Determination of first swing stability limit of multimachine power systems through Taylor series expansions

M.H. Haque, MSc, PhD
A.H.M.A. Rahim, PhD

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Abstract: A new technique of determining first swing stability limit of a power system through checking the existence of peaks of rotor angles of the severely disturbed generators in the postfault period is presented. The existence of peaks is checked by observing the roots of time derivatives of rotor angles of these generators expressed through truncated Taylor series expansions. The stability limit and hence the critical clearing time corresponds to the maximum value of fault duration for which at least one of the roots of the derivative of the angle is real positive. The proposed method has been tested on four different power networks and the critical clearing times obtained were found to be in very good agreement with those obtained by simulation of system dynamics. The method is computationally very fast compared with the existing methods.

1 Introduction

Determination of the limiting value of fault clearing time, for which the generators in a power system can maintain first swing stability, is a major topic in power system analysis. Conventionally this limiting value, termed critical clearing time, is computed through digital simulation of system dynamics. This is computationally unattractive especially for very large systems, leading to extensive research on direct determination of critical clearing time (CCT) over the last two decades.

One of the popular methods for the direct determination of CCT is the Liapunov's second (or direct) method. The direct method eliminates the repeated solution of large numbers of system differential equations to determine CCT for a particular fault on a system. Application of this method to power systems has been investigated exhaustively and reported in a number of publications [1–6]. The method is based on constructing a function V, called the Liapunov function, and determination of a stability region around the postfault stable equilibrium point such that the V function within this region is less than a threshold value, V_{cr}. If the postfault system states lie within this region, then the system is said to be stable. Earlier studies assumed V_{cr} to be the minimum value of V computed at all the possible unstable equilibrium points (UEPs). The estimated CCT was, in general, very conservative [4, 6]. It was recognised in the later studies that V_{cr} should be the value of V at a particular UEP called the controlling UEP for a given disturbance [7, 8]. The UEPs, in general, are to be computed through iterative procedures. The iterative scheme may sometimes not converge to the appropriate equilibrium point. Also, there is no general method of identifying the controlling UEP. Computation of V_{cr} is a major handicap in applying the second method of Liapunov to power systems which are relative large.

Kakimoto [9, 10] proposed an alternative method of determining the stability region by observing the sign of the derivative of kinetic energy component of Liapunov function evaluated along the faulted trajectory. The method does not involve the computation of UEP but gives the exact stability region when transfer conductances are neglected. The transfer conductances of the reduced admittance matrix to the generator internal buses are, in general, not negligible. In the presence of transfer conductances the method requires either a correction term or a modification of the Liapunov function. Several other techniques for determining the stability region using UEPs have also been reported in the literature [11, 12, 13]. But, none of these methods received much attention because of their conservative estimation of CCT.

Rahimi [14] and Xue [15] proposed very simple and efficient methods of direct determination of stability using the idea of the well-known 'equal area criterion' as applied to a single-machine infinite busbar (SMIB) system. The methods first decompose a multimachine system into two subsystems: one consisting of the critical machine(s) and the other of the remaining machines. The two subsystems are then transformed into an SMIB system. Generally speaking, the equivalent SMIB system will not represent the dynamics of the original system accurately, especially when the machines in the subsystems are not coherent.

Determination of CCT, for the faults which are severe in nature, are mainly of interest in power system planning studies. These are necessary in selecting the proper set of relays and circuit breakers such that a given fault is cleared in time without losing system stability. For severe faults, at least one of the generators in the system run out of step at the first swing if the system is unstable. The first swing stability limit may be determined by digital simulation of system dynamics or by direct methods. Unfortunately, all of these involve a great deal of computation.

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Dr. Haque is with the Department of Electrical Engineering, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia

A.H.M.A. Rahim is with the Department of Electrical Engineering and Computer Science, University of Bahrain, Isa Town, Bahrain

Effort should be directed towards finding a simple and computationally efficient method of determining stability limit so that it can be used not only in planning studies but in operation as well.

This paper proposes a method of determining first swing stability limit of a system, following a severe disturbance, by observing the behaviour of postfault rotor angle of the severely disturbed generators in the system. Whether the rotor angle of the generators will peak to a certain value (stable), increase or decrease monotonically (unstable) is determined by a simple check on the roots of a truncated Taylor series. The critical clearing time is determined from the limiting value of fault duration for which at least one root of the series is real positive. The proposed method is tested on four different power networks for a number of fault conditions and the results obtained agree satisfactorily with those obtained by digital simulation. The CPU times required by the proposed method are also comparable with some other reported methods.

