiently unstable system in a fraction of a second, without virtually any oscillation. There is no control action when the fault is on. Figure 6 shows the variation of control. The braking resistor is switched in first

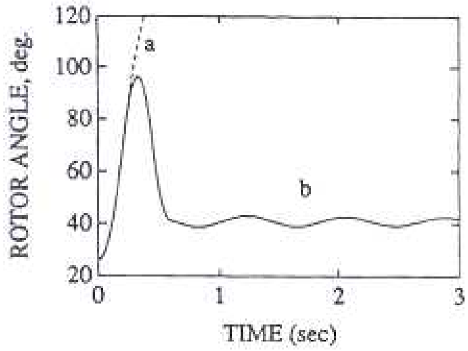

FIGURE 4: varition of rotor angle with time following a three phase fault of .22 sec duration, with (a) no control (b) proposed control.

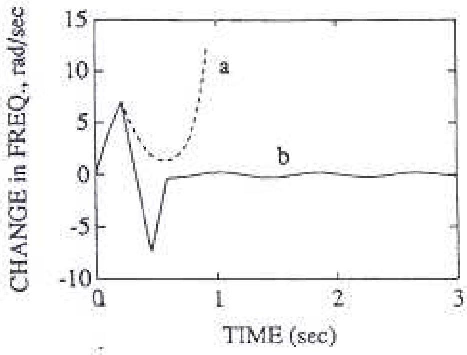

FIGURE 5: varition of the frequency with time following a three phase fault of .22 sec duration, with (a) no control (b) proposed control.

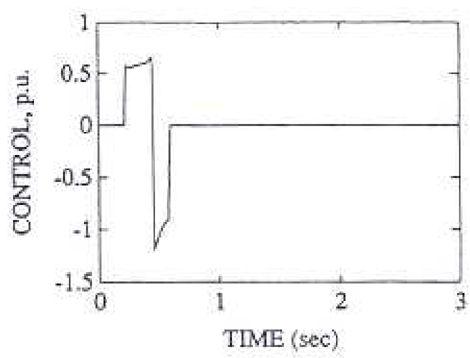

FIGURE 6: variation in the control v corresponding to Fig. 4

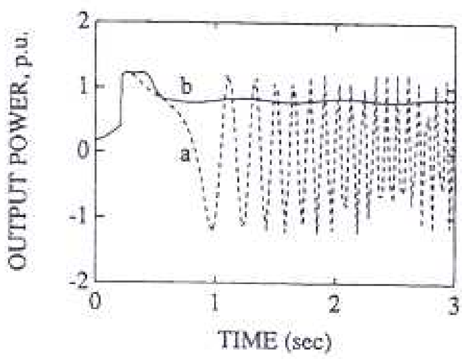

FIGURE 7: variation in the output power with time corresponding to Fig. 3, with (a) no control (b) proposed control

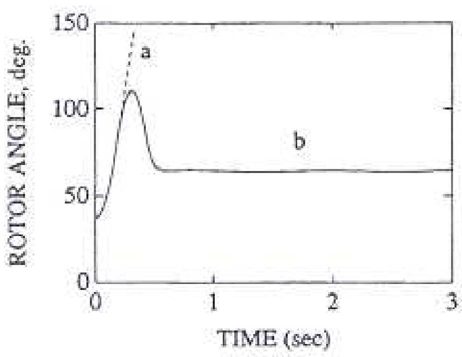

FIGURE 8: varition of rotor angle with time following a three phase fault of .18 sec duration, with (a) no control (b) proposed control.

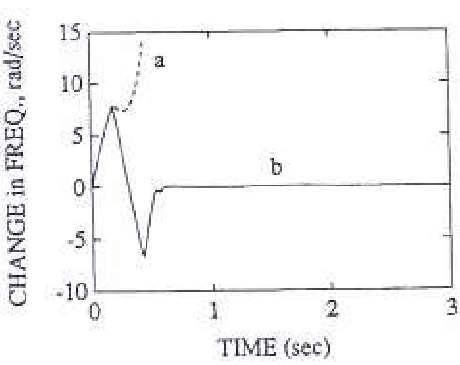

FIGURE 9: varition of the frequency with time following a three phase fault of .18 sec duration, with (a) no control (b) proposed control.

followed by switching of reactors. When the resistor is switched in, reactor is off and vice versa. Notice that the maximum and minimum values of v are not exactly constant because of the dependence of v on the transient power variations dictated by equation (16). This, in essence, introduces a small degree of suboptimality in the algorithms (18) and (19). Figure 7 exhibits the variation of the output power of the machine for both controlled and uncontrolled cases.

