# STABILIZING CONTROLS FOR A DOUBLY FED SYNCHRONOUS-INDUCTION MACHINE

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**Abstract**

The stability of a doubly-fed synchronous-induction machine is investigated through small as well as large perturbation models. Two stabilizing control strategies have been proposed for the small perturbation bilinear model. The first is based on quadratic bilinear regulator theory and the other one provides a sub-optimal control based on minimization of time. The responses are compared with the uncontrolled ones as well as those with conventional linear regulator control. While both the proposed strategies stabilize the system, the minimum-time sub-optimal control has been found to be superior. It has been demonstrated that the minimum-time control can be applied to the large perturbation models also.

## 1. INTRODUCTION

When polyphase excitation is applied to both the rotor and stator of a wound rotor machine it may be made to run synchronously at a speed equal to the sum or difference of supply frequencies depending on the phase sequence of the supply. The special feature of this machine is that a very high torque can be obtained from a small frame size at speeds up to double the synchronous. The machine can be made to generate also by adjusting the excitation.

The doubly fed machine offers an excellent energy conversion device but its use has been very limited because of the problem associated with its starting and also its inherent lack of damping [1]. The total torque of the machine is comprised of three components - the synchronous, stator excited induction and rotor excited induction torques. The rotor excited torque component is always negative and is responsible for the negative damping [2]. Schmitz and Bird [3,4] were some of the early researchers on the analysis and stabilization of doubly fed machines. A recent paper presented a rigorous mathematical description of the machine in the generation mode [5]. Using the general relationships of the polyphase induction motor, Ohi et al [2] derived a linear model of the machine in terms of the components of supply voltage along d and q axes. Stabilization of the variable speed machine through additional speed feedback was considered.

This work shows that the small perturbation model of the doubly fed machine is bilinear with a pair of roots in the right half plane for most of the operating range. Thus the linear theory, as such, cannot be used for analyzing this system. A method of arriving at stabilizing control for such bilinear systems was proposed by Gutman [6]. The method involves iterative calculation and hence is handicapped for online application.

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In this paper, stabilization of the doubly fed machine has been proposed by control of stator and rotor voltages. A simple quadratic control strategy for the bilinear model has been suggested basing on Gutman's work. Also a sub-optimum minimum-time feedback strategy has been used for stabilization. This scheme has been derived considering both the linear and nonlinear models. Responses with the above two strategies (quadratic and minimum-time) were compared with those obtained through linear state regulator control. The response with the minimum time control strategy was found to be superior. Preliminary results were reported by the author in previous publications [7,8].

## 2. THE DYNAMIC MODEL OF A DOUBLY FED MACHINE

Extending the equivalent two phase model of a three phase induction motor presented by Krause [9] and incorporating the second order electromechanical dynamics, the equations of the doubly fed machine can be expressed in the following state form [2]

$$\dot{x} = f[x, V_{ds}, V_{dr}, V_{qs}, V_{qr}] \tag{1}$$

where x is a 6x1 state vector of direct and quadrature axes stator and rotor currents, the rotor speed ($\omega_r$), and the torque angle ($\delta$). $V_{ds}$, $V_{dr}$, $V_{qs}$, $V_{qr}$ are the input voltages to the direct and quadrature axes (d,q) stator and rotor voltages (s,r) respectively. The detailed equations are given in Appendix A.

In the doubly fed mode the rotor voltages will have a phase sequence opposite to that of the stator to fit the signs assumed in equation (1). The component voltages are written in the form

$$\begin{aligned} V_{ds} &= V_s \cos \omega_e t, \quad V_{qs} = V_s \sin \omega_e t \\ V_{dr} &= V_r \cos(s\omega_e t - \delta_o), \; V_{qr} = -V_r \sin(s\omega_e t - \delta_o) \end{aligned} \tag{2}$$

where $V_s$ and $V_r$ are the applied voltages to the stator and rotor, $\omega_e$ is the frequency of the supply voltage and $\delta_o$ is the initial angle.