2 Proposed method

The proposed method is based on the fact that, if a power system is first swing stable, the rotor angles of all the generators in the system, in centre of angle (COA) reference frame, will increase (decrease) until a peak (valley) is reached when the angle starts returning. In an unstable situation, the angle of one or more generators will monotonically increase (decrease). Mathematically, a system can be considered to be first swing stable if the following condition holds:

$$|\theta_i(t_i + \varepsilon)| < |\theta_i(t_i)| \tag{1}$$

for all the generators, where t_i (> clearing time) corresponds to the time at which the rotor angle of generator i reaches the peak and ε is a small positive number.

If a system is first swing stable, it is expected that the system damping, governor, etc. will aid stability in the subsequent swings [16]. Though exception may arise, if a system is first swing stable it is assumed that it will be stable. A method is presented in the following which determines the first swing stability of a system for a certain fault duration by checking eqn. 1. The method requires the computation of real roots of the derivative of rotor angles express through Taylor series expansions given below.

2.1 Power system model

Consider an n-machine power system. The second order dynamics of the ith machine in COA reference frame can be expressed by the following differential equations [6]:

$$\dot{\theta} = \omega_i$$
$$\dot{\omega}_i = \left[P_i - P_{ei} - \frac{M_i}{M_T} P_{COA}\right] \Big/ M_i \tag{2}$$

The expressions for P_i, P_{ei}, P_{COA} and M_T, respectively, are given by

$$P_i = P_{mi} - E_i^2 G_{ii}$$
$$P_{ei} = \sum_{\substack{j=1 \\ \neq i}}^{n} C_{ij} \sin\theta_{ij} + D_{ij} \cos\theta_{ij}$$
$$P_{COA} = \sum_{i=1}^{n} P_i - 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} D_{ij} \cos\theta_{ij} \tag{3}$$
$$M_T = \sum_{i=1}^{n} M_i$$

Here, $C_{ij} = E_i E_j B_{ij}$, $D_{ij} = E_i E_j G_{ij}$; E_i, θ_i, M_i and P_{mi} are the internal voltage, rotor angle, inertia constant and input power, respectively, of the ith machine. G_{ij} and B_{ij} are the real and imaginary parts, respectively, of the reduced admittance matrix between busbars i and j.

The generators are represented by a constant voltage source behind a transient reactance; the input mechanical power is assumed to remain constant and the loads are modelled by constant shunt admittances. Higher order generator model, voltage dependent load models and inclusion of dynamics of the associated generator controls, e.g. exciter and stabiliser, will increase the complexity of the dynamics, particularly when a large number of machines is present in a system. It is to be noted that most of the recent works on transient stability of multimachine system through 'transient energy function' method are based on this model [17].

2.2 Prediction of generator rotor angles through Taylor series expansion

The faulted system states (θ_i, ω_i) of the ith machine in COA reference frame can be expressed by the following Taylor series expansions:

$$\theta_i(t) = \theta_i(0) + \theta_i^{(1)} t + \theta_i^{(2)} \frac{t^2}{2!} + \theta_i^{(3)} \frac{t^3}{3!} + \cdots$$
$$\omega_i(t) = \theta_i^{(1)} + \theta_i^{(2)} t + \theta_i^{(3)} \frac{t^2}{2!} + \theta_i^{(4)} \frac{t^3}{3!} \cdots \tag{4}$$

Where $\theta(0)$ is the prefault angle and $\theta^{(m)}$, $m = 1, 2, 3, \ldots$ is the mth derivative of rotor angle evaluated at $t = 0$ using faulted network parameters. If the fault is cleared at $t = t_{cl}$ the postfault rotor angle of the ith machine can be expressed in terms of another series:

$$\theta_i(t_1) = a_{i0} + a_{i1} t_1 + a_{i2} t_1^2 + a_{i3} t_1^3 + \cdots \tag{5}$$

Where $t_1 = t - t_{cl}$, $a_{i0} = \theta_i(t_{cl})$, $a_{i1} = \omega_i(t_{cl})$ and $a_{im} = \theta_i^{(m)}/m!$, $m = 2, 3, \ldots$. The a coefficients are to be evaluated at $t = t_{cl}$ using postfault network parameters. The expressions for the coefficients of faulted as well as postfault system are given in Appendix 7. The coefficients a_{i0} and a_{i1} correspond to the angle and speed, respectively, at the instant of fault clearing and these can be obtained by substituting t_{cl} for t in eqn. 4. Ribbens-Pavella used the synchronous reference frame to find the rotor angle and frequency deviation of the faulted system only [5].

