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A STRATEGY FOR POWER SYSTEM STABILIZATION BY CONTROL-EFFORT AND RESPONSE-TIME MINIMIZATION

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SUMMARY

A method is described for deriving a quasi-optimal feedback control scheme for stabilizing power systems. This is achieved by minimizing a functional which is a linear combination of response time and control effort. The transient response of a power system obtained using this control strategy is found to be superior to that achieved with a controller based on a linear regulator approach. This scheme, in a slightly modified form, is easily implemented in practice. The modification does not affect the effectiveness of the scheme to control transients. Simulation results are included.

KEY WORDS Electric power system stabilization Reduced-order models Quasi-optimal feedback control Minimum response time and control effort

INTRODUCTION

Use of excitation control for improving power system transient response is well documented in the literature. Since high-gain, fast-acting exciters were found to destabilize the system, several stabilizing control schemes have been proposed over the past fifteen years. These include both optimal and suboptimal strategies.[1–5] In the last couple of years, intensive research has been undertaken to develop power system stabilizers (PSS).[6–9] These stabilizers, or compensators as they are sometimes called, shift the poles or eigenvalues of the linearized system model in an appropriate manner in order to overcome the effect of the negative damping introduced by the excitation system. Although the use of PSS has taken care of the basic instability problem, it was recognized that the enhancement of both dynamic and transient stability through excitation control required further investigation. Two of several strategies which evolved from such an investigation will be mentioned here.

One approach is to use the well-established linear regulator technique of optimal control where the performance index to be minimized is a quadratic functional which takes into account the integral over the control interval of the deviation of the state from the desired value

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and the energy expended.[1,2] The closed-loop control is then found from the steady-state solution of a matrix Riccati equation.[10] This technique requires an explicit knowledge of the system matrices both prior to and after a disturbance appears in the power system that is being controlled.

Another approach used for deriving stabilization strategies is based on the use of time-optimal control techniques, where the objective is to damp out the transients in minimum time.[11,12] Since the power system dynamics are non-linear, it is not easy to obtain a closed-loop time-optimal control law that can be easily implemented. Consequently, efforts have been focused on the second choice of using suboptimal strategies.

A different strategy is presented in this paper. It is based on the minimization of a cost functional or performance criterion made up of a linear combination of the control effort required and the response time. Simulation results are included which show that this method yields a better response compared to that obtained using the linear regulator approach. Also, it is easily implemented in practice in a slightly modified form.

STATE SPACE MODEL OF A SINGLE-MACHINE–INFINITE-BUS SYSTEM

The relationship between the field voltage E_{fd} and the currents i_d, i_q and i_{fd} in a synchronous generator feeding an infinite bus (see Figure 1) can be expressed in terms of the first-order differential equations:

$$pi_{fd} = c_{11}i_{fd} + c_{12}i_d + c_{13}(1+n)i_q + c_{15}\sin\delta + d_1E_{fd} \tag{1}$$

$$pi_d = c_{21}i_{fd} + c_{22}i_d + c_{23}(1+n)i_q + c_{25}\sin\delta + d_2E_{fd} \tag{2}$$

$$pi_q = c_{31}(1+n)i_{fd} + c_{32}(1+n)i_d + c_{33}i_q + c_{35}\cos\delta \tag{3}$$

where the subscripts f, d and q refer to the field, direct axis armature and quadrature axis armature circuits, respectively. p is the operator for differentiation (i.e. $p = d/dt$). The coefficients $c_{11}, c_{12}, \ldots$ depend on the machine (generator) and transmission line parameters. δ is the generator internal power angle with respect to the infinite bus, and n is the per unit deviation

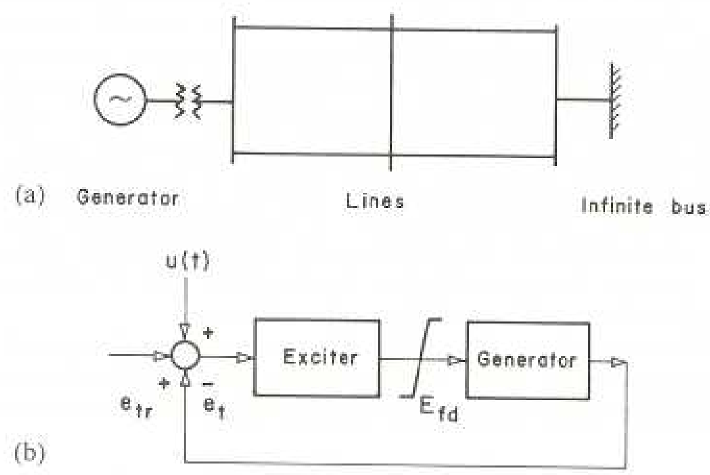

Figure 1. Schematic diagram of a single-machine–infinite-bus power system: (a) power system configuration; (b) the exciter–generator system

in frequency (i.e. $n = \omega - \omega_0/\omega_0$ where ω_0 is the nominal system angular frequency in radians per second). The above equations are obtained from the flux linkage equations given by Kimbark.[13]

The electromechanical swing equation can be written as two equations.

$$p\,\delta = \omega_0 n \tag{4}$$

$$pn = T + c_{41}i_{fd}i_q + c_{42}i_d i_q \tag{5}$$

T is the ratio of the input torque to the rotor inertia constant. The coefficients c_{41} and c_{42} depend on the generator parameters.