### Small Perturbation Linear State Model

If both the stator and rotor are fed from a supply frequency $\omega_e$, the motor will run at an average speed of $2\omega_e$. Considering small perturbation $\Delta\omega_r$ from the rated frequency $2\omega_e$, the slip of the machine can be expressed as

$$s = 1 - \omega_r/\omega_e = -1 - \Delta\omega_r/\omega_e \tag{3}$$

Defining $\Delta\dot{\delta} = \Delta\omega_m$ and $\delta = \delta_o + \Delta\delta$, the rotor voltages in equation (2) can be expressed as

$$V_{dr} = V_r \cos(-\omega_e t - \delta), \quad V_{qr} = -V_r \sin(-\omega_e t - \delta) \tag{4}$$

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The reversal of the phase of the rotor voltages for negative slip condition requires defining a new torque angle $\delta = \delta_{old} - \pi$, where $\delta_{old}$ is the angle $\delta$ considered so far. For the synchronously rotating reference frame, the component voltages are

$$V_{ds} = V_s,\ V_{qs} = 0,\ V_{dr} = V_r \cos\delta,\ V_{qr} = -V_r \sin\delta \qquad (5)$$

Perturbation about the operating points represented by subscript 0 results into the following expressions

$$\Delta V_{ds} = \Delta V_s = u_1,\quad \Delta V_{qs} = 0$$

$$\Delta V_{dr} = [-V_{ro}\sin\delta_o]\,\Delta\delta + [\cos\delta_o - \sin\delta_o\,\Delta\delta]\,u_2 \qquad (6)$$

$$\Delta V_{qr} = [-V_{ro}\cos\delta_o]\,\Delta\delta - [\sin\delta_o + \cos\delta_o\,\Delta\delta]\,u_2$$

where $u_2 = \Delta V_r$. Perturbing equation (1) about normal operating point and including equation (6) for $\Delta V_{ds}$, $\Delta V_{dr}$, $\Delta V_{qr}$ results into the following bilinear equation

$$\dot{x} = Ax + Bu + bx_6 u_2 \qquad (7)$$

where state x now represents the perturbed values. The state $x_6$ is $\Delta\delta$. The expressions for A, B and $b_6$ are given in Appendix A. Note that $u_1 = 0$ under generation mode.

## 3. STABILIZING CONTROL STRATEGIES

### Quadratic Controls for Small Perturbation Models

The A matrix of the bilinear system given in equation (7) has a pair of eigenvalues in the right half plane. Finding a stabilizing control for this system is not easy. Gutman [6] considered a system

$$\dot{x} = Ax + \sum_{i=1}^{m} (b_i x + B_i)\,u_i \qquad (8)$$

where $x \epsilon R^n$, $u \epsilon R^m$ and A, $B_i$, $b_i$ are real matrices with A having arbitrary roots. It was shown that system (8) can be stabilized with controls

$$u_i = -\alpha_i (b_i x + B_i)^T Px, \qquad i = 1,2,\ldots m,\quad \alpha_i > 0 \qquad (9)$$

if there exists a P matrix $P = P^T > 0$ such that

$$(b_i x + B_i)^T Px \neq 0, \quad i = 1,2,\ldots m \qquad (10)$$

in the set

$$\{x | x \neq 0,\ x^T(PA + A^T P)\,x \geq 0\} \qquad (11)$$

It was suggested that the numerical computation of P matrix can be carried out through an iterative technique suggested by Polak [10].

Besides the fact that the control strategy involving iterative calculation is expensive in terms of computation, on-line realization of such control is extremely handicapped. Basing on Gutman's work an alternative method of determining control for system (7) which is similar to (8), from direct calculation of P matrix is proposed as follows:

The control strategies

$$\text{a.}\quad u_i = -\alpha_i (B_i + b_i x)^T Px,\quad i = 1,2,\ldots m \qquad (12)$$

for the set $\{x | x \neq 0,\ x^T(PA + A^T P)\,x \geq 0$

$$\text{b.}\quad u_i = 0,\ \text{for the set}\ \{x | x \neq 0,\ x^T(PA + A^T P)\,x < 0\} \qquad (13)$$

will stabilize the bilinear system (8) where P is obtained from the solution of the Matrix Riccati Equation

$$PA + A^T P - PBB^T P + Q = 0 \qquad (14)$$

Note that P is the solution of the steady state Riccati equation corresponding to equation (7) with $b \equiv 0$ which can be solved by any standard technique (for example, by the method suggested by Potter [11]). Q is a positive semidefinite matrix which penalizes excursion of state variables as in conventional state regulator formulation. This could be selected such that equation (12) holds for almost all $x \epsilon R^n$ thus stabilizing the system. When the trajectory is sufficiently close to the origin, a linear regulator control of the form $u = -B^T Px$ can be switched to.