The authors showed in a recent article that eqns. 4 and 5 can predict the system states fairly accurately for a duration of 0.4–0.5 s, considering terms up to 4th order derivatives [18]. However, if prediction of system states for longer duration is desired, a second or multistep Taylor series expansion in both the fault as well as post-fault period may be used, if necessary [19].

2.3 Identification of severely disturbed generators

Normally, for a given disturbance, only a few generators in the system are severely disturbed and these are, in general, responsible for the first swing instability. Thus stability can be determined by checking eqn. 1 only for these severely disturbed generators (SDGs). Fortunately for stable situation, the peak of rotor angle of the SDGs in many systems will occur within a very short period of time after the fault has been cleared. For this relatively small period, the rotor angle can easily be predicted by the truncated Taylor series of eqn. 5 considering only a few terms in the series. A typical variation of rotor angles (swing curves) of generators 2, 5, 9, 10 and 16 of the IEEE test system (Fig. 2 in Section 3) are shown in Fig. 1. The swing curves are generated by numerical integration of system dynamics given in eqn. 2 for a three-phase fault

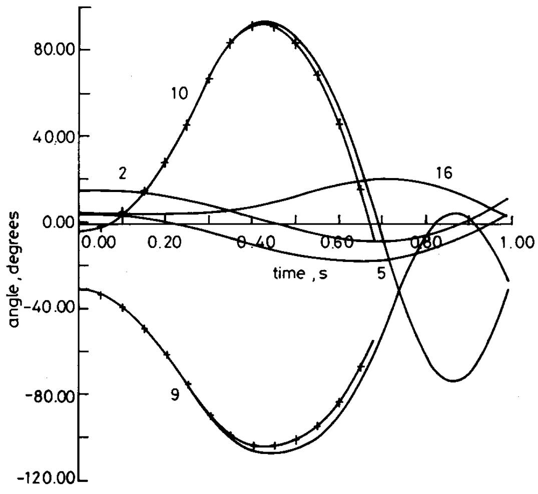

Fig. 1 *Swing curves of generators 2, 5, 9, 10 and 16 for a three-phase fault on busbar 49 of the IEEE system*
—— digital simulation
×—× Taylor series expansion

on bus 49 cleared at 0.29 s. The critical clearing time for this fault as obtained by simulation is 0.31–0.32 s. Fig. 1 shows that generators 9 and 10 swing very fast compared to some others (generators 2, 5, 16 etc.) and the peaks appear in less than 0.20 s after the fault is cleared. For a fault of 0.32 s, generator 9 runs out of step first and thus it is responsible for the first swing instability. It may also be observed from Fig. 1 that the peaks of the less disturbed generators (2, 5, 16 etc.) occur at a much later time. A single step TSE of eqn. 5 in the postfault period may not be able to predict the existence of peaks correctly for such a long duration. Since these generators are, in general, not responsible for first swing instability, they may be excluded in the stability check without affecting the results. A procedure to identify the most severely disturbed generators is proposed in the following:

(i) Compute $J_i = |\theta_i(t) - \theta_i(0)|$ for $i = 1, 2, \ldots, n$. Here, the angle $\theta_i(t)$ is the faulted angle at any arbitrary time $t(\leq t_{cl})$ and $\theta(0)$ is the prefault angle. $\theta_i(t)$ is calculated from the faulted Taylor series expansion given by eqn. 4.

(ii) Find the maximum value of J corresponding to the kth generator, say J_k^*.

(iii) Define generator j to be one of the severely disturbed if

$$J_j/J_k^* > \sigma \tag{6}$$

is satisfied. Here σ is a predetermined quantity.

2.4 Stability determination

As mentioned, a power system can be considered to be first swing stable if eqn. 1 is satisfied for only the severely disturbed generators. The stability criterion (eqn. 1) or the existence of peak of post fault rotor angles can be checked either by computing the rotor angles from eqn. 5 or by observing the roots of the time derivative of rotor angle given in eqn. 5. If there exists a real positive root of $\dot{\theta}(t_1) = 0$, this guarantees that the predicted rotor angle has a peak value. The system may be considered to be stable if this is true for all the severely disturbed generators. However, if there is no real root or all the roots are negative, it implies that the predicted rotor angle of the corresponding generator does not have a peak and that the system is unstable.