Figure 8 gives the angle variation for a 0.18 sec fault duration on high voltage bus. The machine has a slightly higher load in this case (operating point 2 in Appendix B). The corresponding frequency variation is shown in Fig. 9. Variation of control in Fig. 10 shows that again only one set of (optimal) switiching of the resistor and reactor stabilizes the transiently unstable system in a very short time.

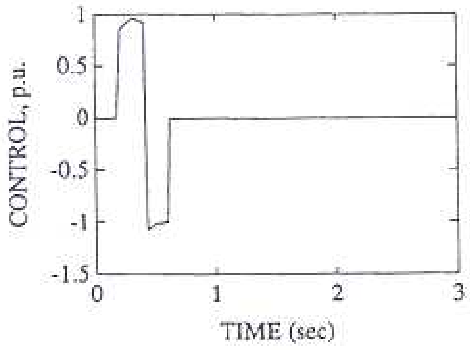

FIGURE 10: variation in the control v corresponding to Fig. 8.

7. CONCLUSIONS

A coordinated dynamic braking-resistor-shunt-reactor switching strategy has been used to stabilize a transiently unstable single machine infinite bus power system. The control algorithms have been derived by converting the nonlinear dynamic model to a linear one through a novel technique. The minimum time switching strategy for the transient stability problem is then obtained from simple linear control techniques. The resistor-reactor switching control was so effective that only one optimum switching of each stabilized the system in a fraction of a second eliminating the oscillations almost completely.

The switching strategy was obtained considering an approximate dynamic model of the system. Application of the method considering higher order dynamics and also multi-machine system studies are under consideration.

8. REFERENCES

1. W.H. Croft and R.H. Hartley, "Improving Transient Stability by use of Dynamic Braking", IEEE Trans. on Power App. & systems, Vol. PAS-59, pp 17-26, 1962.

2. K. Yoshida, "Development of System Damping Resistor for Stabilization of Bulk Power Transmission", Elect. Eng. in Japan, Vol. 91 No. 93, pp. 79-90, 1971.

3. U.O. Aliyu and A.H. El-Abiad, "A Local Control Strategy for Power Systems in Transient Emergency State, Part I: Functional Design", IEEE Trans. Power App. & System, Vol.PAS-101, pp. 4245-4253, 1982.

4. A.H.M.A. Rahim and D.H. Alamgir, "A closed-loop Quasi-optimal Dynamic Braking Resistor and Shunt Reactor Control Strategy for Transient Stability", IEEE Trans. Power System, Vol.3, No.3, pp. 879-886, 1988.

5. A.J. Jakubczyk, "On Linearization of Control Systems", Bull. Acad. Polon.Sci.Math., Vol 28, pp.517-522, 1980.

6. R. Su, "On the Linear Equivalents of Nonlinear Sytems", Syst. Contr. Lett., Vol.2, pp.48-52, 1982.

7. L.R. Hunt, R. Su, and G. Meyer, "Gloval Transformations of Nonlinear Systems", IEEE Tran. Automat. Contr., Vol. AC-28, No.1, pp. 24-31, 1983.

8. R.W. Brockett, "Feedback Invariants for Nonlinear Systems", IFAC Cong., Helsinki, Finland, pp 1115-1120, 1978.

9. H.A. In-Joong, A.K., Tugcu, and N.M. Boustamy, "Feedback Linearization Control of Vehicle Longitudinal Acceleration", IEEE Tran. Automat. Contr. Vol. AC-34, No.7, pp. 689-698, 1989.