The proposed strategy (12) - (14) has been found to provide and excellent stabilizing control not only for the bilinear system equations of the doubly fed machine, it has been found equally useful for other such systems. An example is given in Appendix B to demonstrate this. Gutman showed that the system was unstable for any constant control [6].

### A Minimum-time Sub-optimal Control for Small Perturbation Models

Consider the electromechanical torque equation of the machine given in Appendix A (corresponding to the 5th term in A.4). Differentiate this equation to get (neglect the damping term)

$$p^2\Delta\omega_r = a_{51} p\Delta i_{ds} + a_{52} p\Delta i_{qs} + a_{53} p\Delta i_{dr} + a_{54} p\Delta i_{qr} \qquad (15)$$

substitute the expressions for $p\Delta i_{ds}$, $p\Delta i_{qs}$, $p\Delta i_{dr}$ and $p\Delta i_{qr}$ in (15) and simplify to get

$$p^2\Delta\omega_r = L(x) + m_1(x)u_1 + m_2(x)u_2 \qquad (16)$$

Assume that control vector is constrained in magnitude such that $|u_i| \leq 1$, $i = 1,2$. The minimum time control $u_1$ and $u_2$ can be found by assuming L, $m_1$, $m_2$ to remain constant in each time step by the relationship

$$u_i = -\text{sgn}\,\Sigma \qquad i = 1,2 \qquad (17)$$

where,

$$\Sigma = \Delta\omega_r - \frac{\Delta\dot{\omega}_r^2}{2[L(x) - m_1(x)\text{sgn}\{\Delta\dot{\omega}_r\} - m_2(x)\text{sgn}\{\Delta\dot{\omega}_r\}]}$$

The steps to arrive at control strategy (17) are similar to those for a synchronous generator control scheme given in reference [12]. Expression (16) is for positive $m_1(x)$ and $m_2(x)$ and will have to be modified for negative signs. This quasi-optimal control is of bang bang nature and poses problem in terms of realization. An alternative suboptimal control proportional to the switch function is given as

$$u_1 = -K_1 \Sigma, \qquad u_2 = -K_2 \Sigma \tag{18}$$

This strategy will drive the control to the limits only when the perturbations are relatively large. $K_1$ and $K_2$ have to be chosen judiciously. Since the control is proportional to the switch function it is also called proportional control strategy. Here again $u_1 = 0$ in the generation mode.

## A Minimum-time Sub-optimal Control for Large Perturbation Models

The control strategies so far discussed are valid only for small perturbations. A discrepancy of these control schemes is that they are valid for a particular operating point around which the perturbations were made. From implementation viewpoint it is only desirable that a control strategy is derived which does not explicitly depend on the operating points. The minimum time strategy discussed can be extended to the nonlinear machine model as follows:

Consider the nonlinear torque equation in Appendix A (combining 5th and 6th equations in A.3). Differentiate it and substitute the expressions for $pi_{ds}$, $pi_{qs}$, $pi_{dr}$, $pi_{qr}$ as in the small perturbation case to arrive at an equation of the form

$$p^3\delta = L^{'}(x)+m_1^{'}(x)\, u_1+m_2^{'}(x)u_2 \tag{19}$$

The terms $L^{'}$, $m_1^{'}$, and $m_2^{'}$ are functions of the states and do not depend on a fixed operating point as in (16). The time optimal control $u_1$ and $u_2$ can be obtained from the assumption that $L^{'}$ and $m_1^{'}$, $m_2^{'}$ remain constant over each integration step. The switching strategy involving $\delta$, $\dot{\delta}(\omega_r)$, $\ddot{\delta}(\dot{\omega}_r)$ is similar to that for a synchronous machine reported in reference [13] and is not included here.

## 4. RESULTS

The quadratic and minimum-time (proportional) control strategies proposed in the previous section were tested through digital simulation of an induction machine model. Necessary bilinear and nonlinear model of a 3-$\phi$, 15 hp (1 p.u.) singly fed induction motor given in reference [14] were simulated considering that it is fed both from the stator and the rotor terminals. Studies were carried out considering the machine to be under both generation and motoring mode for a variety of loading conditions. Results for only a few salient cases are presented in the following. The pertinent parameters of the machine and the nominal values of the state variables for a sample operating point are given in Appendix A.

The developed torque ($T_e$) of the motor as it varies with the torque angle ($\delta$) for a slip s = -1 in the doubly fed mode is shown in Fig. 1. It can be observed that the doubly fed machine can develop 3 to 4 times its rated torque depending on whether it is motoring or generating.