A question may arise here that how many terms should be considered in eqn. 5 to predict the nature of the postfault rotor angle fairly accurately. It might appear though, that inclusion of more and more terms in eqn. 5 will give better estimates of the nature of rotor angle variation for longer duration, but because of the oscillatory nature of power swings, this is not so. In fact, it has been observed that a third or fourth order expansion gives better estimates compared to the higher order ones [18]. If a higher degree of accuracy is desired, it can be obtained by updating the a coefficients more frequency, say every 0.20 s, if necessary.

Fig. 1 also shows the swing curve of severely disturbed generators (9 and 10) generated through the proposed multistep Taylor series expansions. It is assumed that 4th order expansion for a duration of 0.25 s in the faulted period and 3rd order expansion for a duration of 0.20 s in the postfault period can predict the system states fairly accurately. It can be observed from Fig. 1 that the predicted angle are very close to the actual values (obtained by digital simulation). Thus the stability of the system may be determined from the truncated Taylor series given by eqn. 5.

2.4.1 Third order expansion

Consider the truncated Taylor series expansion of eqn. 5 having only up to third order derivatives

$$\theta(t_1) = a_0 + a_1 t_1 + a_2 t_1^2 + a_3 t_1^3 \tag{7}$$

The time derivative of this equation when set to zero gives

$$\dot{\theta}(t_1) = a_1 + 2a_2 t_1 + 3a_3 t_1^2 = 0 \tag{8}$$

The roots of eqn. 8 which correspond to the time t_p at which the postfault rotor angle has a peak are given as

$$t_p = \frac{-2a_2 \pm \sqrt{(4a_2^2 - 12a_1 a_3)}}{6a_3} \tag{9}$$

If the roots are negative or complex for any of the SDGs then the system is considered to be unstable. However, existence of a real positive root for all the SDGs indicates a stable system.

2.4.2 Fourth and higher order expansions

The existence of peaks can also be determined by considering terms up to fourth order derivatives in eqn. 5. The time derivative of $\theta(t)$ when set to zero gives

$$a_1 + 2a_2 t_1 + 3a_3 t_1^2 + 4a_4 t_1^3 = 0 \tag{10}$$

Eqn. 10 can have either one or three real roots. Computation of complex roots is not necessary in this study. The expression for the real roots of eqn. 10 can be written as [20]

$$(-1)^j 2\sqrt{\left(\frac{q}{3}\right)} \cos\left[\frac{\phi + (j-2)\pi}{3}\right] - \frac{a_3}{4a_4}; \quad j = 1, 2, 3 \tag{11}$$

where

$$\phi = \cos^{-1}\left[(3/q)^{3/2}\,\frac{r}{2}\right]$$

$$q = \frac{3a_3^2}{16a_4^2} - \frac{a_2}{2a_4} \qquad (12)$$

$$r = \frac{a_2 a_3}{8a_4^2} - \frac{a_3^3}{32a_4^3} - \frac{a_1}{4a_4}$$

Eqn. 11 is valid when $27r^2 - 4q^3 < 0$. Otherwise, only one root of eqn. 10 is real and is given as

$$2\ \mathrm{sgn}\,(r)\sqrt{\left(\frac{q}{3}\right)}\cosh\left(\frac{\phi}{3}\right) - \frac{a_3}{4a_4} \qquad (13)$$

Here

$$\phi = \cosh^{-1}\left[(3/q)^{3/2}\,\frac{r}{2}\right], \quad q > 0$$

For $q < 0$, cosh terms are replaced by sinh and q is replaced by $-q$.

When higher order terms are included in eqn. 5, derivative of $\theta(t)$ set to zero does not give any closed form solution. In this case, an iterative solution is required. As mentioned earlier, the inclusion of higher-order terms may not necessarily give better predictions.

2.5 Algorithm for computation of CCT

The computational procedure involved in computing the critical clearing time is given in the following:

(*i*) Obtain the Taylor series expansions of eqn. 4 for the faulted rotor angle and speed deviation.

(*ii*) Identify the severely disturbed generators using eqn. 6.

(*iii*) Select a clearing time t_{cl}. Start with a reasonably smaller value.

(*iv*) Compute the coefficients of the post fault Taylor series expansion a_1, a_2, etc. of eqn. 5 for the severely disturbed generators.

