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AGGREGATION OF INDUCTION MOTOR LOADS FOR TRANSIENT STABILITY STUDIES

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Abstract - This study investigates the dynamic representation of a group of induction motors by one or more equivalent motors. The unknown parameters of the equivalent are estimated through a weighted-least-squares procedure. Verification of simulated results with experimental ones suggests that a fifth order dynamic model of the aggregate with a ninth order parameter model is appropriate. The least- squares technique was found to provide good convergence characteristics subject to satisfaction of the following conditions good initial estimate of the parameters, proper selection of acceleration factors and homogenity of the motor group.

INTRODUCTION

In the present day power system transient stability analyses, induction motors are often represented as constant impedance loads. The reason for doing so is, obviously, to minimize the computational effort. This was also justified since the induction motor contribution to fault currents could be neglected because of the relatively smaller time constants involved. However, with the modern trend of very fast acting circuit interrupting devices, the protective gear must interrupt larger currents, a good fraction of which could be coming from induction motor loads. This, obviously, warrants a better (dynamic) representation of the induction motor loads. Inclusion of induction motor dynamics in power system transient stability studies is computationally feasible only if the vast number of motors in the system can be represented by a reasonably smaller number of equivalent machines.

A good number of dynamic models for individual induction motor loads are reported in the literature, the earliest ones are those by Stanley [1], and Maginnis and Schultz [2]. More detailed dynamic representations were reported by a number of investigators [3-12]. Reduced order dynamics for individual motors, neglecting stator transients, were reported by Krause [13]. Results of simplified studies were also presented by a number of researchers, basically based on Krause's work [6, 14-18]. Representation of a group of motors by an equivalent machine, from circuit reduction viewpoint, were reported by Hakim and Berg [19], Ilecito and Capasso [20], Subramaniam [6] and various other researchers. Richards and Tan [21] used the least squares method for estimating the parameters of an equivalent reduced order machine model. The method determines the reference output on the basis of a detailed model and then smoothes it by filtering the 60 Hz component. Application of such a technique for equivalencing the motor group is, naturally very complicated.

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This article examines the various induction motor models reported and compares the simulated result with the reported experimental ones [6] and from this decides on the equivalent motor dynamics. The weighted-least-squares technique [21-24] is then used to estimate the unknown parameters of the assumed equivalent. The various factors which affect the convergence of the algorithm have been identified. Some of the preliminary results of this study were presented by the authors earlier [24].

THE DYNAMIC MODEL OF THE AGGREGATE

The first step in the aggregation procedure is to choose a dynamic model of the equivalent motor which will reflect the terminal behavior of the group of motors. A survey of the existing dynamic models for individual motors as well as motor groups is carried out. Five different models reported in the literature were simulated. The first two are for individual motor dynamics and the group response is obtained by summing individual outputs. The rest three are approximate aggregate models. The simulated models are:

A. Two axis detailed model. The dynamic equations are derived from basic induction motor equations considering all rotor and stator unit flux linkage equations, electromechanical equations etc. Each motor is represented by a fifth order dynamic model.

B. Stator transients are neglected in this model. The number of dynamic equations for each motor is three.

C. A group of motors connected to the same bus is represented by an equivalent machine, the parameters of which are obtained through circuit reduction methods. Only electromechanical dynamics are retained.

D. Dynamics for an equivalent machine is obtained by retaining electromechanical and rotor transients (similar to B for individual machines) but parameters of the equivalent machine are assumed to be weighted mean of those of the individual machines.

E. Model for dynamic equivalent of a group which includes electromechanical equations, rotor transients and stator loss. Parameters of the equivalent motor are obtained through circuit reduction procedures similar to those in model C.

Examination of the simulated results show that model A provides best fit of the transients with the reported experimental results [6], the others in descending order being models B, E, C and D. The real power input to the motor group for the best (A) and worst (D) models is shown in Fig. 1. The experimental results are shown by dashed lines.

Since model A provides the best fit with experimental results, it was decided to represent the group of motors with one equivalent motor dynamics of which will be similar to that of model A. This can be expressed as follows:

$$\dot{X} = f\,[X, u, \alpha] \quad (1)$$

$$Y = g\,[X, u, \alpha] \quad (2)$$

where X is a 5x1 state vector, u and Y are input and output vectors respectively; α is a 9x1 vector of unknown parameters. The detailed equation of model A are given in Appendix A.

THE WEIGHTED-LEAST-SQUARES METHOD

Assuming that a group of induction motors can be replaced by an equivalent machine whose dynamics is given by equations (1) and (2), the question is what should be the values of parameters of the equivalent so that the terminal behavior of the group and the equivalent are the same? These unknown parameters can be estimated from a record of the outputs through a weighted-least-squares method [21-24] presented in the following briefly.

