DUAL-AXIS SUPPLEMENTARY CONTROLLERS FOR MULTI-MACHINE POWER SYSTEMS

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ABSTRACT

The effect of an additional field winding on the quadrature axis of the rotor of a synchronous generator in suppressing undesirable sustained oscillations experienced in multi-machine power system is investigated. A small-perturbation model for interconnected power systems with dually excited synchronous machines is developed. A two-level optimal strategy incorporating controls both on the direct and quadrature axes field windings is developed. A multi-machine system was tested with the proposed controls as well as with power system stabilizers. From a number of case studies it has been observed that an appropriate additional control in the quadrature axis enhances the stability of the system substantially.

1. INTRODUCTION

The problem of dynamic stability of synchronous machines has been explored extensively and widely reported in the literature. The effects of excitation controls on the dynamic stability and the use of supplementary signals acting through the direct-axis field winding have also been demonstrated in a number of investigations [1-4]. However, experience indicates that controlled direct-axis excitation is only effective in enhancing the power system damping over the normal loading range, and is less effective at light or no-load conditions. Enhancing power system damping characteristics over the whole range of operating conditions is desirable. Recent studies have shown that providing a synchronous machine with a controlled additional winding on the quadrature axis of the rotor can improve dynamic stability limits at any loading conditions [5-6]. In this regard, a weighted speed plus power supplementary controller acting simultaneously on both the d-axis and the q-axis was shown to be effective at light and no-load conditions [7]. The work however dealt exclusively with a single machine-infinite bus system. It is well known that the choice of PSS's for a multi-machine system is not simple. The PSS suitable for a single machine-infinite bus model may give adverse response if used for an actual multi-machine power system because of the interactions of the large number of generators in the system. The problem will definitely be more complicated when two excitation systems are involved.

This paper investigates the effect of controlled dual-axis excitation on system damping in the case of multi-machine power systems. A comprehensive perturbation model for the multi-machine interconnected system is developed. Two control strategies are then tested. The first involves the use of power- actuated stabilizers on both the d-axis and the q-axis of each generator. The second adopts a two-level approach whereby local control signals are derived at the lower level in a completely decentralized environment, and a global controller at a higher level takes the effect of interactions into account. A comparison of the application of the two strategies in a multi-machine power system for direct-axis, quadrature-axis, and dual-axis excitations is presented.

2. MATHEMATICAL MODEL

General configuration of a multi-machine power system is shown in Figure 1. It consists of an N-machine synchronous power system and a transmission network. Each of the synchronous generators is described by its set of Park's equations, two field circuits: one in the d-axis and the other in the q-axis, and the swing equations. The network is represented by a constant admittance matrix which is reduced by eliminating non-generator buses. Loads are represented by constant admittances and are included in the network admittance matrix. Prime mover torque is kept constant. Excitation controls are also represented in the model. The small-perturbation behavior of the power system in the vicinity of a given operating condition can be described to first order by a set of linear time-invariant differential equations in the form:

$$S: \quad \dot{\underline{x}} = A\underline{x} + B\underline{u} \qquad (1)$$

where the n-dimensional state vector $\underline{x}$ represents the perturbation of the system state from its nominal value at the chosen operating condition, and the m-dimensional control vector $\underline{u}$ represents perturbations of the inputs. The state vector comprises of the frequency, angle, direct and quadrature axes internal and field voltages for each machine. The control vector comprises of the additional excitation on the two axes. The derivation of the equations for the multi-machine system is described in Appendix I. Figure 2 shows a complete block diagram of the i^{th} machine.

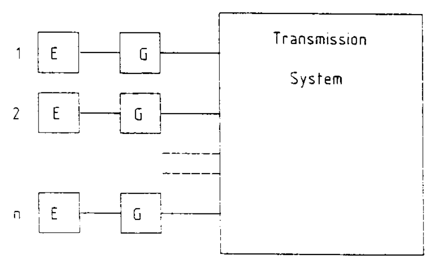

Fig. 1 System configuration, E - excitation system
G - synchronous machine

3. TWO-LEVEL OPTIMAL CONTROL STRATEGY

In recent years there has been considerable interest in the application of optimal control to develop multi-machine stabilization techniques [8-9], with the objective of extending the dynamic stability boundaries. The computation of an optimal controller becomes difficult and time consuming as the order of the system increases. One of the means to overcome these anomalies is to use multi-controller structures, in which each controller acts on a different part of the information space of the system under control. In this paper, the overall power system is decomposed into separate subsystems, each subsystem comprising of one generator. At the subsystem level, an optimal feedback controller, whose model is obviously of lower order than the overall system, is derived for each generator. The controller thus determined at the subsystem level is based on local information pertaining to the particular generator. It is important to realize that as far as an individual generator is concerned, the interactions among various generators are reflected exclusively on the local information monitored at that generator. In order, however, to take into account the interaction between the different subsystems, a global controller is designed at a higher level [10-11].