(*v*) Determine the stability of the SDGs using the roots given by eqns. 9 or 11 and 13 corresponding to third or fourth order expansions.

(*vi*) If the system is stable, increase the clearing time t_{cl} by a suitable increment and return to step iv. If one of the SDGs has been idenfitied to be unstable, record the time t_{cl}. This is the critical clearing time.

3 Simulation results

The proposed method of estimating the critical clearing time through prediction of postfault swing curves was tested on four different power networks for a number of fault conditions in each network. The systems studied were:

(*i*) The 20-machine IEEE system
(*ii*) The 17-machine Iowa system
(*iii*) The 10-machine New England system
(*iv*) The 7-machine CIGRE system.

In all the systems, the severely disturbed generators were identified by using the criterion on angle given in eqn. 6 where J_i was calculated to be $|\theta_i(0.25\ \mathrm{s}) - \theta_i(0)|$. From a number of studies, a unified value of $\sigma = 0.70$ was selected for all the systems. The critical clearing time obtained by the proposed method were compared with the actual values obtained by digital simulation of the corresponding system dynamics. A comparison of the computational times was also made.

3.1 IEEE Test system

The IEEE test system consists of 20 machines and 118 busbars. The single-line diagram is shown in Fig. 2. Unit numbers (encircled) are shown on the diagram, units not assigned any number have been represented by fixed impedances. The system data are given in reference [21]. Faults at five different locations were considered in this system. Table 1 gives a listing of the prefault angles, faulted angles at $t = 0.25$ s and the angle deviation J_i for each machine in the system for a 3-phase fault on busbar

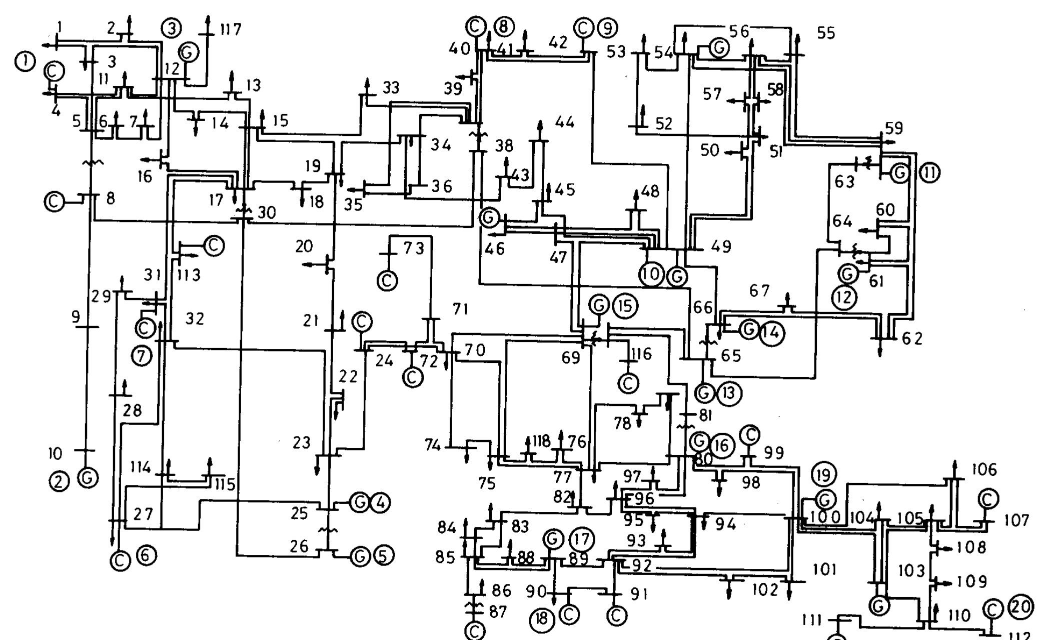

Fig. 2 *Single-line diagram of the IEEE system*

Table 1: Generator rotor angles and deviations for a three-phase fault on busbar 49 of IEEE system

Gen. no.	Prefault angle	Faulted angle at $t_{cl} = 0.25$ s	Angle deviation J_i
1	−20.69	−27.66	7.07
2	14.67	9.34	5.31
3	−15.00	−23.02	7.02
4	3.25	−2.75	6.00
5	3.41	−2.61	6.02
6	−20.57	−28.13	7.55
7	−20.38	−30.46	8.88
8	−30.83	−59.80	28.96
9	−30.20	−73.43	43.23
10	−3.83	45.63	49.46
11	−5.36	5.51	10.87
12	−0.83	7.51	8.34
13	5.40	15.70	10.30
14	5.52	25.60	20.08
15	7.54	13.40	5.86
16	4.42	4.45	0.03
17	17.76	12.15	5.60
18	−7.46	−13.10	5.64
19	2.57	−1.92	4.49
20	−22.66	−30.50	7.83

49. It can be seen that generators 9 and 10 have angle deviations of 43.23° and 49.46°, respectively, and satisfy the angle criterion (eqn. 6). Hence, these can be considered to belong to the severely disturbed generators set.