Consider a system which can be represented by equations (1) and (2). Choose α so as to minimize the cost index

$$J = \frac{1}{2}\int_{t_o}^{t_f} [Y_m - Y_c(\alpha)]^T W [Y_m - Y_c(\alpha)]\, dt \qquad (3)$$

where Y_m is the measured output and $Y_c(\alpha)$ is the calculated output for a particular choice of α. W is a positive semidefinite weighting matrix chosen on the basis of engineering judgement. Linearizing the state and output equations, one gets,

$$\dot{X} = A(\alpha)\, X + B(\alpha)\, u \qquad (4)$$

$$Y = H(\alpha)\, X \qquad (5)$$

where X, Y, u now represent the perturbed quantities from their respective equilibrium values. A is the Jacobian, B and H are the respective coefficient matrices. Expanding $Y_c(\alpha)$ in a Taylor series and substituting in (3), the partial derivative

$$\frac{\partial J}{\partial(\Delta\alpha)} = 0 \qquad (6)$$

gives the relationship

$$D\Delta\alpha = N \qquad (7)$$

where

$$N = \int_{t_o}^{t_f} \left[\frac{\partial Y_c(\alpha)}{\partial \alpha}\right]^T_{\alpha_o} W[Y_m - Y_c(\alpha_o)]\, dt \qquad (8.a)$$

$$D = \int_{t_o}^{t_f} \left[\frac{\partial Y_c(\alpha)}{\partial \alpha}\right]^T_{\alpha_o} W \left[\frac{\partial Y_c(\alpha)}{\partial \alpha}\right]_{\alpha_o} dt \qquad (8.b)$$

$$\left(\frac{\partial Y_c}{\partial \alpha}\right)_{\alpha_o} = H(\alpha_o)\left(\frac{\partial X}{\partial \alpha}\right)_{\alpha_o} + \left(\frac{\partial H}{\partial \alpha}\right)_{\alpha_o} X(\alpha_o) \qquad (8.c)$$

$$\frac{d}{dt}\left[\frac{\partial X}{\partial X}\right]_{\alpha_o} = A(\alpha_o)\left(\frac{\partial X}{\partial \alpha}\right)_{\alpha_o} + \left[\frac{\partial A(\alpha)}{\partial \alpha}\right]_{\alpha_o} X(\alpha_o) + \left[\frac{\partial B}{\partial \alpha}\right]_{\alpha_o} u \qquad (8.d)$$

Starting with an initial estimate of α, solution of equations (4), (5), (8a-d) and (7) sequentially provide an iterative scheme for successive estimation of α. The recursion formula for computing the estimates is

$$\alpha^k = \alpha^{k-1} + G\Delta\alpha^k \quad ; \; k = 1,2, \ldots \qquad (9)$$

where k is the iteration count and G is a matrix of acceleration factors which are selected on the basis of computational judgement. For simplicity G is assumed to be a diagonal matrix in this study.

The quantity Y_m is the measured value of the output selected. This, in actual implementation, can be directly measured. Or in simulation studies like the present one, can be obtained through equation (2) when the system of equations (1) are solved for the specific input. As can be noted, the estimate of the parameters will depend on the accuracy of the measured output Y_m. Possible measurable outputs in the induction motor aggregation study are the real and reactive power outputs, frequency deviation etc. To simulate a worse case, the estimation algorithm is developed considering that only one output, namely, real power input to the motor group is available for measurement. The real power input to the motor group has been obtained by calculating the individual motor input power from expression (A.13) in Appendix A for the individual motors and then summing them up.

Selection of the elements of the acceleration factor matrix is a difficult task. The factors were estimated from a test case where the final values of the parameters are known a-priori. This is discussed in the next section. The acceleration factors were determined in two levels. These are

1. $\Delta\alpha_i$ (i = 1,2,... 9) is calculated from equation (7) and is compared with fixed values C_i (i = 1,2,... 9) such that if $|\Delta\alpha_i| > C_i$, α_i is updated as

$$\alpha_{i(new)} = \alpha_{i(old)} + k_{ii}|\alpha_{i(old)}/\Delta\alpha_i|\Delta\alpha_i \qquad (10)$$

where k_{ii} are the diagonal elements of matrix K chosen so as to put a ceiling on the variation of α.