By decomposing the system S into N interconnected subsystems, the dynamics of each subsystem S_i, each of dimension n_i, is described by:-

$$S_i:\quad \dot{\underline{x}}_i = A_{ii}\underline{x}_i + B_{ii}\underline{u}_i + h_i(\underline{x}),\quad i = 1,2,\ldots N \tag{2}$$

$$\text{Where}\quad \underline{x} = [\underline{x}_1^T, \underline{x}_2^T \ldots \underline{x}_N^T]^T,\ \text{and}\ \sum_{i=1}^{N} n_i = n \tag{3}$$

A_{ii} and B_{ii} are controllable and observable block diagonal matrices of the matrices A and B. The coupling vector $h_i(\underline{x})$ can be expressed as:

$$h_i(\underline{x}) = \sum_{j=1, j\neq i}^{N} A_{ij}\underline{x}_i \tag{4}$$

The performance of each subsystem is measured when the quadratic cost:

$$j_i = \frac{1}{2}\int_0^\infty (\underline{x}_i^T Q_i \underline{x}_i + \underline{u}_i^T R_i \underline{u}_i)dt \tag{5}$$

attains its minimum value when an optimal control vector $\underline{u}_i^*$ is applied to each subsystem.
Q_i and R_i are symmetric positive semidefinite and positive definite matrices respectively.

It is assumed that the overall performance of the system S given by (1) is :

$$j = \sum_{i=1}^{N} j_i \tag{6}$$

Note that $\sum_{i=1}^{N} j_i < j$ implies a detrimental interconnection; otherwise the interconnection is beneficial.
In order to control the overall system S, we adopt a two -level strategy of the form

$$\underline{u}_i = \underline{u}_i^l + \underline{u}^g \tag{7}$$

where $\underline{u}_i^l$ is a local feedback control vector assuming no interactions between subsystems, (when $h_i(\underline{x}) = 0$), and $\underline{u}^g$ represents a global control signal that neutralizes the degradation of the performance due to coupling between individual generating units.
The local optimal feedback control $\underline{u}_i^l$ can be determined as follows: Assuming no interactions at the local level, the modified form of equation (2) is given by:

$$\dot{\underline{x}}_i = A_{ii}\underline{x}_i + B_{ii}\underline{u}_i^l,\quad i = 1,2,\ldots N \tag{8}$$

represents N decoupled systems. The optimal control $\underline{u}_i^l$ minimizing (5) is :

$$\underline{u}_i^l = -R_i^{-1}B_{ii}^T P_i \underline{x}_i \tag{9}$$

where P_i is the solution of the Ricatti equation

$$P_i A_i + A_i^T P_i - P_i B_{ii} R_i^{-1} B_{ii}^T P_i + Q_i = 0 \tag{10}$$

The global signal $\underline{u}^g$ is determined from satisfying the relationship

$$B\underline{u}^g + C\underline{x} = 0 \tag{11}$$

$$\text{where,}\quad C_{ij} = A_{ij},\quad i \neq j$$
$$= 0,\quad i = j \tag{12}$$

Let B' be the pseudo-inverse of B, defined as

$$B' = [B^T B]^{-1} B^T \tag{13}$$

The global control signal is therefore given by:

$$\underline{u}^g = -B'C\underline{x} \tag{14}$$

Hence, the optimal control law can be determined in a two-level scheme as follows:
First level: determine the local control signal $\underline{u}_i^l$ from eqns. (8)-(10)
Second level: calculate the global signal from eqns. (13) and (14).

A dynamical system which is built-up by the interconnection of simpler subsystems is termed weakly coupled when the degree of interaction between the constituent subsystems is not large. If the subsystems are in some sense weakly coupled, then the behavior of the system as a whole will be related to the behavior of that system which consists simply of the sum of the individual subsystems. Therefore, in the case of weakly coupled subsystems, the global signal need not be used [12].

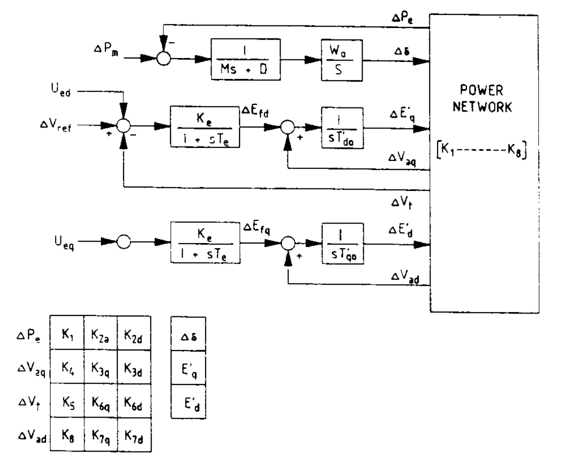

Fig. 2 Block diagram of machine i (Δ Va signal proportional to armature reaction)

4. APPLICATION TO A MULTI-MACHINE POWER SYSTEM

The proposed control strategies are tested on a typical multi-machine power system. In addition to the two-level optimal control strategy, the response of the system with PSS controls on both d and q axes were studied. The one-line diagram of a nine-bus three machine study system is shown in Figure 3 [13]. The system contains all the main features found in a general and complete multi-machine case. The system data are given in Appendix II.