The results of the stability (or root) test applied to these generators for different clearing time t_{cl} (for different post-fault Taylor series coefficients) are given in Table 2 for a third order expansion. It can be observed that the

Table 2: Roots of eqn. 8 for severely disturbed generators for a three-phase fault on busbar 49 of IEEE system

Clearing time, s	Roots of eqn. 8	
	Generator 9	Generator 10
0.25	0.081, −4.680	0.095, −1.352
0.27	0.093, 1.20	0.108, −12.0
0.29	0.120, 0.407	0.130, 1.280
0.30	0.179, 0.216	0.151, 0.721
0.31	$0.155 \pm j0.092$	0.192, 0.425
0.32	$0.125 \pm j0.112$	$0.234 \pm j0.107$

roots are real and at least one positive up to $t_{cl} = 0.30$ s. At $t_{cl} = 0.31$ s, the roots for generator 9 become complex followed by generator 10 at the next instant. From this it can be concluded that the critical clearing time is in the range 0.30–0.31 s. However, a more precise value can be arrived at by taking smaller intervals of time.

The estimates of CCT obtained by the proposed method and by digital simulation for all the faults studied in this system are given in Table 3. Comparison of the

Table 3: Summary of results for IEEE system

Fault location Busbar no.	Line tripped between buses	Severely disturbed generators	CCT, s obtained by	
			proposed method	digital simulation
49	49–66	9, 10	0.30–0.31	0.31–0.32
25	25–26	4	0.27–0.28	0.27–0.28
61	61–64	12	0.44–0.45	0.43–0.44
65	65–66	13	0.33–0.34	0.33–0.34
59	59–63	11	0.39–0.40	0.39–0.40

results clearly indicates that the critical clearing times obtained by the proposed method have excellent agreement with the digitally simulated ones. The maximum error was found to be 0.01 s. Table 4 shows the CPU time required to compute CCT by the proposed method and by digital simulation of system dynamics. In digital simulation a fourth order Runge-Kutta method was

Table 4: CPU time required to compute CCT for IEEE system using IBM 3090 computer

Fault location busbar no.	CPU time, s required by	
	proposed method	digital simulation
49	1.04	47.30
25	0.86	28.38
61	1.65	113.52
65	1.15	66.20
59	1.32	94.60

employed to solve the sets of eqns. 2 and 3 up to a period of 1 s. The CPU time required for one simulation was approximately 9.46 s. For a fault on busbar 49, five estimates of clearing times give a total CPU time of 47.3 s, while the time required by the proposed method for the five attempts is only 1.04 s. It is to be noted that both the methods (the proposed and digital solution through Runge-Kutta method) required calculation of reduced admittance matrix. The CPU times shown in Table 4 do not include the time required for computing reduced admittance matrix in either of the methods.

3.2 Iowa system

This system consists of 17 machines, 163 busbars and 284 lines which represents the reduced power network of the state of Iowa. The system data and the initial operating conditions are given in Reference 22. Faults at four different locations were considered in this system. These faults were previously studied and reported in References 8 and 22. In these reports, complex modes of instability were encountered where more than one generator was running out of step at the controlling unstable equilibrium point. This happened for all the cases considered. Since the proposed method does not involve the identification and iterative computation of the controlling unstable equilibrium point, it determines the critical clearing time very quickly and reliably. Table 5 shows the estimated CCT

Table 5: Summary of results of Iowa system

Fault location busbar no.	Line tripped between buses	Severely disturbed generators	CCT, s obtained by	
			proposed method	digital simulation
372	372–193	6	0.20–0.21	0.19–0.20
436	436–771	10, 12	0.21–0.22	0.20–0.21
773	773–779	10, 12, 16	0.36–0.37	0.35–0.36
6	6–439	2, 17	0.23–0.24	0.22–0.23

obtained by the proposed method along with those from digital simulation. Comparison of the results indicates that the estimated critical clearing times obtained by the proposed method are slightly pessimistic. However, the deviations are within 0.01 s.