2. If $|\Delta\alpha_i| \leq C_i$ (i = 1,2,... 9), then α_i is updated as

$$\alpha_{i(new)} = \alpha_{i(old)} + g_{ii}\,\Delta\alpha_i \qquad (11)$$

where g_{ii} are the diagonal elements of Matrix G stated in equation (10). The normalized cost function

$$\bar{J} = J/J_o \qquad (12)$$

where

$$J_o = \frac{1}{2}\int_{t_o}^{t_f} Y_m^2\, dt$$

is calculated and compared with a predetermined small number 6. If $\bar{J} \leq \sigma$ then the process is terminated. If convergence is not attained in a prespecified maximum number of iterations, the process is discontinued and the whole process is started with a new set of initial estimates for α. In this study, the initial choice for α was made from one of the approximate equivalent models, model B. The initial value of the vector $[\partial X/\partial X]_{\alpha_o}$ was assumed to be zero. The weighting matrix W was considered to be an identity matrix.

TESTING OF THE ALGORITHM

The estimation algorithm developed was tested for its convergence characteristics on a single motor. Since the final values of the parameters to which the algorithm will converge is known, it was relatively easier to select the acceleration factors. A disturbance of 30% dip in terminal voltage for 6 cycles was simulated. A 10 hp motor, data of which is given in Appendix B (motor # 2), was used for this study. Normalized cost index against iteration count are

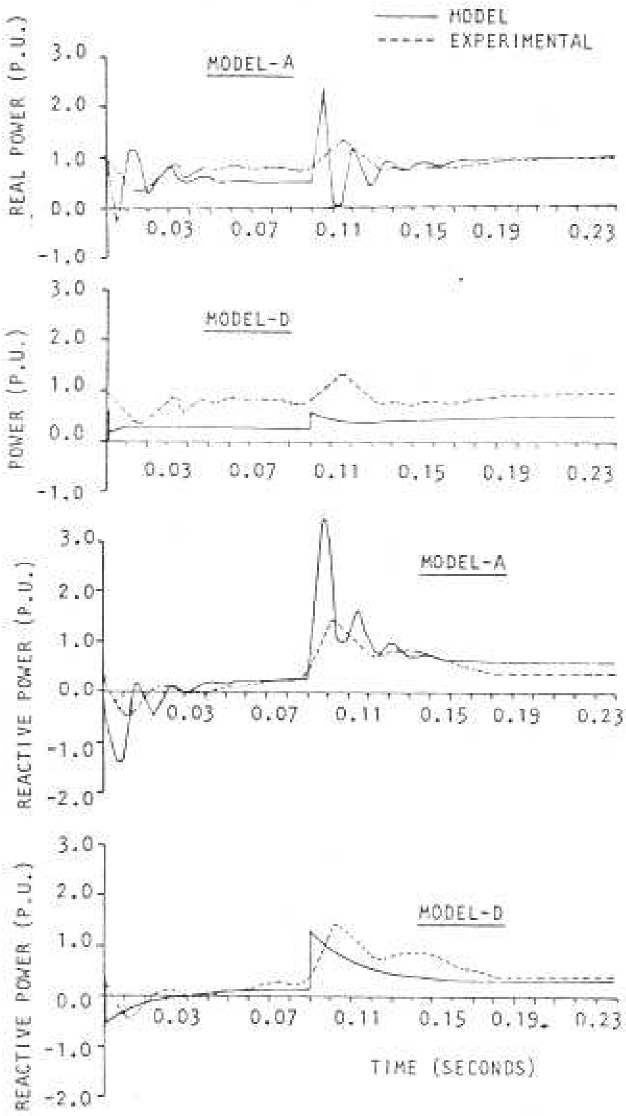

Fig. 1 Comparison of the simulated (model) results with the experimental ones.

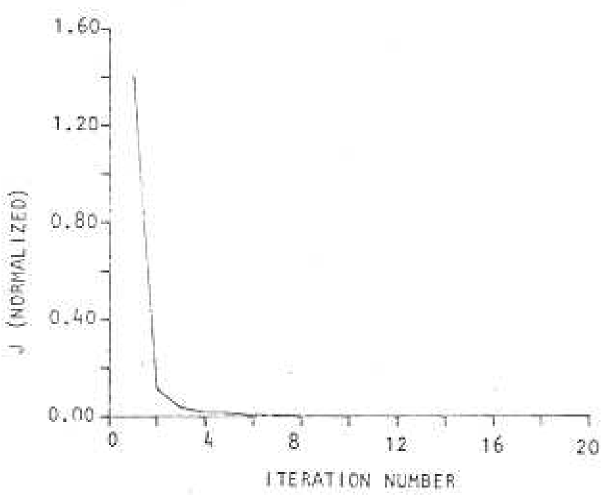

Fig. 2 Normalized cost index vs. iteration count for one motor, initial estimate of parameters 50% below nominal.