4.1 Dual PSS

As mentioned, the PSS's designed for single machine-infinite bus system may not, generally, be satisfactory for multi-machine systems. DeMello presented a useful technique that could be used in coordinating the application of PSS's among the various machines in the system [14]. The general form of the PSS used in this study is given as

$$\frac{k_s s}{(1+sT_a)(1+sT_b)} \tag{15}$$

The input to the stabilizer, in both the d and q axes, is assumed to be the power deviation of the individual units in the system. Also, it is further assumed that all the generators have the provision of dual supplementary controls. The stabilizers were tuned based on the transfer function for each individual generating unit. A few samples of the dynamic response of the system for a 10% step change in input

power to machine 1 are given in Figures 4 through 8. Generator 1 is considered to be the reference machine. Figures 4 and 5 exhibit the rotor angle characteristics of machines 2 and 3 respectively with various controls. Without control, the system is dynamically stable. However, the response is quite oscillatory. Application of PSS control on direct and quadrature axes separately does improve the dynamic response. But a coordinated application of the control in both the axes provides a very smooth response as is evidenced in the recorded responses. Figures 6,7 and 8 show that the additional PSS on the q axis effectively controls the internal and field voltages of the various machines respectively to give desired dynamic performance.
It can be inferred from the case studies that the two supplementary PSS's assist each other in their effort to damp out system oscillations under disturbance conditions.

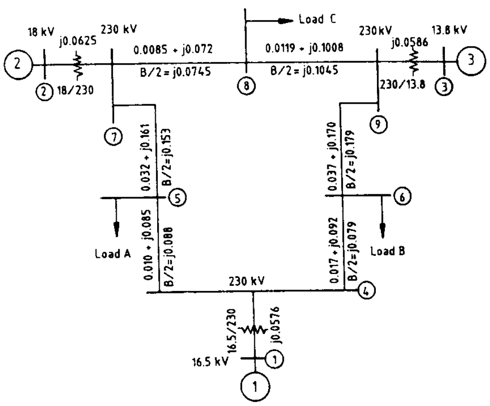

Fig. 3 One-line diagram of the study system.

4.2 Optimally-controlled dual excitation

The control scheme derived from the two-level strategy discussed in section 3 is then applied to the multi-machine system described in Figure 3. The overall system is decomposed into three subsystems. The state variables for the i[th] subsystem are

$$x_i = [\Delta\omega_i, \Delta\delta_i, \Delta E'_{qi}, \Delta E'_{e}, \Delta E_{fdi}, \Delta E_{fqi}]^T \quad (16)$$

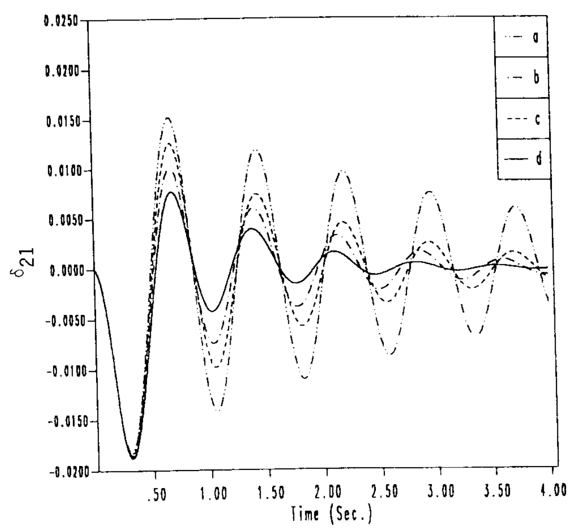

Fig. 4 Angle variation of unit #2,δ_{21} following a 10% step change in unit #1 input torque.
a- without control b- PSS control on d axis
c- PSS control on q axis d- PSS control on both axes

An optimal controller is designed for each subsystem, based on local information only, for a selected set of state and control weighting matrices. The global control signal was also calculated and the response compared with those obtained from considering only local information. The system response with the proposed controls for a 0.1 pu step change in the mechanical input torque of generator 1 is given in Figures 9 to 12. All the cases shown are with controls on both direct and quadrature axes. The dotted lines represent the response with local control strategy only while the solid lines are those with the two-level strategy.