The CPU times required to compute CCT by the proposed method and by direct method as suggested by Fouad [8, 22, 23], for all the faults studied in this sytem, are given in Table 6. For these faults the controlling generators were considered to be the same as those identified in Reference 8. The Davidon-Fletcher-Powell (DFP) iterative method was used to compute the controlling UEP. Table 6 shows that the proposed method is very fast compared to the direct method. Also the method has no

Table 6: CPU time, s, required in computing CCT of Iowa system using IBM 3090 computer

Fault location busbar no.	CPU time required by	
	proposed method	direct method
372	0.53	19.34
436	0.54	21.57
773	0.90	22.49
6	0.62	22.11

convergence problems because it does not involve any iteration.

3.3 New England system

The single line diagram of the 10 machine, 39 busbar New England system is shown in Fig. 3. The system data

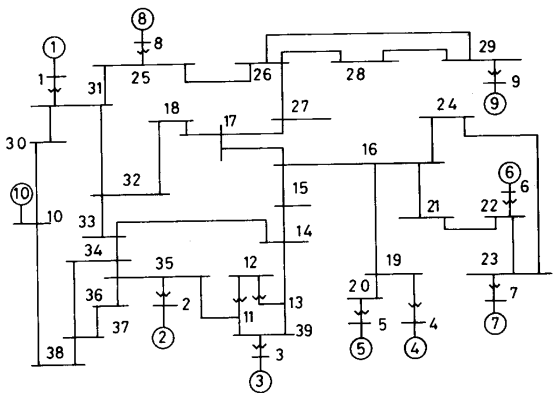

Fig. 3 *Single line diagram of the New England system*

are given in Reference 21. This system represents the 345 kV bulk transmission network of New England. Generator 10 is an equivalent power source representing parts of the US–Canadian interconnection system, having an inertia constant of $H = 500$. The inertia constants of all other generators in the system are within 42 s. A summary of results obtained for various fault studies on the system is given in Table 7.

Table 7: Estimation of CCT for New England system

Fault location busbar no.	Line tripped between buses	Severely disturbed generators	CCT, s, obtained by	
			proposed method	digital simulation
35	35–11	2, 3	0.22–0.23	0.21–0.22
15	15–16	4, 6, 7	0.21–0.22	0.22–0.23
39	39–13	3	0.24–0.25	0.22–0.23
26	26–27	9	0.14–0.15	0.13–0.14
5	none	5	0.20–0.21	0.19–0.20
2	none	2	0.23–0.24	0.23–0.24

Because of the large value of the inertia constant of generator 10, the rotor angles in the center of angle reference frame vary slowly. For most of the faults studied, the peak value of rotor angles for the SDGs occurs at time greater than 0.20 s. That is, for a stable situation the minimum value of real positive root(s) of eqn. 8 is greater than 0.20 s. A single-step Taylor series expansion in the post-fault period for such a long time may not be adequate. Thus a second-step Taylor series expansion in the post-fault period was used to predict the nature of rotor angle correctly.

3.4 CIGRE system

The single line diagram of the 7 machine, 11 busbar CIGRE system is shown in Fig. 4. The system data are

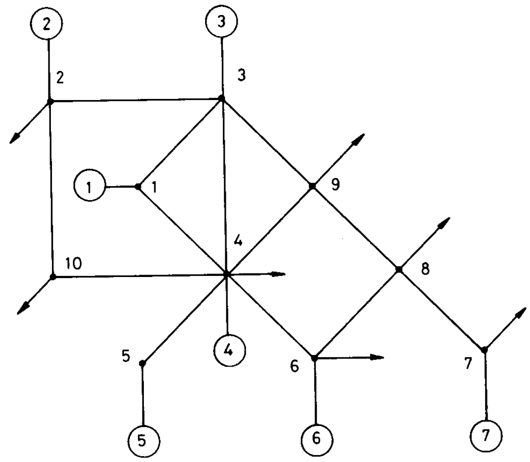

Fig. 4 *Single line diagram of the CIGRE system*

given in Reference 6. Three phase fault at six different locations were considered. The estimates of CCT obtained by the proposed method and by digital simulation are given in Table 8. There is a very good agreement

Table 8: Estimation of CCT for CIGRE system

Fault location busbar no.	Line tripped between buses	Severely disturbed generators	CCT, s, obtained by	
			proposed method	digital simulation
1	1–3	1	0.33–0.34	0.34–0.35
2	2–3	2	0.38–0.39	0.39–0.40
3	3–9	3	0.37–0.38	0.39–0.40
6	6–8	6	0.46–0.47	0.47–0.48
5	none	5	0.34–0.35	0.35–0.36
7	none	7	0.34–0.35	0.33–0.34

between the results, the maximum error was found to be 0.02 s, for a fault on bus 3.

