in-tube nutation dampers," in *Proc. Symp. Attitude Stabilization and Control of Dual-Spin Spacecraft,* (Aug. 1–2, 1967); also Aerospace Corp., El Segundo, Calif., Rep. TR-0158(3307-01)-16, pp. 81–90, Nov. 1967.
[3] G. J. Cloutier, "Stable rotation states of dual-spin spacecraft," *J. Spacecr. Rockets,* vol. 5, pp. 490–492, Apr. 1968.
[4] P. W. Likins, G. T. Tseng, and D. L. Mingori, "Stable limit cycles due to nonlinear damping in dual-spin spacecraft," presented at the AAS/AIAA Astrodynamics Conf., Santa Barbara, Calif., Aug. 1970, AIAA Paper 70-1044.
[5] A. K. Sen, "A nutation damper for a dual-spin spacecraft," *IEEE Trans. Aerosp. Electron. Syst.,* vol. AES-6, pp. 764–769, Nov. 1970.
[6] ——, "Stability of a dual-spin satellite with a four-mass nutation damper," *AIAA J.,* vol. 8, pp. 822–823, Apr. 1970.
[7] ——, "On the determination of global asymptotic stability of spinning space vehicles," unpublished.
[8] T. M. Spencer, "Cantilevered-mass nutation damper for a dual-spin spacecraft," in *Proc. Symp. Attitude Stabilization and Control of Dual-Spin Spacecraft* (Aug. 1–2, 1967); also Aerospace Corp., El Segundo, Calif., Rep. TR-0158(3307-01)-16, pp. 91–109, Nov. 1967.
[9] P. W. Likins, "Attitude stability for dual-spin spacecraft," *J. Spacecr. Rockets,* vol. 4, pp. 1638–1643, Dec. 1967.

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Short Papers

Evaluation of a Quasi-Optimal State Feedback Excitation Strategy for Power System Stability

A. H. M. A. RAHIM AND D. H. KELLY

***Abstract*—A method is presented for finding time-optimal (or suboptimal) excitation control for power system stability. The control is found directly as a function of the states of the system, which can be measured. A closed-loop control is found for the linearized system first, then the method is extended to the nonlinear model. The results obtained for the single-machine infinite bus problem are compared with those obtained by a steepest descent method. For small disturbances where linearity holds, the closed-loop scheme gives optimal solutions, while, for large disturbances, it yields a suboptimal solution.**

I. Introduction

A major aspect to be considered in the reliable production and transmission of electrical energy is the stability of the system. The growth of loads as well as increased demand for reliability have led to the development of complex and highly interconnected power systems. Disturbances on such systems can propagate, resulting in major failures of the power supply. In order to minimize the effects of such disturbances, it is necessary to develop new and faster methods to stabilize the system. The advent of high-speed excitation systems has introduced the problem of dynamic instability in addition to transient stability due to disturbances. On the other hand, the use of high-speed excitation, controlled by feedback signals derived from the rotor velocity and acceleration of a generator, has been found useful for maintaining stability [1]. Optimal excitation control has been investigated [2] for particular disturbances in the system. But, as the disturbances which may occur are not known in advance, the implementation of optimal control is difficult or, in general, not possible, so that suboptimal schemes have to be used.

In this paper, which extends a basic principle set out in a previous paper [3], a quasi-optimal control is found directly as a function of the system states, resulting in a closed-loop scheme. Examples considering a single machine connected to an infinite bus are studied for linearized and nonlinear cases, and the method of steepest descent is used to provide a basis for comparison.

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II. Analysis

The dynamic equations of a synchronous generator connected to an infinite bus system can be expressed as

$$\dot{X} = f(X,u) \tag{1}$$

where the state X is a vector of rotor stator currents, frequency deviation, and rotor angular position. The control $u(t)$ is the output of the exciter, which is constrained in magnitude so that $|u(t)| \leq 1$. For stable operation of system (1), the final velocity and acceleration should decrease to zero following a disturbance, while the final rotor angle should remain constrained between 0 and $\pi/2$ rad.