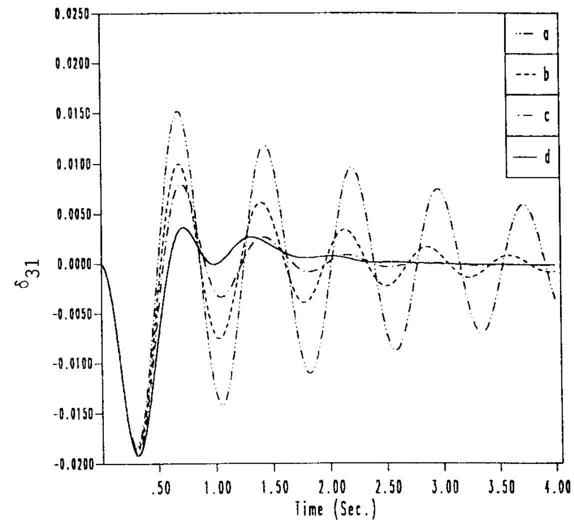

Fig. 5 Angle variation of unit #3, δ_{31}following a 10% step change in unit #1 input torque.
a- without control b- PSS control on d axis
c- PSS control on q axis d- PSS control on both axes

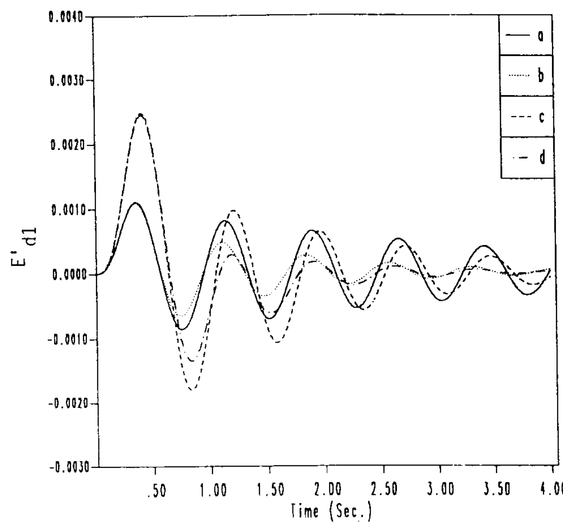

Fig. 6 Direct axis internal voltage variation of unit #1,E'_{d1} following a 10% step change in unit #1 input torque.
a- without control b- PSS control on d axis
c- PSS control on q axis d- PSS control on both axes

Figures 9 and 10 show the rotor angle behavior of machines 2 and 3. Comparison of responses with PSS controls given in Figures 4 and 5 (plot d) indicates that both the proposed optimum strategies provide much superior transient performance. The two-level strategy, as expected, gives a slightly better response over the local control alone. However, the response with the local control is definitely much improved compared to that with PSS control. The same conclusions can be arrived at from the examination of direct and quadrature axes voltage variations given in Figures 11 and 12. Although the local control is slightly inferior to the two-level strategy in terms of transient control, it has the advantage that it can be realized relatively at ease from measurements of local quantities only.

5. CONCLUSIONS

The effectiveness of controlled quadrature-axis excitation in enhancing damping of undesirable sustained oscillations experienced in multi-machine power systems has been investigated. A small-perturbation model for interconnected power systems with dual-axis excited synchronous machines has been developed. Two different control strategies have been proposed: the first scheme uses PSS's on both the direct and quadrature axes, the second one is based on a two-level optimal control approach. At the lower level of the two -level strategy, controllers are designed using local information only, while a global controller is designed at the higher level to minimize the effects of interactions.

Simulation results clearly indicate that dual-axis supplementary control provides a much improved response compared to control on the direct axis alone. Though both PSS's and optimal control strategies improve the transient performance, the optimal local and two-level controllers are much superior in terms of transient control. The optimal local control strategy has the additional advantage of being amenable to practical implementation since it requires only local measurements within each generating unit.

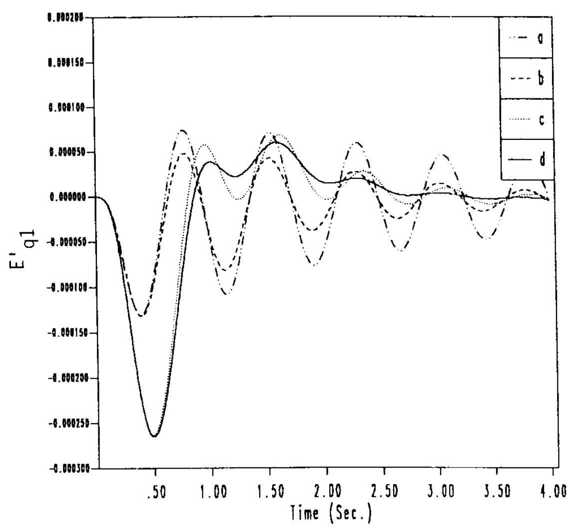

Fig. 7 Quadrature axis internal voltage variation of unit #1, E'_{q1} following a 10% step change in unit #1 input torque.
a- without control b- PSS control on d axis
c- PSS control on q axis d- PSS control on both axes