4 Conclusions

A novel method of estimating critical clearing time for a fault in multimachine power system is proposed. The stability or instability of the system for a certain fault is obtained by examining the roots of truncated Taylor series expansions. The critical clearing time corresponds to the fault duration for which the roots of the derivative of rotor angle for the severely disturbed generators cease to be positive real.

The determination of stability by the proposed method is very simple. It requires calculation of few coefficients which depend on quantities such as angle, speed, power, etc. of the post-fault system. Compared to the existing methods of calculating critical clearing times (simulation of system dynamics and Liapunov's method, etc.) the proposed method is extremely fast in terms of computation. Comparison with the standard techniques shows that the estimates from the proposed method are very reliable. The maximum error, for a number of fault conditions, was found to be of the order of 0.02 s. The proposed method may be considered to be an additional tool for determining stability, particularly in planning studies where quick determination of CCT may be of interest.

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7 Appendix

7.1 Coefficients of Taylor series expansion

The expressions for $\theta_i^{(m)}$, mth derivative of θ_i, are derived from swing equation (eqn. 2) which can be rewritten as follows:

$$\theta_i^{(2)} = \frac{1}{M_i}\left[P_{mi} - \sum_{j=1}^{n}(C_{ij}\sin\theta_{ij} + D_{ij}\cos\theta_{ij}) - \frac{M_i}{M_T}\sum_{k=1}^{n}P_k + 2\frac{M_i}{M_T}\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}D_{kj}\cos\theta_{kj}\right] \quad (14)$$

Define

$$\begin{aligned} A_{ij} &= (C_{ij}\sin\theta_{ij} + D_{ij}\cos\theta_{ij})/M_i \\ B_{ij} &= (-C_{ij}\cos\theta_{ij} + D_{ij}\sin\theta_{ij})/M_i \\ F_{ij} &= 2D_{ij}\cos\theta_{ij}/M_T \\ H_{ij} &= 2D_{ij}\sin\theta_{ij}/M_T \end{aligned} \quad (15)$$

The time derivatives of A, B, F and H are given by

$$\begin{aligned} \frac{d(A_{ij})}{dt} &= -B_{ij}\theta_{ij}^{(1)}; \quad \frac{d(B_{ij})}{dt} = A_{ij}\theta_{ij}^{(1)} \\ \frac{d(F_{ij})}{dt} &= -H_{ij}\theta_{ij}^{(1)}; \quad \frac{d(H_{ij})}{dt} = F_{ij}\theta_{ij}^{(1)} \end{aligned} \quad (16)$$

The second and higher derivatives of θ can be obtained from eqn. 14, as can its successive differentiation with regard to time. These derivatives of θ in terms of the parameters A, B, F and H are given as follows:

$$\begin{aligned} \theta_i^{(2)} &= \frac{P_{mi}}{M_i} - \sum_{j=1}^{n}A_{ij} + d_1 \\ \theta_i^{(3)} &= \sum_{j=1}^{n}B_{ij}\theta_{ij}^{(1)} + d_2 \\ \theta_i^{(4)} &= \sum_{j=1}^{n}[A_{ij}\theta_i^{(1)2} + B_{ij}\theta_{ij}^{(2)}] + d_3 \end{aligned} \quad (17)$$

where

$$\begin{aligned} d_1 &= -\sum_{k=1}^{n}\frac{P_k}{M_T} + \sum_{k=1}^{n-1}\sum_{j=k+1}^{n}F_{kj} \\ d_2 &= -\sum_{k=1}^{n}\sum_{j=k+1}^{n}H_{kj}\theta_{kj}^{(1)} \\ d_3 &= -\sum_{k=1}^{n-1}\sum_{j=k+1}^{n}[F_{kj}\theta_{kj}^{(1)2} + H_{kj}\theta_{kj}^{(2)}] \end{aligned} \quad (18)$$

and

$$\theta_{ij}^{(m)} = \theta_i^{(m)} - \theta_j^{(m)}$$