A time-optimal control for system (1) is obtained by differentiating the nonlinear "swing equation" with respect to time and substituting the voltage and current relations from (1) to give

$$\dddot{\delta}/\omega_0 = L(t) + b(t)u(t) \tag{2}$$

where δ is the rotor angular position and ω_0 is the rated angular velocity. $L(t)$ and $b(t)$ are functions of the states and the disturbances which are generally not known in advance. The constraint imposed on $L(t)$ and $b(t)$ is that

$$|L(t)/b(t)| \leq 1, \qquad \text{for all } t \in [t_0, t_f]. \tag{3}$$

Supposing $L(t)$ and $b(t)$ constant at their initial values L_0 and b_0 and assuming $\delta/\omega_0 = y_1$, $\dot{y}_1 = y_2$, $\dot{y}_2 = y_3$, the switching surfaces passing through the points $\{y_1, y_2, y_3\} = [\pi/2\omega_0, 0, 0]$ and $\{0,0,0\}$, respectively, are

$$\Sigma_1 = y_1 - \frac{\pi}{2\omega_0} - \frac{y_2 y_3}{m} + \frac{y_3^3}{3m^2} = 0 \tag{4}$$

$$\Sigma_2 = y_1 - \frac{y_2 y_3}{m} + \frac{y_3^3}{3m^2} = 0 \tag{5}$$

where

$$m = L_0 + b_0 \operatorname{sgn}\left[y_2 - \frac{y_3^2}{2[L_0 + b_0 \operatorname{sgn}\{y_3\}]}\right]. \tag{6}$$

The control sequence for $b_0 < 0$ is

$$u(t) = 1, \quad \text{if } \Sigma_1 \geqslant 0 \tag{7}$$

$$u(t) = -1, \quad \text{if } \Sigma_2 \leqslant 0 \tag{8}$$

$$u(t) = \operatorname{sgn}\left[y_2 - \frac{y_3^2}{2[L_0 + b_0 \operatorname{sgn}\{y_3\}]}\right], \quad \text{otherwise.} \tag{9}$$

The procedure is continued, recalculating $L(t)$ and $b(t)$ at the end of each integration step until the desired final states are reached. Details of the above scheme are given in [3] and [4].

For small disturbances, the rotor angle does not traverse far beyond the operating point. Consequently, the variation of other states from the operating point is also small. In such a case, the equations of system (1) can be linearized and the restriction on the final value of angle is not necessary. The control scheme can be obtained simply from (9). However, for large disturbances the angular deviation of the rotor is large. In such cases the linear model which is obtained by considering the first variation of sin δ, cos δ, and other nonlinear terms, will give a very poor approximation of the original system equations (1). It is possible to linearize the system around an operating point which is then updated at each step. In this case control is applied on the basis of the linear model. It is then necessary to consider the constraint on the rotor angle.

In this formulation the governor and exciter dynamics are not taken into consideration. In stability studies, the torque variations due to prime mover action are generally not considered since the time under consideration is quite small compared to the governor and prime-mover time constants. For multimachine systems, where relative velocity and acceleration between the machines are to be brought to zero, it may happen that the system stabilizes slightly away from its rated frequency. In such cases, governor action returns the frequency of the system to the normal value. The time constants of modern static excitation systems are negligibly small (of the order of 10–20 ms) compared to the machine time constants, so the neglect of exciter dynamics causes little error. However, a proportional control, based on the quasi-optimal scheme as suggested in [4], takes the exciter dynamics into consideration.