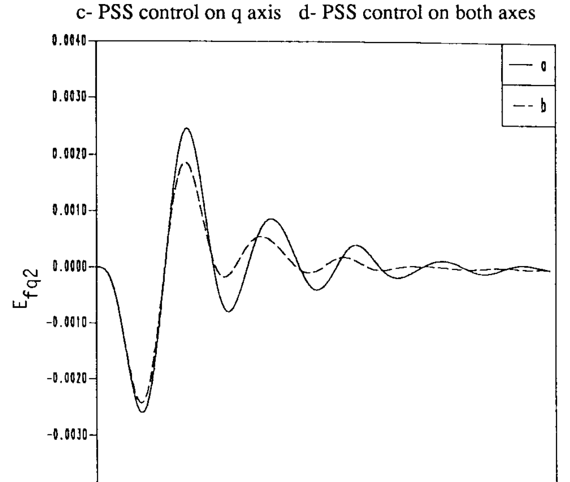

Fig. 8 Quadrature axis field voltage variation of unit #2, E_{fq2} following a 10% step change in unit #1 input torque.
a- PSS control on q axis b- PSS control on both axes

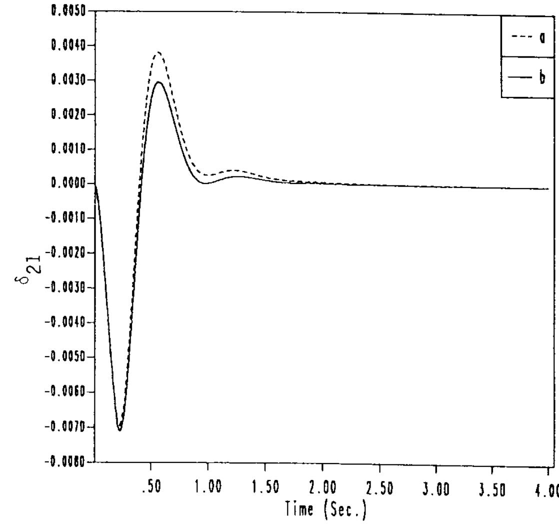

Fig. 9 Angle variation of unit #2, δ_{21} following a 10% step change in unit #1 input torque.
a- optimal local control b- optimal two-level control

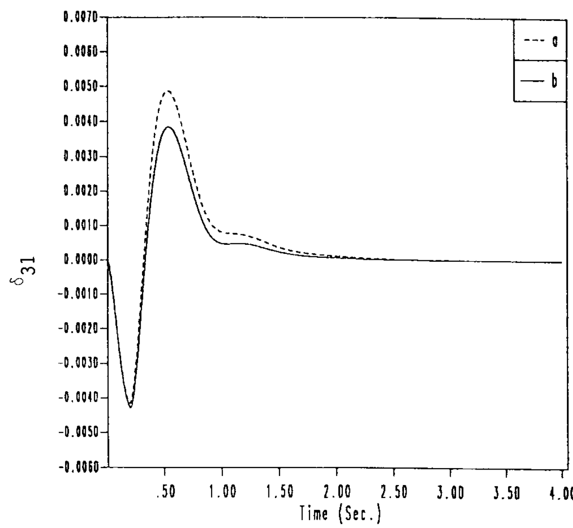

Fig. 10 Angle variation of unit #3, δ_{31} following a 10% step change in unit #1 input torque.
a- optimal local control b- optimal two-level control

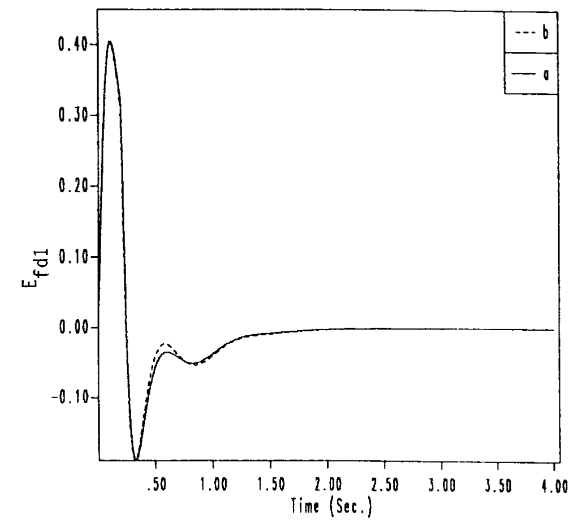

Fig. 11 Direct axis field voltage variation of unit #1,E_{fd1} following a 10% step change in unit #1 input torque.
a- optimal local control b- optimal two-level control