Figure 2 shows the angle-time characteristics in the motoring mode when the input torque is increased by a step of 0.3 p.u. The operating angle $\delta_o$ is $-35^o$. The eigenvalues of the A matrix for this operating condition are -65±j373.12, -35 ± j372.89, 0.0064 ± j9.533. The last two eigenvalues are responsible for the growing response when there is no stabilizing control (curve a). With the control derived from the proposed bilinear control, the system is stable (curve b).

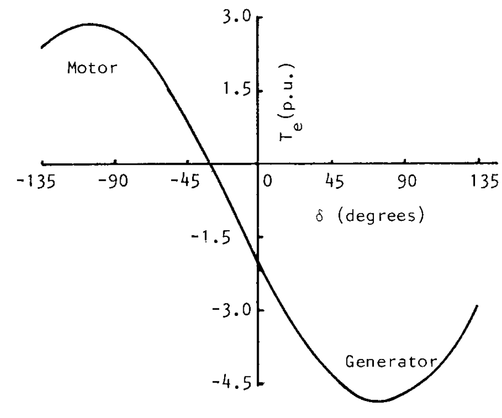


Fig. 1 Variation of angle dependent torque of the doubly fed machine at double synchronous speed.

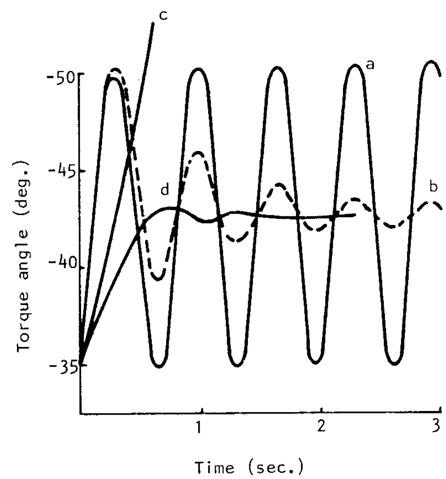


Fig. 2 Torque angle time characteristics for a step input torque of 0.3 p.u. (a) No control (b) bilinear regulator control (c) linear regulator control (d) sub-optimal proportional control.

With conventional linear regulator control $u = -B^TPx$ and neglecting term b in equation (7), the system is unstable (curve c). The best transient response (curve d) is exhibited by the proposed sub-optimal proportional control given by equation (18).

In the linear and bilinear formulation the Q matrix was selected to be

diag[0.1, 0.1, 0.1, 0.1, 10, 10]

For the proportional strategy, $K_1$ and $K_2$ each were found to be 2.5 for best performance.

Figure 3 gives the angle-time characteristics of the machine under generation mode. In this mode only control $u_2$ is present. The change considered was again an input torque step of 0.3 p.u. The operating angle was $-20^o$. The responses with bilinear and sub-optimal proportional controls are compared against the uncontrolled case. It can be seen that the response with the proportional control is superior over

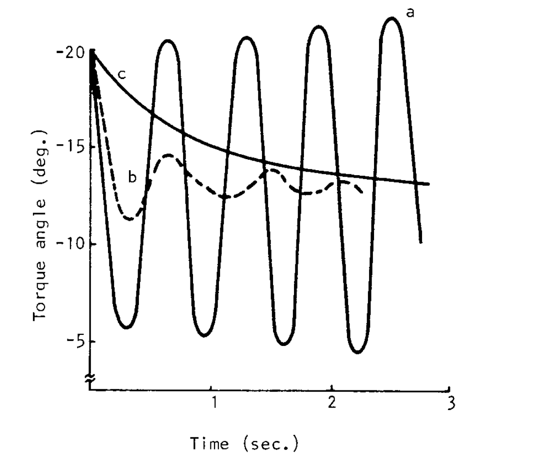


Fig. 3 Angle time characteristics for an input torque step of 0.3 p.u. under generation mode, (a) with no control (b) with bilinear regulator control (c) sub-optimal proportional control.