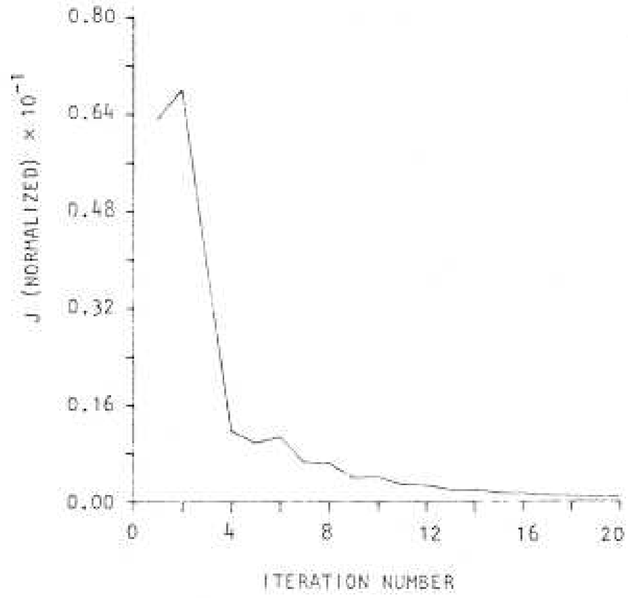

Fig. 3 Normalized cost index vs. iteration count for one motor, initial estimates of the parameters are random.

plotted for two initial estimates of the parameters - 50% below the nominal ones and a random guess. These are given in Figs. 2 and 3 respectively. It can be seen that the convergence characteristics are very good. A number of studies were made with different disturbances and various initial estimates. The following acceleration factors were found to give good convergence characteristics for all the cases studied. The numbers are rounded to the third place of decimal

$$G = \mathrm{Diag}\,[0.1\;\; 0.1\;\; 1.0\;\; 1.0\;\; 1.0\;\; 1.5\;\; 0.01\;\; 0.01\;\; 0.5]$$

$$K = \mathrm{Diag}\,[0.01\;\; 0.01\;\; 0.01\;\; 0.01\;\; 0.01\;\; 0.01\;\; .001\;\; 0.001\;\; .001]$$

$$C^T = [0.01\;\; 0.01\;\; 0.05\;\; 0.05\;\; 0.05\;\; 1.0\;\; 5.0\;\; 5.0\;\; .005] \qquad (13)$$

THE AGGREGATION STUDY

The estimation algorithm developed and tested for single motor case was then used to determine the unknown parameters of the aggregate of the motor group. A number of cases were studied considering various motor sizes (7.5 h.p. - 100 h.p.), loadings etc. for groups of two and more induction motors. Appendix B lists the data of the motors used in a study involving three motors. Appendix C summarizes the initial guesses and final converged values of parameters of a few test cases. The variation of normalized cost indices and those of the parameters for two test cases involving group of two motors (case 1 and 3 in Appendix C) are shown in Figs. 4-7. The corresponding plots for a three motor group (case 9) are given in Figs. 8 and 9 respectively.

From the large number of studies performed, it was observed that the convergence of the nonlinear algorithm depends heavily on the following factors:

1. The initial guess of the parameters (α_o)

2. The choice of proper acceleration factors

3. Homogenity of the motor group

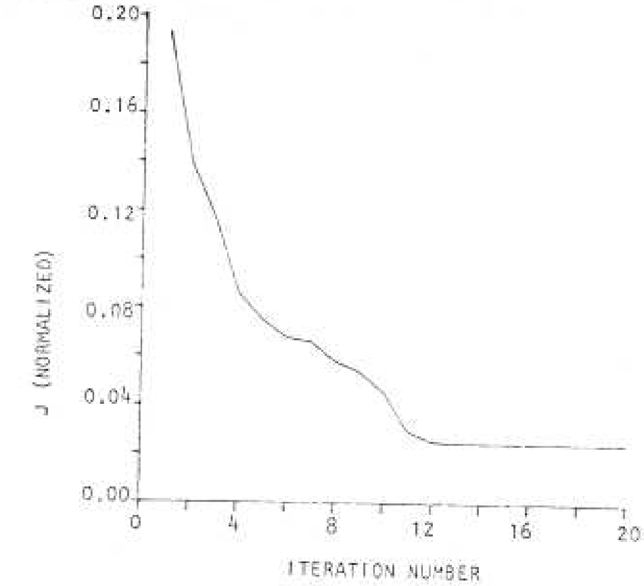

Fig. 4 Normalized cost index variation for a two motor group (case 1).