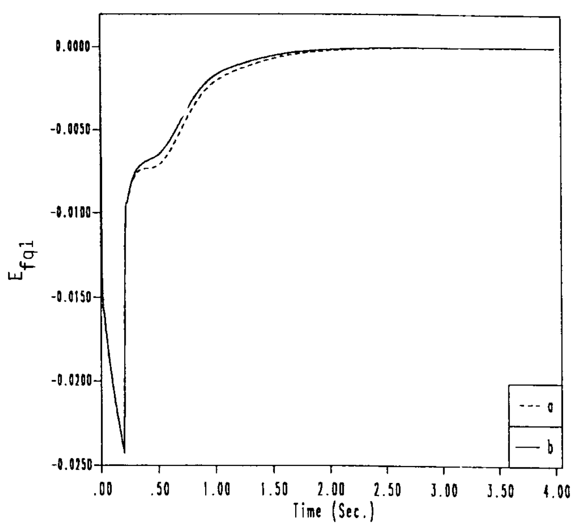

Fig. 12 Quadrature axis field voltage variation of unit #1,E_{fq1} following a 10% step change in unit #1 input torque.
a- optimal local control b- optimal two-level control

6. ACKNOWLEDGMENTS

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8. LIST OF SYMBOLS

v_d, v_q	terminal voltages on direct and quadrature axes of machines
i_d, i_q	direct and quadrature axes currents on machine
v_D, v_Q	network terminal voltage expressed with respect to network reference axes
i_D, i_Q	network terminal currents expressed with respect to network reference axes
G, B	real(imaginary) component of a network self or mutual admittance
δ	rotor angle of machine in electrical radians
E'_q	internal voltage on q-axis proportional to field flux linkage
E'_d	internal voltage on d-axis proportional to field flux linkage
x_d, x_q	direct(quadrature) axis synchronous reactance
x'_d, x'_q	direct(quadrature) axis transient reactance
ω_o	rated angular frequency, in electrical radian per second
T'_{do}, T'_{qo}	direct(quadrature) axis open-circuit time constant
M, D	inertia and damping coefficients
K_e, T_e	Exciter gain and time constant

E_{fd}, E_{fq}	direct(quadrature) axis field voltage
P_e	electrical power of synchronous machine
v_t	terminal voltage
Δ	incremental operator
$[\,]^T$	transpose

9. APPENDIX I

The linearized model of a multi-machine power system is derived by expanding the system variables around an operating point in the Parks two-axis reference frame. The following small-perturbation equations represent the basis for the construction of the linearized model:

$$[i_N] = [Y_N]\,[v_N], \quad [\Delta i_N] = [Y_N]\,[\Delta v_N]$$
$$[v_N] = [T]\,[v_m], \quad [\Delta v_N] = [T]\,[\Delta v_m] + [I]\,[\Delta\delta]$$
$$[i_m] = [T]^T\,[i_N], \quad [\Delta i_m] = [T]^T\,[\Delta i_N] + [J]\,[\Delta\delta]$$
$$[v_m] = [E'] - [X_r]\,[i_m], \quad [\Delta v_m] = [\Delta E'] - [X_r]\,[\Delta i_m]$$
$$[P] = [V]\,[i_m], \quad [\Delta P] = [V]\,[\Delta i_m] + [C]\,[\Delta v_m]$$
$$[\Delta v_t] = [VT]\,[\Delta v_m]$$
$$[\Delta \dot{E}_{fd}] = [KT]\,[\Delta v_t] + [TE]\,[\Delta E_{fd}] - [KT]\,[u_{ed}]$$
$$[\Delta \dot{E}_{fq}] = [TE]\,[\Delta E_{fq}] - [KT]\,[u_{eq}]$$
$$[\Delta P] = [K_1]\,[\Delta\delta] + [K_{2q}]\,[\Delta E'_q] + [K_{2d}]\,[\Delta E'_d]$$
$$[\Delta v_t] = [K_5]\,[\Delta\delta] + [K_{6q}]\,[\Delta E'_q] + [K_{6d}]\,[\Delta E'_d]$$
$$[i_N] = [i_{D1}, i_{Q1}, \ldots i_{Dn}, i_{Qn}]^T, \quad [v_N] = [v_{D1}, v_{Q1}, \ldots v_{Dn}, v_{Qn}]^T$$
$$[i_m] = [i_{d1}, i_{q1}, \ldots i_{dn}, i_{qn}]^T, \quad [v_m] = [v_{d1}, v_{q1}, \ldots v_{dn}, v_{qn}]^T$$
$$[v_t] = [v_{t1}, \ldots v_{tn}]^T, \quad [P] = [P_{e1}, \ldots P_{en}]^T$$
$$[E'] = [E'_{d1}, E'_{q1}, \ldots E'_{dn}, E'_{qn}]^T$$
$$[E_{fd}] = [E_{fd1}, \ldots E_{fdn}]^T, \quad [E_{fq}] = [E_{fq1}, \ldots E_{fqn}]^T$$
$$[\omega] = [\omega_1, \ldots \omega_n]^T, \quad [\delta] = [\delta_1, \ldots \delta_n]^T$$