III. Steepest Descent Method

In order to evaluate the effectiveness of the closed-loop scheme presented in the previous section, some of the results obtained by it are compared with those obtained by a steepest descent method. The right-hand side of (1) is linear in the control variable u. By Pontryagin's maximum principle, the time-optimal control for such problems is bang-bang, that is, either $+1$ or -1. The optimal control for such problems can be parametized by the switching times of u. The steepest descent algorithm constructed by Bryson and Denham [5], [6] and later modified by Vachino [7] to take into consideration control problems with inequality constraints has been used here.

The linearized equations of system (1) can be written as [3]

$$\dot{X} = AX + du. \tag{10}$$

The performance index is

$$\phi = -t_f. \tag{11}$$

The stopping condition is also chosen as a function of final time only,

$$\Omega = k - t_f \tag{12}$$

where k is the amount of time required for stabilization, obtained by the closed-loop scheme. The terminal constraints are

$$\psi_1 = -x_4 = 0 \tag{13}$$

$$\psi_2 = -\dot{x}_4 = -a_{41}x_1 - a_{42}x_2 - a_{43}x_3 = 0 \tag{14}$$

where x_4 is the velocity and $\dot{x}_4$ acceleration.

The initial guess for the control and switch times is made from the closed-loop scheme. The values assumed are

$$\begin{aligned} u(t) &= 2, & t_0 &\leqslant t \leqslant t_1 \\ u(t) &= -4, & t_1 &\leqslant t \leqslant t_2 \\ u(t) &= 2, & t_2 &\leqslant t \leqslant t_f. \end{aligned} \tag{15}$$

Using Heaviside step function [7], the equation of variation for (10) can be written as

$$\delta\dot{X}(t) = A\delta X + L\delta\omega \tag{16}$$

where

$$\delta\omega = [\delta t_1, \delta t_2, \delta t_f]^T \tag{17}$$

$$L = \begin{bmatrix} 6d_1\Delta(t-t_1) & -6d_1\Delta(t-t_2) & 2d_1\Delta(t-t_f) \\ 6d_2\Delta(t-t_1) & -6d_2\Delta(t-t_2) & 2d_2\Delta(t-t_f) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{18}$$

$\Delta(t - t_s)$ is a delta function occurring at $t = t_s$.

The adjoint equations are

$$\dot{\lambda} = -A^T\lambda. \tag{19}$$

Choosing the terminal conditions on the adjoint vector,

$$\lambda_\phi(t_f) = \left.\frac{\partial\phi}{\partial X}\right|_{t_f}$$

$$\lambda_\psi(t_f) = \left.\frac{\partial\psi}{\partial X}\right|_{t_f}$$

$$\lambda_\Omega(t_f) = \left.\frac{\partial\Omega}{\partial X}\right|_{t_f}. \tag{20}$$

The variation of the control ω (the switch times) is obtained by the relation

$$\delta\omega(t) = \sum_{s=1}^{3} W^{-1}L_s^T(t)\lambda_{\psi\Omega}(t)I_{\psi\psi}^{-1}\, d\psi \tag{21}$$

where

$$I_{\psi\psi} = \sum_{s=1}^{3}\int_{t_{s-1}}^{t_s} \lambda_{\psi\Omega}^T(t)L_s(t)W^{-1}L_s^T(t)\lambda_{\psi\Omega}\, d\tau \tag{22}$$

$$\lambda_{\psi\Omega}^T(t) = \lambda_\psi^T(t) - \frac{\dot{\psi}(t_f)}{\dot{\Omega}(t_f)}\lambda_\Omega^T(t) = \lambda_\psi^T(t) \tag{23}$$

W is a nonnegative-definite weighting matrix.

A similar procedure is followed for the nonlinear machine model, but the exciter ceiling is raised to 5 pu from 3 pu.