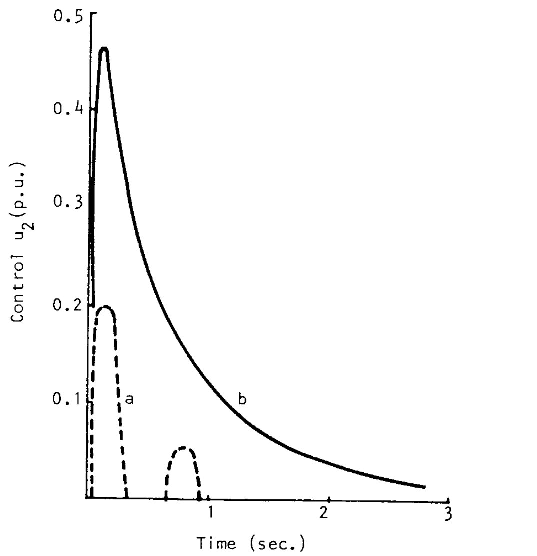


Fig. 4 Variation of rotor voltages corresponding to Fig. 3, with (a) bilinear strategy and (b) sub-optimal proportional strategy.

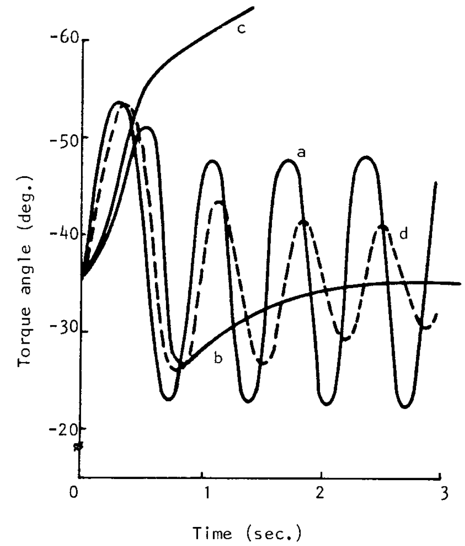


Fig. 5 Torque angle characteristics following a 0.4 p.u. torque pulse for 0.5 s duration. Response recorded are with (a) no control, (b) sub-optimum proportional control, (c) linear regulator control, and (d) bilinear regulator control.

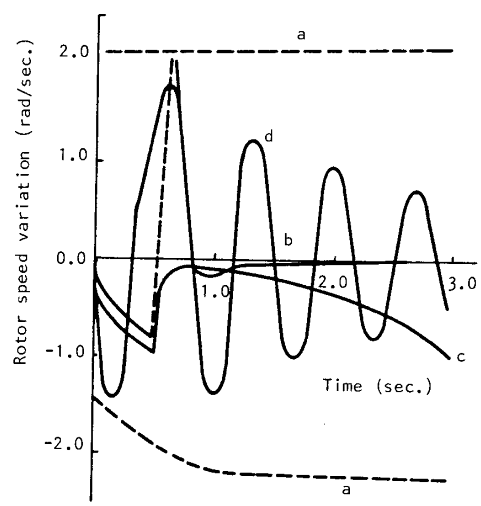


Fig. 6. Rotor speed variation corresponding to Fig. 5 with (a) no control, (b) sub-optimum proportional control, (c) linear regulator control, and (d) bilinear regulator control.

the other one (b). Figure 4 shows the variation of control voltages with the two schemes. As expected the proposed bilinear control is discontinuous.

The response of the system was studied for a relatively larger disturbance through the large perturbation (nonlinear) model of the machine given in equation (1). A torque pulse of 0.4 per unit for a duration of 0.5 sec was considered in this case. Figures 5 and 6 exhibit the torque angle ($\delta$) and rotor speed variation ($\Delta\omega_r$) when the nominal value of torque angle is $-35^{\circ}$. The response without any control is given by curve a while curve b is with the sub-optimal proportional control derived in equation (19) considering the large perturbation nonlinear model. Note that in all the previous cases, Figs. 2 through 4, the bilinear model of the system given by equation (7) were considered.

For finding the response with the linear regulator and bilinear quadratic control strategies, the small perturbation model had to be simulated. The corresponding responses are shown by curves c and d respectively in both Figs. 5 and 6. The linear regulator control drives the system unstable. This can be expected since the linear model (b = 0) is not valid for this large perturbation. Both the proposed strategies stabilize the system; however, the proportional strategy is superior in terms of transient performance. Note that a very large disturbance could not be considered because in this case the bilinear representation and the linear and bilinear control strategies would be meaningless.

## 5. CONTROL STRATEGIES AND THEIR IMPLEMENTATION

The two control strategies proposed were tested through simulation of a doubly fed machine model. The proposed bilinear quadratic control is valid for small perturbations while the proportional control is valid for small as well as large perturbations. Both the controls are obtained as functions of state variables, given by equation (12) for the bilinear and by equation (18) for the proportional schemes. It is assumed that in actual implementation proper logic circuits will be available to measure the different state variables which are the different currents in the stator and rotor circuits, the rotor speed variation ($\Delta\omega_r$) and its integral the torque angle ($\delta$).