10. W.M. Boothby, An introduction to Differential Manifolds and Riemmanian Geometry, New York: Academic, 1975.

11. M. Athans, and P.L. Falb, Optimal Control, New York: McGraw-Hill, 1966.

9. APPENDICES

APPENDIX A

Theorem: The system (1) is transformable to system (2) if and only if

i)the matrix $\{ g, [f, g], \ldots, (ad^{n-1}f, g)\}$has rank n in some neighborhood of the origin R^n, and

ii)the set of vector field $\{ g, [f, g], \ldots, (ad^{n-2}f, g)\}$is involutive in some neighborhood of the origin R^n.

where, the Lie bracket of the two vector fields f and g is defined as

$$[f, g] = (\partial g/\partial x)f - (\partial f/\partial x)g \tag{A.1}$$

with $\partial g/\partial x$, and $\partial f/\partial x$ denoting n x n Jacobain matrices. Also,

$$(ad^0 f, g) = g$$

$$(ad^1 f, g) = [f, g]$$

$$(ad^2 f, g) = [f, [f, g]]$$

$$\vdots$$

$$(ad^k f, g) = [f, (ad^{k-1} f, g)]. \tag{A.2}$$

A set of vector fields $\{f_1, f_2, \ldots, f_m\}$ on R^n is said to be involutive [10] if there exist scalar functions $\alpha_{ijk}(x)$ such that

$$[f_i, f_j](x) = \sum_{k=1}^{m} \alpha_{ijk} f_k(x), \qquad 1 \le i, j \le m, \quad i \ne j. \tag{A.3}$$

It was shown in [6] that if the desired transformation, $T=(T_1, T_2, \ldots, T_n, T_{n+1})$, exist the following partial differential equations must hold:

$$\begin{aligned} <dT_i, g> &= 0 \\ <dT_i, f> &= T_{i+1} \qquad i = 1, \ldots, n-1 \\ <dT_n, f+gu> &= T_{n+1} \end{aligned} \tag{A.4}$$

where dT_i is the gradient of T_i with respect to the vector of independent variables x, and < , > is the inner product. Note that $T_1, T_2, \ldots, T_n$ are functions of x only while T_{n+1} is function of x and u this leads to a set of n-1, first order partial differential equations for T_1

$$<dT_1, (ad^k f, g)> = 0 \qquad k = 0, \ldots, n-2 \tag{A.5}$$

with the additional property

$$<dT_1, (ad^{n-1} f, g)> \ne 0 \tag{A.6}$$

A solution of the preceding set of partial differential equation will give the required transformation. Normally this solution is not unique since no initial conditions are specified. Equations (*A.5*) and (*A.6*) can be solved by reducing them to systems of first order partial differential equations [7]. This can be achieved by introducting the parameters s, t_1, t_2, ... , t_{n-1} as follows. For all $s \in R$ we solve

$$\frac{dx}{ds} = (ad^{n-1} f, g) \tag{A.7}$$

with initial conditions $x(0) = 0$. Then for every $t_1 \in R$ we solve

$$\frac{dx}{dt_1} = (ad^{n-2} f, g) \tag{A.8}$$

with initial conditions $x(s,0) = x(s)$. This argument is repeated until the last step is reached, by solving

$$\frac{dx}{dt_{n-1}} = g \tag{A.9}$$

with initial conditions $x(s, t_1, \ldots, t_{n-2}, 0) = x(s, t_{,1}, \ldots, t_{n-2})$.

By letting T_1 to be any infinitely differentible function of s which vanishes at (0,0, ... ,0) and nonvanishing derivative will give a solution to the system of equations (*A.5*) and (*A.6*) provided that the following Jacobian matrix is nonsingular

$$\begin{pmatrix} \frac{\partial x_1}{\partial s} & \frac{\partial x_1}{\partial t_1} & \cdot & \cdot & \cdot & \frac{\partial x_1}{\partial t_{n-1}} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{\partial x_n}{\partial s} & \frac{\partial x_n}{\partial t_1} & \cdot & \cdot & \cdot & \frac{\partial x_n}{\partial t_{n-1}} \end{pmatrix} \tag{A.10}$$

The implication of the nonsingularity of the preceding Jacobian matrix is that we can solve for $s, t_1, t_2, \ldots, t_n$ as functions of $x_1, x_2, \ldots, x_n$. Clearly that will give a solution for T_1 interms of $x_1, x_2, \ldots, x_n$.

APPENDIX B

The parameters of the single machine infinite bus system are

Machine rating 46 MVA, 13.8 KV

x_d =0.2 p.u.

x_{trans} =0.1 p.u.

x_{line} =0.5 p.u.

V =1.0 p.u.

D=0.0055 p.u.

M=3.0

f=60 HZ

The various operating points considered are

1) P_e =1.0 p.u.

   $\delta = 36.8°$

   E =1.0 p.u.

2) P_e =0.8 p.u.

   $\delta = 24.7°$

   E =1.15 p.u.

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