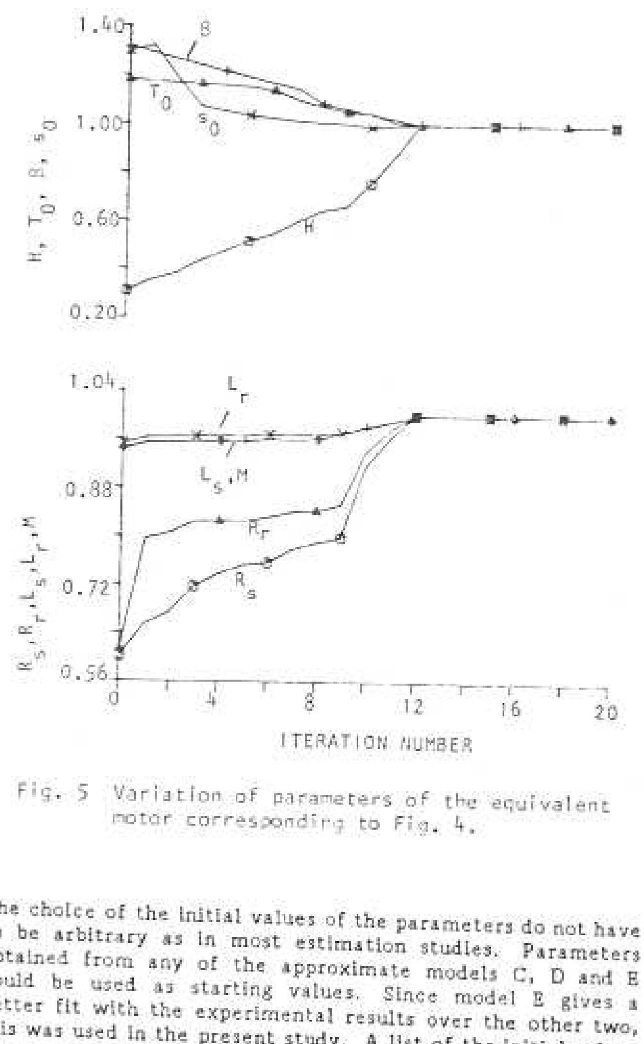

Fig. 5 Variation of parameters of the equivalent motor corresponding to Fig. 4.

The choice of the initial values of the parameters do not have to be arbitrary as in most estimation studies. Parameters obtained from any of the approximate models C, D and E could be used as starting values. Since model E gives a better fit with the experimental results over the other two, this was used in the present study. A list of the initial values for the different cases is given in Appendix C.

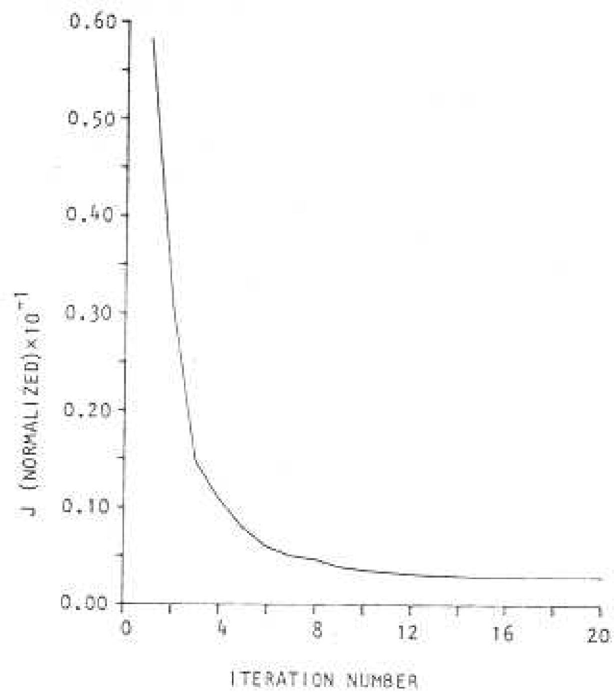

Fig. 6 Normalized cost index variation for a two motor group (case 3).

The acceleration factors obtained from the test phase were in general not satisfactory in multimotor studies. However, these provided good starting estimates. The factors were tuned to suit individual cases and a range of factors were identified. These are

$$[g_{ii}]_{max} = [0.2\ \ 0.25\ \ 1.0\ \ 1.0\ \ 1.0\ \ 1.5\ \ 0.01\ \ 0.01\ \ 0.5]$$

$$[g_{ii}]_{min} = [0.1\ \ 0.1\ \ 0.5\ \ 0.5\ \ 0.5\ \ 0.8\ \ 0.005\ \ 0.005\ \ 0.2] \qquad (14)$$

C and K were kept the same. The range found though is narrow enough, one must be careful in selecting the numbers because improper choice of some or all of them may lead to slow convergence or sometimes the algorithm may diverge, particularly if the initial guess of α is bad.