The bilinear control strategy given by equation (12) is expressed in terms of P matrix which is the solution of matrix Riccati equation (14). Solution of this equation is straightforward. However, since matrix A depends on the operating values of the currents, etc. the P matrix is not unique for the entire range of operation. This implies that a set of P matrices have to be arrived at a-priori and the appropriate value used for the specific operating point. This is one of the serious disadvantages of linear and bilinear regulator controls, and the control strategy is thus handicapped in terms of implementation.

The bilinear model was derived through a Taylor series expansion of the nonlinear equations given in (A.3). Such a model is valid only if the perturbations are small when the higher order terms can be neglected. For very large deviations in the system, such as large load changes, this model will be inadequate and the control strategy may not be very effective as is shown in Figs. 5 and 6.

Two versions of the sub-optimal proportional scheme were considered. One is given in equation (18) where the control is obtained from the reduced second order equation (16) considering the perturbations to be small. For large perturbations the control strategies were derived from equation (19).

Control strategies derived through third order model (19) are obtained directly as a function of the state variables and the nonlinearity of the original system equation (A.3) is retained. This strategy would be suitable for implementation on an actual machine provided the requisite circuitry for feeding back the state signals to the controller is available.

The minimum time control required the control to be either the maximum or minimum value depending on the switch function $\Sigma$ given in equation (17). Since realization of such a control which requires changing it from maximum to minimum value in zero time is almost impossible, the sub-optimum scheme given by (18) is used. The sub-optimum scheme requires the control to be proportional to the switch function. The result is a continuous control function. Also another advantage of this scheme is that when the deviations of the rotor speed etc. are small, the magnitude of the control is also reduced. Proper choice of $K_1$ and $K_2$ have to be made. As indicated earlier, the best transient response was found to have resulted when $K_1$ and $K_2$ were both 2.5. In the simulation studies a ceiling voltage of 1.00 per unit was arbitrarily chosen. The maximum voltage to be applied to the motor, in general, will depend on the level of the disturbance. In practice, the ceiling voltage will be dictated by physical considerations. Notice that the large voltage variations are only very transient in nature and driving the voltages to large values for a short duration does not interfere with normal operation. Choice of a ceiling voltage of 2.00 per unit does not mean that the voltages will reach that level always. Figure 4 shows that the maximum control excursion as dictated by the particular disturbance for the gain selected was less than 0.5 p.u. In most cases a value of about 150% for the ceiling were observed to be satisfactory.

## 6. CONCLUSIONS

The small perturbation model of a doubly fed machine is bilinear with a pair of roots of the A matrix in the right half plane for most of the operating range. The conventional quadratic bilinear control for such system has to be determined through an iterative procedure. The method suggested in this paper obtains the control directly and hence saves a great deal of computation. For very small perturbations, of course, the control strategy derived through linear regulator theory is sufficient to stabilize since in this case the effect of coupling terms (of state and control variables) is negligible. A superior closed loop stabilizing control termed as the proportional control has been obtained through a minimum-time formulation. This control eliminates the transients effectively and is also simple in terms of realization. The advantage of this strategy is that it can be used to stabilize the doubly fed machine either in generator or motor mode even when the disturbance is large.

## 7. ACKNOWLEDGEMENT

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## LIST OF SYMBOLS

| | |
|---|---|
| $V_s$, $V_r$ | Stator and rotor applied voltages |
| $V_{ds}$, $V_{dr}$ | Direct axis component of stator and rotor voltages. |
| $V_{qs}$, $V_{qr}$ | Quadrature axis stator and rotor voltages. |
| $i_{ds}$, $i_{dr}$ | Direct axis stator and rotor currents. |
| $i_{qs}$, $i_{qr}$ | Quadrature axis stator and rotor currents. |
| $\Psi_{ds}$, $\Psi_{dr}$ | Direct axis flux linkages. |
| $\Psi_{qs}$, $\Psi_{qr}$ | Quadrature axis flux linkages. |
| $R_s$, $R_r$ | Stator and rotor resistances. |
| $X_s$, $X_r$ | Stator and rotor reactances. |
| $X_m$ | Mutual reactance between stator and rotor. |
| H | Inertia constant of machine (sec.). |
| D | Damping coefficient of machine (p.u.). |
| $\omega_b$ | Base frequency. |
| $\delta$ | Torque angle. |
| $\omega_e$ | Frequency of supply voltage. |
| $\omega_m$ | Rotor angular speed. |