IV. Discussion of Results

For a 10-percent torque step (Figs. 1 and 2), it can be seen that there is virtually no difference in response between the closed-loop and optimal rchemes except near the point where control switches. There is no doubt that the closed-loop scheme gives optimal control when $L(t)$ is zero. For small disturbances such as the case considered,

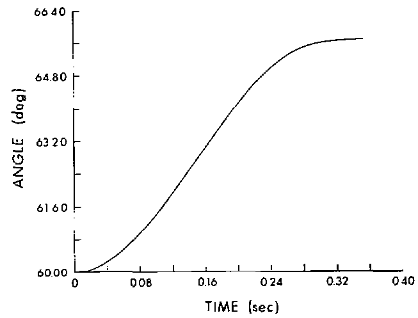

Fig. 1. Rotor angle time characteristics for 10-percent torque step, both optimal and closed-loop control.

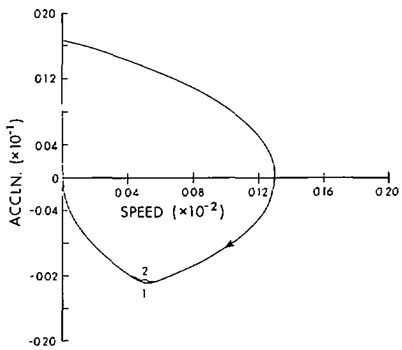

Fig. 2. Phase plane plot corresponding to Fig. 1, where curve 1 stands for optimal control and curve 2 stands for closed-loop control.

TABLE I
10-Percent Torque Step (Linear System)

Scheme	Step Size	Switch Time	Total Cost (s)	Percent of Error	Remarks
Steepest descent	0.001	0.2656	0.3415	—	
Closed loop	0.0001	0.2716	0.3427	0.352	Switching chatter (SC)
Closed loop	0.0002	0.272	0.3428	0.382	Less SC
Closed loop	0.0005	0.287	0.343	0.44	Still less SC
Closed loop	0.001	{0.268, 0.350}	0.366	6.7	Almost no SC

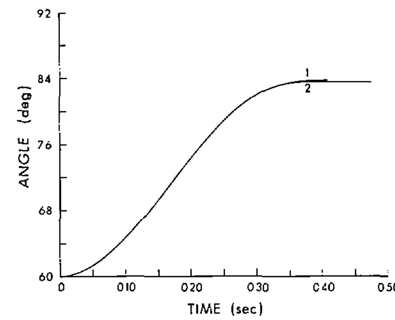

Fig. 3. Rotor angle time characteristics for 30-percent torque step, where curve 1 stands for optimal control and curve 2 stands for closed-loop control.

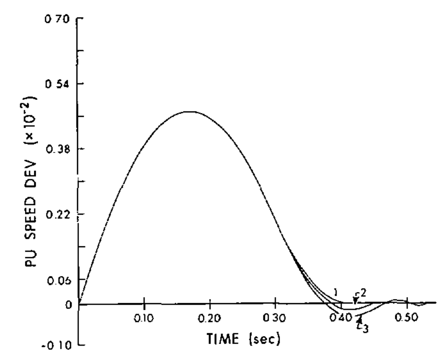

Fig. 4. Velocity-time characteristics, where curve 1 stands for optimal control, curve 2 stands for closed-loop control, and curve 3 stands for closed loop with deadzone.

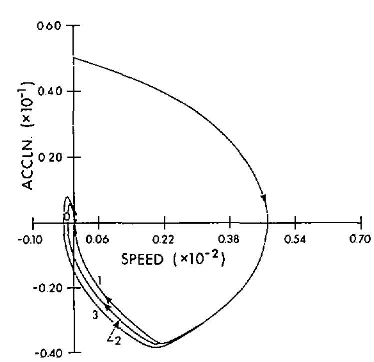

Fig. 5. Phase plane plot corresponding to Figs. 3 and 4, where curve 1 stands for optimal control, curve 2 stands for closed-loop control, and curve 3 stands for closed loop with deadzone.

the term $L(t)$ in (3) is quite small compared to the input yielding an almost optimal control on piecewise constant approximation. The term $b(t)$ in a linear formulation is constant [3].