No serious convergence problem was encountered in the present studies. However, the rate of convergence was found to vary over a wide range. It was found that the convergence is very fast if the machines in the group satisfy the homogenity criterion [20]. The motor group is said to be homogeneous if the H's of the individual motors are such that

$$H_i > \frac{1}{2} T'_{doi} \qquad (15)$$

where i = 1,2,... n; n is the no. of motors in the group.

Convergence characteristics were poor or sometimes there was no convergence when some of the dominant motors violated the homogenity criterion. Both the motors satisfy condition (15) in case 3 (Appendix C) while in Case 1 one of the motors ($H = 0.15$ sec, $T'_{do} = 0.4$ sec) violates it. Convergence characteristics in this case is not so good compared to the cases where motors are homogeneous. Again in Case 5, one machine violates the homogenity condition slightly and the convergence characteristics are poor. Though satisfaction of requirement (15) is helpful in terms of good convergence, absolute adherence to it is not a must. The homogenity criterion specified may be slightly relaxed if the power ratings of the machines are different.

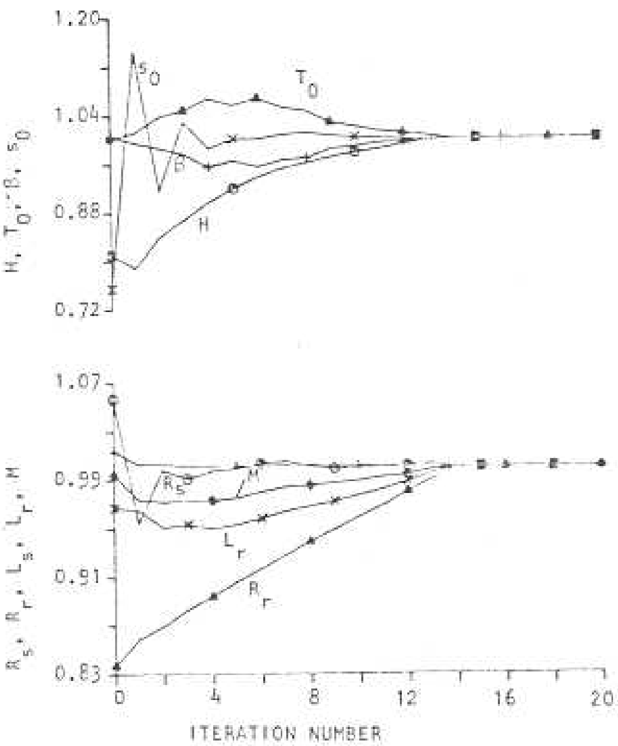

Fig. 7 Variation of parameters of the equivalent motor corresponding to Fig. 6.

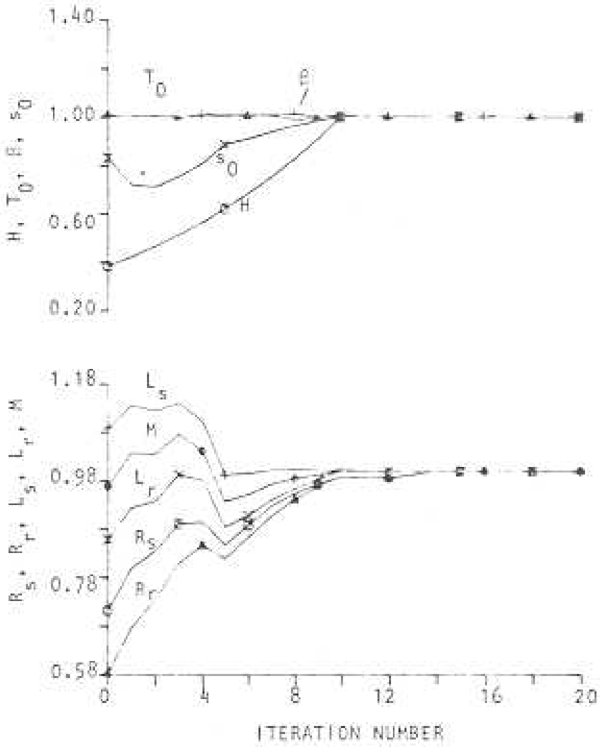

Fig. 9 Variation of parameters of the equivalent machine corresponding to Fig. 8.