## APPENDIX A

The equations of a doubly fed machine in the synchronously rotating frame of reference are

$$V_{ds} = \frac{p}{\omega_b}\Psi_{ds} - \frac{\omega_e}{\omega_b}\Psi_{qs} + R_s i_{ds}$$

$$V_{qs} = \frac{p}{\omega_b}\Psi_{qs} + \frac{\omega_e}{\omega_b}\Psi_{ds} + R_s i_{qs}$$

$$V_{dr} = \frac{p}{\omega_b}\Psi_{dr} - \frac{(\omega_e - \omega_r)}{\omega_b}\Psi_{qr} + R_r i_{dr}$$

$$V_{qr} = \frac{p}{\omega_b}\Psi_{qr} + \frac{(\omega_e - \omega_r)}{\omega_b}\Psi_{dr} + R_r i_{qr} \qquad (A.1)$$

$$T_e = \frac{2H}{\omega_b} p^2\delta + Dp\delta + T_\ell$$

$$T_e = \Psi_{qr} i_{dr} - \Psi_{dr} i_{qr}$$

where the relationships between currents and fluxes are

$$\Psi_{ds} = X_s i_{ds} + X_m i_{dr}$$

$$\Psi_{qs} = X_s i_{qs} + X_m i_{qr}$$

$$\Psi_{dr} = X_r i_{dr} + X_m i_{ds} \qquad (A.2)$$

$$\Psi_{qr} = X_r i_{qr} + X_m i_{qs}$$

Substituting (A.2) in (A.1) and breaking the torque equation into two first order equations, the dynamic relationships of the doubly fed machine can be written in the form

$$p\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \\ \omega_r \\ \delta \end{bmatrix} = \frac{\omega_b}{\sigma}\begin{bmatrix} -\frac{R_s}{X_s} & \sigma+k^2(1-s) & \frac{k^2R_r}{X_m} & \frac{(1-s)X_m}{X_s} & 0 & 0 \\ -[\sigma+k^2(1-s)] & -\frac{R_s}{X_s} & \frac{-(1-s)X_m}{X_s} & \frac{k^2R_r}{X_s} & 0 & 0 \\ \frac{k^2R_s}{X_m} & \frac{-(1-s)X_m}{X_r} & -\frac{R_r}{X_r} & (\sigma-1+s) & 0 & 0 \\ \frac{(1-s)X_m}{X_r} & \frac{k^2R_r}{X_m} & -(\sigma-1+s) & -\frac{R_r}{X_r} & 0 & 0 \\ \frac{-\sigma X_m i'_{qr}}{2\omega_o H} & \frac{\sigma X_m i'_{dr}}{2\omega_o H} & \frac{\sigma X_m i_{qs}}{2\omega_o H} & \frac{-\sigma X_m i_{ds}}{2\omega_o H} & \frac{\sigma D}{\omega_o} & 0 \\ 0 & 0 & 0 & 0 & \frac{\sigma}{\omega_o} & 0 \end{bmatrix}\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \\ \omega_r \\ \delta \end{bmatrix} + \frac{\omega_o}{\sigma}\begin{pmatrix} \frac{V_{ds}}{X_s} - \frac{X_m}{X_sX_r}V_{dr} \\ \frac{V_{qs}}{X_s} - \frac{-X_m}{X_sX_r}V_{qr} \\ -\frac{X_m}{X_sX_r}V_{ds} + \frac{1}{X_r}V_{dr} \\ -\frac{X_m}{X_sX_r}V_{qs} + \frac{1}{X_r}V_{qr} \\ \frac{T_e\,\sigma}{\omega_o} \\ 0 \end{pmatrix} \qquad (A.3)$$

All rotor quantities are referred to the stator side. Here $k = X_m(X_sX_r)^{-1/2}$, $\sigma = 1-k^2$, $s = 1-\omega_r/\omega_b$. The base frequency $\omega_b$ assumed to be equal to $\omega_e$. Standard nomenclature for voltage current, flux, torque etc. are used.