The closed-loop scheme depends on the integration step size. Table I shows that, as the integration step size is reduced, the time required for stabilization is less. For online application it implies that the effectiveness of the scheme depends on the frequency of sampling. The closed-loop scheme is based on the information available at the present moment. It does not foresee the correct switch point, so that it is quite likely that the control switches several times near the correct switch point (Fig. 2). In the tables, the switch times provided are those at the end of the switching chatter period. The control sequences in both tables are $\{+1, -1\}$, etc.

For large disturbances, the results obtained with the closed-loop scheme do not agree very closely with optimal schemes as shown in Figs. 3–5 and Table II, which are for a 30-percent torque step and integration step size of 0.001 s. However, the use of a smaller step size should show a considerable improvement. The switching function Σ_1 is greater than zero in the time interval [0, 0.055] s, about 3 cycles. The control is decided by relation (9) for the rest of the period. The term $|L(t)/b(t)|$ is less than the control term for both the cases considered.

The problem of multiple switching can be avoided by taking large integration step sizes. This will effect the accuracy of the numerical scheme. Also in this case, the "switch miss" may be quite large resulting in at least another extra switch. The results in Tables I and II show this. The best way to avoid the switching chatter is to provide a "deadzone" so that the control does not switch if the magnitude of the switching function Σ becomes smaller than a certain predetermined quantity ϵ. The response for the 30-percent torque step case is shown in curve 3 of Figs. 4 and 5 for a value of $\epsilon = 5 \times 10^{-5}$. As is shown in Fig. 4, the system takes more time to stabilize because of the "error" in switching. The angle time characteristics (Fig. 3) are almost the same as those for the closed-loop scheme without deadzone.

The success of the closed-loop scheme depends on the behavior of the term $L(t)/b(t)$. As discussed, if the magnitude of this term is sufficiently small compared to $u(t)$, the scheme yields an almost optimal solution. If this term becomes greater than the input for a short period of time followed by a long "controllable section," the scheme yields a suboptimal solution, as was observed in [8]. Such a case, considered in [4], is a multiple disturbance, a three-phase fault

TABLE II
30-PERCENT TORQUE STEP (NONLINEAR SYSTEM)

Scheme	Step Size	Switch Time	Total Cost (s)	Percent of Error	Remarks
Steepest descent	0.001	0.2977	0.40991	—	
Closed loop	0.001	$\begin{Bmatrix}0.31\\0.434\end{Bmatrix}$	0.4788	16.8	Switching chatter

at the terminal of the machine, cleared by opening a line. The system stabilized in about 1 s. From the power system reliability viewpoint, this is quite an improvement over existing methods. However, if $|L(t)/b(t)|$ remains greater than the input term for all $t \in [t_0,t_f]$, the scheme fails to give any information about the optimal or suboptimal control. Another factor to consider is the rate of variation of $L(t)/b(t)$. If this variation is rapid, or $L(t)/b(t)$ happens to have jump discontinuities, results obtained with the closed-loop scheme will obviously be erroneous.

V. CONCLUSION

An important aspect of the closed-loop scheme is that the control is obtained directly as a function of the states. While it is not optimal for large disturbances, it is suited to on-line controls requiring relatively little hardware, either digital or analog, as it is not necessary to solve the system differential equations. The amount of computation required for steepest descent methods precludes on-line control, requiring a predetermined switching strategy which may give worse results than the closed-loop scheme for other than the design disturbance.

For the application of steepest descent methods, the closed-loop method offers computational advantages in that the results provide a good set of initial switching times for the steepest descent algorithms, so that convergence is rapid.

The disadvantage of switching chatter in the closed-loop scheme may be eliminated in simulation by a deadzone and, in on-line applications, by the finite response speeds of system components.