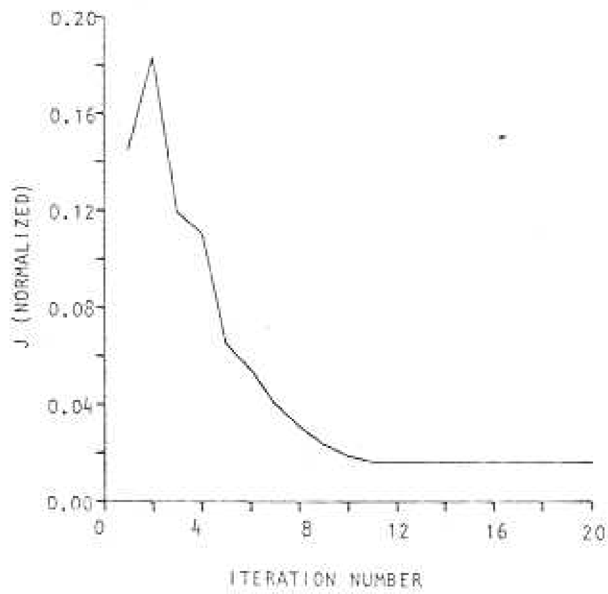

Fig. 8 Normalized cost index variation for a three motor group (case 5).

Specifically, H's may vary over a range of 20 - 25% when the power ratios of the largest to the smallest motor in the group is less than 2. A maximum variation of H upto 50% is acceptable if the power ratios are between 2 and 15.

Motor groups having heterogeneous property may be split into a number of homogeneous groups and then the equivalencing procedure continued. Obviously, in such a case there will be more than one equivalent machine.

CONCLUSIONS

Comparison of the reported experimental results with the simulated ones show that a fifth order detailed model of the equivalent is more appropriate than the reported aggregate models. The non-linear weighted-least-squares method is found to be a suitable technique for estimating the unknown parameters of the equivalent motor. The convergence characteristics were found to depend very much on the initial estimates of the parameters, choice of suitable acceleration factors and homogenity characteristics of the motor group. Model B, one of the approximate aggregate model, provided the starting value of the parameters. The acceleration factors found from testing phase of the algorithm were properly tuned for multi-motor studies. One very important factor which affected convergence of the algorithm was the homogenity of the motor group. Non homogeneous motor groups should be divided into homogeneous subgroups, if possible, and each subgroup should be replaced by its equivalent.

Algorithms presented in this article appear to provide one attractive method for off-line computation of dynamic equivalent for a group of induction motor loads. Through such an equivalent, , a large number of induction motors can be accurately represented in transient stability studies without excessive computation.

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APPENDIX A: DYNAMICS OF THE DETAILED INDUCTION MOTOR MODEL

The equivalent two phase model of a three phase induction motor in the arbitrary reference frame can be expressed as [14]

$$V_{ds} = \frac{p}{\omega_b}\psi_{ds} - \frac{\omega}{\omega_b}\psi_{qs} + R_s i_{ds} \qquad (A.1)$$

$$V_{qs} = \frac{p}{\omega_b}\psi_{qs} + \frac{\omega}{\omega_b}\psi_{ds} + R_s i_{qs} \qquad (A.2)$$

$$V_{dr} = \frac{p}{\omega_b}\psi_{dr} - \frac{(\omega - \omega_r)}{\omega_b}\psi_{qr} + R_r i_{dr} \qquad (A.3)$$

$$V_{qr} = \frac{p}{\omega_b}\psi_{qr} + \frac{(\omega - \omega_r)}{\omega_b}\psi_{dr} + R_r i_{qr} \qquad (A.4)$$

where

$$\psi_{ds} = X_{\ell s} i_{ds} + X_m (i_{ds} + i_{dr}) \qquad (A.5)$$

$$\psi_{qs} = X_{\ell s} i_{qs} + X_m (i_{qs} + i_{qr}) \qquad (A.6)$$

$$\psi_{dr} = X_{\ell r} i_{dr} + X_m (i_{ds} + i_{dr}) \qquad (A.7)$$

$$\psi_{qr} = X_{\ell r} i_{qr} + X_m (i_{qs} + i_{qr}) \qquad (A.8)$$

where the subscripts d, q, s, r refer to direct and quadrature axes, stator and rotor variables respectively. V, i, ψ represent the voltage, current and flux linkages of the various circuits. R, X and ω are the resistance, reactance and angular frequency respectively.

The electromechanical equation of motion is

$$p\left(\frac{\omega_r}{\omega_o}\right) = \frac{1}{2H}(T_e - T_m) \qquad (A.9)$$

where

$$T_e = \psi_{qr} i_{dr} - \psi_{dr} i_{qr} \qquad (A.10)$$

$$T_m = T_o \left(\frac{\omega_r}{\omega_o}\right)^{\beta} \qquad (A.11)$$

ω_r is the rotor angular speed and $\omega_b' = \omega_o$ is the base frequency.