The expression for the A matrix in terms of machine parameters is

$$\frac{\omega_b}{\Delta}\begin{bmatrix} R_sX_s & (\Delta-\omega_{ro}X_m^2/\omega_o) & -R_rX_m & -X_mX_r\omega_{ro}/\omega_o & -X_m(X_mi_{qso}+X_ri_{qro})/\omega_o & -X_mV_{ro}\mathrm{Sin}\delta_o \\ -(1-\omega_{ro}X_m^2/\omega_o) & R_sX_s & X_mX_r\omega_{ro}/\omega_o & R_rX_m & X_m(X_mi_{dso}+X_ri_{dro})/\omega_o & -X_mV_{ro}\mathrm{Cos}\delta_o \\ -R_sX_m & X_mX_s\omega_{ro}/\omega_o & R_rX_s & (\Delta+X_sX_r\omega_{ro}/\omega_o) & X_s(X_mi_{qso}+X_ri_{qro})/\omega_o & X_sV_{ro}\mathrm{Sin}\delta_o \\ X_mX_s\omega_{ro}/\omega_o & -R_sX_m & -(\Delta+X_sX_r\omega_{ro}/\omega_o) & R_rX_s & -X_s(X_mi_{dso}+X_ri_{dro})/\omega_o & X_sV_{ro}\mathrm{Cos}\delta_o \\ -X_mi_{qro}\Delta/(2H\omega_o) & X_mi_{dro}\Delta/(2H\omega_o) & X_mi_{qso}\Delta/(2H\omega_o) & -X_mi_{dso}\Delta/(2H\omega_o) & D\Delta/(2H\omega_o) & 0 \\ 0 & 0 & 0 & 0 & \Delta/\omega_o & 0 \end{bmatrix} \quad \text{(A.4)}$$

The matrices B and $b_6$ are respectively

$$\frac{\omega_o}{\Delta}\begin{bmatrix} X_r & -X_m\mathrm{Cos}\delta_o \\ 0 & X_m\mathrm{Sin}\delta_o \\ X_m & X_s\mathrm{Cos}\delta_o \\ 0 & -X_s\mathrm{Sin}\delta_o \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \frac{\omega_o}{\Delta}\begin{bmatrix} X_m\mathrm{Sin}\delta_o \\ X_m\mathrm{Cos}\delta_o \\ -X_s\mathrm{Sin}\delta_o \\ -X_s\mathrm{Cos}\delta_o \\ 0 \\ 0 \end{bmatrix} \quad \text{(A.5)}$$

where $\Delta = X_m^2 - X_sX_r$

The machine parameters are (in p.u.)

$R_s$=0.04373 $X_s$=3.418 $X_m$=3.289 D=0.002
$R_r$=0.024 $X_r$=3.418 H=7.54 (sec)

## APPENDIX B

Gutman considered the following system (example 3.5, reference [6] which is unstable for any constant control strategy

$$\dot{x} = Ax+(B_1+b_1x)u_1+(B_2+b_2x)u_2 \quad \text{(B.1)}$$

with

$$A = \begin{bmatrix} 1/6 & 1 \\ 0 & 1/6 \end{bmatrix}, b_1 = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}, b_2 = \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} -3 \\ -2 \end{bmatrix}, B_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix} \quad \text{(B.2)}$$

The P matrix obtained from the linear part of (B.1) is

$$\begin{bmatrix} 0.100 & 0.059 \\ 0.059 & 0.187 \end{bmatrix} \quad \text{(B.3)}$$

Corresponding to a Q of diag [0.1, 0.1].

Figure 7 shows the response of the second order system with the proposed stabilizing control for two sets of initial conditions. The controls for the two cases Fig. 7(a) and 7(c) are given in Fig. 7(b) and 7(d) respectively. The system has been stabilized. The same can be shown to hold for any initial condition.

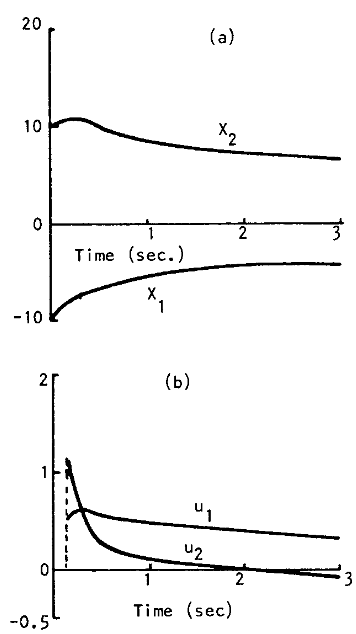


Fig.7. Response of the second order (Gutman) system and the control signals for an arbitrary initial condition.