Each generating unit is modeled by six first order differential equations. Selecting the state vector for the i[th] subsystem as in eqn. 16, the system coefficient matrix may be constructed as follows:

$$\begin{pmatrix} [D] & [M]\,[K_1] & [M]\,[K_2q] & [M]\,[K_2d] & 0 & 0 \\ [\omega_o] & 0 & 0 & 0 & 0 & 0 \\ 0 & [TD]\,[K_4] & [TD]\,[K_3q] & [TD]\,[K_3d] & [TD] & 0 \\ 0 & [TQ]\,[K_8] & [TQ]\,[K_7q] & [TQ]\,[K_7d] & 0 & [Tq] \\ 0 & [KT]\,[K_5] & [KT]\,[K_6q] & [KT]\,[K_6d] & [TE] & 0 \\ 0 & 0 & 0 & 0 & 0 & [TE] \end{pmatrix}$$

where $K_1..K_8$ are load-dependent $n \times n$ matrices defined as follows

$$[K_1] = [V]\,[\tilde{B}] - [C]\,[X_r]\,[\tilde{B}]$$
$$[K_{2q}] = \text{Even columns of } [[V]\,[\tilde{A}] + [C] - [C]\,[X_r]\,[\tilde{A}]]$$
$$[K_{2d}] = \text{Odd columns of } [[V]\,[\tilde{A}] + [C] - [C]\,[X_r]\,[\tilde{A}]]$$
$$[K_{3q}] = -[[U] + [X_D]\,[\alpha]]\,;\ [\alpha] = \text{Even columns and odd rows of } [\tilde{A}]$$
$$[K_{3d}] = -[[X_D]\,[\gamma]]\,;\ [\gamma] = \text{Odd columns and odd rows of } [\tilde{A}]$$
$$[K_4] = -[[X_D]\,[\beta]]\,;\ [\beta] = \text{odd rows of } [\tilde{B}]$$
$$[K_5] = -[[VT]\,[X_r]\,[\tilde{B}]]$$
$$[K_{6q}] = \text{Even columns of } [[VT] - [VT]\,[X_r]\,[\tilde{A}]]$$
$$[K_{6d}] = \text{Odd columns of } [[VT] - [VT]\,[X_r]\,[\tilde{A}]]$$
$$[K_{7d}] = [[-U] + [X_Q]\,[\mu]]\,;\ [\mu] = \text{Odd columns and even rows of } [\tilde{A}]$$
$$[K_{7q}] = [[X_Q]\,[\lambda]]\,;\ [\lambda] = \text{Even columns and even rows of } [\tilde{A}]$$
$$[K_8] = [[X_Q]\,[\eta]]\,;\ [\eta] = \text{even rows of } [\tilde{B}]$$

Other matrices are defined as follows ($i = 1, \ldots n$) :

$$[D] = diag\left(\frac{-D_i}{M_i}\right), \quad [M] = diag\left(\frac{-1}{M_i}\right), \quad [U] = diag(1.0)$$
$$[TD] = diag\left(\frac{1}{T'_{doi}}\right), \quad [TQ] = diag\left(\frac{1}{T'_{qoi}}\right), \quad [KT] = diag\left(\frac{-k_{ei}}{T_{ei}}\right)$$
$$[X_D] = diag(x_{di} - x'_{di}), \quad [X_Q] = diag(x_{qi} - x'_{qi}), \quad [TA] = diag\left(\frac{-1}{T_{ai}}\right)$$
$$[\omega_o] = diag(\omega_{oi}), \quad [TE] = diag\left(\frac{1}{T_{ei}}\right), \quad [TB] = diag\left(\frac{1}{T_{bi}}\right)$$

$$[\tilde{A}] = [\,[Y]\,[X_r] + [U]\,]^{-1}\,[Y], \quad [\tilde{B}] = [\,[Y]\,[X_r] + [U]\,]^{-1}\,[Z]$$
$$[Y] = [T]^T\,[Y_N]\,[T], \quad [Z] = [T]^T\,[Y_N]\,[I] + [J]$$