Expressing the fluxes in terms of currents and defining slip to be $(\omega - \omega_r)/\omega$, the dynamic equations can be written in the following state notation

$$p\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} = \frac{\omega_o}{\sigma}\left\{\begin{bmatrix} \frac{V_{ds}}{L_s} \\ \frac{V_{qs}}{L_s} \\ -\frac{K^2 V_{ds}}{M} \\ -\frac{K^2 V_{qs}}{M} \end{bmatrix} + \begin{bmatrix} -\frac{R_s}{L_s} & \sigma+K^2(1-s) & \frac{K^2 R_r}{M} & \frac{(1-s)M}{L_s} \\ -[\sigma+K^2(1-s)] & -\frac{R_s}{L_s} & -\frac{(1-s)M}{L_s} & \frac{K^2 R_r}{M} \\ \frac{K^2 R_s}{M} & -\frac{(1-s)M}{L_r} & -\frac{R_r}{L_r} & \sigma-1+s \\ \frac{(1-s)M}{L_r} & \frac{K^2 R_r}{M} & -(\sigma-1+s) & -\frac{R_r}{L_r} \end{bmatrix}\begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix}\right. \quad (A.12)$$

$$p\left(\frac{\omega_r}{\omega_o}\right) = \frac{M}{2H}[i_{qs}\, i_{dr} - i_{ds}\, i_{qr}] - \frac{1}{2H}\, T_o\left(\frac{\omega_r}{\omega_o}\right)^{\beta}$$

where

$$K = M\,(L_s L_r)^{-1/2}$$

$$\sigma = 1 - K^2$$

The active power input to the motor is expressed as

$$P = V_{ds}\, i_{ds} + V_{qs}\, i_{qs} \quad (A.13)$$

APPENDIX B: MOTOR DATA

Parameters	Motor 1 (15 hp)	Motor 2 (10 hp)	Motor 3 (7.5 hp)
R_s	0.04373	0.05467	0.07157
R_r	0.02405	0.04374	0.03936
L_s	3.41892	4.78615	3.78258
L_r	3.41892	4.78615	3.78258
M	3.28939	4.63704	3.63694
T_0	0.50396	0.43521	0.17283
β	2.2	1.164	1.970
s_0	0.01355	0.02115	0.00745

APPENDIX C: INITIAL AND FINAL CONVERGED VALUES OF PARAMETERS.

Case	(α_1)	(α_2)	(α_3)	(α_4)	(α_5)	(α_6)	(α_7)	(α_8)	$(\alpha_9)^*$
Initial Values									
1	0.02463	0.01601	1.99371	1.99371	1.92432	1.618	0.93917	1.65367	0.01676
2	0.03517	0.02872	2.39956	2.39956	2.28374	3.66	0.83520	0.84706	0.03315
3	0.02398	0.01388	1.75532	1.75532	1.67286	0.1767	0.9840	2.02261	0.01595
4	0.02733	0.02187	2.39307	2.39307	2.31852	3.82	0.8352	1.54734	0.02008
5	0.02802	0.01541	1.79600	1.79600	1.72722	0.2934	0.4539	1.10454	0.00715
6	0.01428	0.01022	1.52302	1.52302	1.41094	1.97	0.65391	1.09393	0.00808
7	0.00724	0.00537	0.65865	0.65865	0.58601	2.072	1.0110	0.9643	0.00724
8	0.00544	0.00403	0.54912	0.54912	0.45751	2.9188	0.7620	1.41462	0.00461
9	0.01869	0.01145	1.30553	1.30553	1.25846	2.9375	1.1120	1.9027	0.01413
Final Values									
1	0.0410	0.0261	2.1112	2.0920	2.0359	5.2534	0.7951	1.2556	0.0129
2	0.0368	0.0254	2.3539	2.3102	2.2169	10.4515	2.6953	0.5662	0.0245
3	0.0227	0.0166	1.7306	1.8158	1.6836	0.2182	0.9820	2.0115	0.0211
4	0.0416	0.0268	2.4959	2.5400	2.4436	6.8756	1.1535	0.9145	0.0238
5	0.0283	0.0207	1.7221	2.3594	1.9264	0.4528	0.4632	1.1156	0.0059
6	0.0143	0.0101	1.5361	1.5095	1.4106	3.8927	0.5992	1.1732	0.0080
7	0.0188	0.0054	0.6800	0.6001	0.5776	3.6463	1.0039	1.0070	0.0071
8	0.0166	0.0038	0.6481	0.3292	0.3983	3.4040	0.7618	1.2198	0.0076
9	0.0262	0.0196	1.1983	1.5218	1.2981	7.6191	1.1009	1.8946	0.0170

* $\alpha = [R_s \;\; R_r \;\; L_s \;\; L_r \;\; M \;\; H \;\; T_o \;\; \beta \;\; s_0]^T$