Preliminary investigations indicate that the closed-loop scheme is effective in a multimachine system, although the use of a proportional control rather than bang-bang may be best from system performance viewpoint, at the cost of a longer settling time. These investigations also indicate that time delay in the control, up to approximately 0.05 s for severe disturbances, are tolerable. Further study is indicated concerning such factors as the inclusion of governor response and control and the effects of parameter variation.

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REFERENCES

[1] J. L. Dineley, A. J. Morris, and C. Preece, "Optimal transient stability from excitation control of synchronous generators," *IEEE Trans. Power App. Syst.*, vol. PAS-87, pp. 1696–1705, Aug. 1968.
[2] O. J. M. Smith, "Optimal transient removal in a power system," *IEEE Trans. Power App. Syst.*, vol. PAS-84, pp. 361–374, May 1965.
[3] A. A. Rahim and D. H. Kelly, "Optimal excitation control for power system stability," *IEEE Trans. Automat. Contr.* (Tech. Notes and Corresp.), vol. AC-16, pp. 379–381, Aug. 1971.
[4] D. H. Kelly and A. A. Rahim, "Closed-loop optimal excitation control for power system stability," *IEEE Trans. Power App. Syst.*, vol. PAS-90, pp. 2135–2141, Sept./Oct. 1971.
[5] A. E. Bryson, Jr., and W. F. Denham, "A steepest ascent method for solving optimum programming problems," *Trans. ASME (J. Appl. Mech.)*, ser. E, vol. 29, pp. 247–257, 1962.
[6] W. F. Denham and A. E. Bryson, Jr., "Optimum programming problems with inequality constraints II: Solution by steepest descent," *AIAA J.*, vol. 2, pp. 25–34, 1964.
[7] R. F. Vachino, "Steepest descent with inequality constraints on the control variables," *SIAM J. Contr.*, vol. 4, no. 1, pp. 245–261, 1966.
[8] R. Oldenburger, *Optimal Control*. New York: Holt, Rinehart and Winston, 1966.
[9] M. Althaus and P. Falb, *Optimal Control*. New York: McGraw-Hill, 1966.
[10] P. L. Dandeno, A. N. Karas, K. R. McClymont, and W. Watson, "Effect of high-speed rectifier excitation systems on generator stability limits," *IEEE Trans. Power App. Syst.*, vol. PAS-87, pp. 190–201, Jan. 1968.
[11] F. W. Smith, "Design of quasi-optimal minimum-time controllers," *IEEE Trans. Automat. Contr.*, vol. AC-11, pp. 71–77, Jan. 1966.

Optimum Scheduling of Power Systems Using Functional Analysis

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***Abstract*—Optimum scheduling of a hydrothermal power system with variable head hydroplants is discussed. Functional analysis is used to obtain the optimum solution for the scheduling problem. The problem is formulated as a minimum norm problem. The optimum solution found is unique and is easily shown to agree with the solution found for a simpler problem formulated earlier. An example is given to illustrate the results obtained.**

I. INTRODUCTION

This paper deals with the problem of the short-range optimum scheduling of power generation in a hydrothermal system with variable head hydroplants. The schedules obtained are designed to meet a specified power demand and, simultaneously, to satisfy constraints imposed on the water discharged through the hydroplants during the optimization interval. Various optimization techniques have been used in the past [1], [2], [4].

In this paper the problem is solved by use of the functional analytic minimum norm formulation. The optimum solution for this problem subject to a bounded linear transformation is given in [3]. The advantage of this approach is the elimination of the multipliers associated with each linear constraint in the control vector. Further, this formulation places some restrictions on the unknown multipliers associated with the nonlinear constraints. This simplifies the computational aspects in implementing the optimum solution.

In [5], the authors reported what seems to be the first attempt to apply functional analytic optimization techniques to this problem.

II. THE MINIMUM NORM PROBLEM

In [3], minimum norm problems were defined and their optimum solutions were given. Following is the version applied to the power system problem.

Theorem: Let B and D be Banach spaces and T, a bounded linear

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