$$[Y_N] = \begin{pmatrix} G_{11} & -B_{11} & \cdots & G_{1n} & -B_{1n} \\ B_{11} & G_{11} & \cdots & B_{1n} & G_{1n} \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ G_{n1} & -B_{n1} & \cdots & G_{nn} & -B_{nn} \\ B_{n1} & G_{n1} & \cdots & B_{nn} & G_{nn} \end{pmatrix}$$

$$[X_r] = \begin{pmatrix} 0 & -x'_{q1} & \cdots & 0 & 0 \\ x'_{d1} & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & 0 & -x'_{qn} \\ 0 & 0 & \cdots & x'_{dn} & 0 \end{pmatrix}$$

$$[C] = \begin{pmatrix} i_{d1} & i_{q1} & \cdots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & i_{dn} & i_{qn} \end{pmatrix} \qquad [V] = \begin{pmatrix} v_{d1} & v_{q1} & \cdots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & v_{dn} & v_{qn} \end{pmatrix}$$

$$[I] = \begin{pmatrix} -v_{d1}\sin\delta_1 - v_{q1}\cos\delta_1 & \cdots & 0 \\ +v_{d1}\cos\delta_1 - v_{q1}\sin\delta_1 & \cdots & 0 \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ 0 & \cdots & -v_{dn}\sin\delta_n - v_{qn}\cos\delta_n \\ 0 & \cdots & +v_{dn}\cos\delta_n - v_{qn}\sin\delta_n \end{pmatrix}$$

$$[T] = \begin{pmatrix} \cos\delta_1 & -\sin\delta_1 & \cdots & 0 & 0 \\ \sin\delta_1 & \cos\delta_1 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & \cdots & \cos\delta_n & -\sin\delta_n \\ 0 & 0 & \cdots & \sin\delta_n & \cos\delta_n \end{pmatrix}$$

$$
[J] = \begin{pmatrix} -i_{D1}\sin\delta_1 + i_{Q1}\cos\delta_1 & \dots & 0 \\ -i_{D1}\cos\delta_1 - i_{Q1}\sin\delta_1 & \dots & 0 \\ \cdot & \dots & \cdot \\ \cdot & \dots & \cdot \\ 0 & \dots & -i_{Dn}\sin\delta_n + i_{Qn}\cos\delta_n \\ 0 & \dots & -i_{Dn}\cos\delta_n - i_{Qn}\sin\delta_n \end{pmatrix}
$$

When power system stabilizers of the type given in section 4.1 are used, the state vector of each machine will be given by:

$$
[\Delta\omega, \Delta\delta, \Delta E'_q, \Delta E'_d, \Delta E_{fd}, \Delta E_{fq}, \Delta w, \Delta u_e]^T
$$

The stabilized system matrix becomes:

$$
\begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & [KT] \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & [KT] \\ [KR]\,[F_1] & [KR]\,[F_2] & [KR]\,[F_3] & [KR]\,[F_4] & [KR]\,[F_5] & [KR]\,[F_6] & [TA] & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & [TB] & [-TB] \end{pmatrix}
$$

Where

$[F_1] = [K_1]\,[\omega_o]$

$[F_2] = [K_2q]\,[TD]\,[K_4] + [K_2d]\,[TQ]\,[K_8]$

$[F_3] = [K_2q]\,[TD]\,[K_3q] + [K_2d]\,[TQ]\,[K_7q]$

$[F_4] = [K_2q]\,[TD]\,[K_3d] + [K_2d]\,[TQ]\,[K_7d]$

$[F_5] = [K_2q]\,[TD]$, $\quad [F_6] = [K_2d]\,[TQ]$

$$
[KR] = diag\left(\frac{k_{si}}{T_{ai}}\right)
$$

10. APPENDIX II

<u>Generator parameters in p.u.</u>

Parameter	Gen #1	Gen #2	Gen #3
Type	hydro	steam	steam
Rated MVA	247.5	192.0	128.0
kV	16.5	18.0	13.8
Power factor	1.0	0.85	0.85
x_d	0.1460	0.8958	1.3125
x'_d	0.0608	0.1198	0.1813
x_q	0.0969	0.8645	1.2578
x'_q	0.0969	0.1969	0.25
x_l leakage	0.0336	0.0521	0.0742
$T'_{d0}\,[s]$	8.96	6.00	5.89
$T'_{q0}\,[s]$	1.0	0.535	3.1
$H(MW.S/100MVA)$	23.64	6.4	3.1

<u>Loads(kVA)</u>

load A = 125+j50
Load B = 90+j30
Load C = 100+j35

<u>Exciters and PSS's</u>

$K_{ei} = 50$

$T_{ei} = 0.02s$

$k_{si} = 0.2553$

$T_{ai} = 0.22s$

$T_{bi} = 0.6s$