The voltage regulator and exciter can be described by the differential equation

$$pE_{fd} = \frac{K_r}{T_r}e_t - \frac{1}{T_r}E_{fd} + \frac{(E_{fdo} - K_r e_{tr})}{T_r} - \frac{K_r}{T_r}u(t) \tag{6}$$

where K_r and T_r are the gain and time constant, respectively, for the voltage regulator and exciter combined. e_t, e_{tr} and $u(t)$ are the generator terminal voltage, terminal reference voltage, and the additional control input to the excitation system, respectively. The terminal voltage can be expressed in terms of the machine direct and quadrature axis voltages

$$e_t = \sqrt{(e_d^2 + e_q^2)} \tag{7}$$

where

$$e_d = h_{11}i_{fd} + h_{12}i_d + h_{13}(1+n)i_q + h_{15}\sin\delta + f_1 E_{fd} \tag{8}$$

$$e_q = h_{21}i_{fd} + h_{22}(1+n)i_d + h_{23}i_q + h_{25}\cos\delta \tag{9}$$

The parameters h_{ij} can be expressed in terms of the generator and transmission line parameters.

Equations (1)–(6) can be linearized about a nominal operating point and cast into the state space form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}\mathbf{u}(t) \tag{10}$$

where

$$\mathbf{x}(t) = [\Delta i_{fd}, \Delta i_d, \Delta i_q, n, \Delta\delta, \Delta E_{fd}]^{\mathrm{T}} \tag{11}$$

is a 6×1 state vector whose components represent deviations or perturbations of the variables from their nominal values. $\mathbf{A}$ is a 6×6 matrix, and $\mathbf{b}$ is a 6×1 vector. The elements of $\mathbf{A}$ and $\mathbf{b}$ are constants and can be determined in terms of the parameters of the generator and transmission lines in the system as well as the time constants of the voltage regulator and exciter.

DERIVATION OF A CONTROL STRATEGY FOR POWER SYSTEM STABILITY

A power system is said to be stable if, following a disturbance, it returns to a state of equilibrium. Although in theory the system can take an infinite amount of time to return to equilibrium, it is necessary that this happens in a finite interval of time in practice. By an equilibrium state we mean a condition where the angular velocity and acceleration are both zero. Also, the steady-state torque angle of each generator connected to the system must be less than 90° with respect to the chosen frame of reference. The linearized (state space) model of the power system (10) represents deviations from such a stable equilibrium state.

Although the most desirable objective would be to restore the power system to its equilibrium state in the shortest possible time, it is also necessary that the control effort expended be minimized. However, these two objectives cannot be achieved at the same time. Consequently, a realistic and practical goal is to achieve a compromise. In mathematical terms this can be stated as follows. Given a power system described by (10) find a control $u^*(t)$ which takes the state $\mathbf{x}(t)$ from an initial state $\mathbf{x}(t_0)$ to a final state $\mathbf{x}(t_f) = \mathbf{0}$ such that the performance criterion (cost functional)

$$J = \int_{t_0}^{t_f} [k + |u(t)|] \, dt \tag{12}$$

is minimized, where k is a positive constant. t_0 and t_f are the initial and final instants of the control interval. Consequently, $t_f - t_0$ is the transit time required by the state $\mathbf{x}(t)$ to go from $\mathbf{x}(t_0)$ to $\mathbf{x}(t_f)$. The term $\int_{t_0}^{t_f} |u(t)| \, dt$ is a measure of the control effort expended. Equation (12) represents, therefore, a linear combination of the transit time and control effort expended. The value of k determines the relative importance assigned by the designer to the two components of the performance criterion. In most practical cases the control $u(t)$ is also subject to a magnitude constraint of the form $|u(t)| \leq M$. For convenience we can set $M = 1$.

The solution is obtained by using the calculus of variations and Pontryagin's minimum principle. The optimal control sequence $u^*(t)$ is given by[14]

$$u^*(t) = \begin{cases} 1, \text{ for } \boldsymbol{\lambda}^T(t)b < -1 \\ 0, \text{ for } -1 < \boldsymbol{\lambda}^T(t)\mathbf{b} < +1 \\ 1, \text{ for } \boldsymbol{\lambda}^T(t)\mathbf{b} > 1 \end{cases} \tag{13}$$

$\boldsymbol{\lambda}(t)$ is the costate vector and is governed by the differential equation

$$\dot{\boldsymbol{\lambda}}(t) = -\left(\frac{\partial H}{\partial x}\right) \tag{14}$$

where H is called the Hamiltonian and is defined as

$$H = k + |u(t)| + \boldsymbol{\lambda}^T(t)[\mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t)] \tag{15}$$

The optimal control law (13) is in terms of the unknown costate vector. It can be determined by an iterative solution of a two-point boundary value problem. Since this law is an open-loop law, it is not useful. For practical implementation, we need to obtain a closed-loop control law in terms of the state vector $\mathbf{x}(t)$ (equation 11)). However, this is an almost impossible task, since six states are involved and will be difficult to implement.

A simpler closed-loop control law will therefore be derived. This law will depend on a reduced-order model which will be updated during the control interval. The linearized differential equation describing the behaviour of frequency deviation is obtained from (10) as

$$pn = \dot{n} = a_{41} \, \Delta i_{fd} + a_{42} \, \Delta i_d + a_{43} \, \Delta i_q \tag{16}$$

This equation is differentiated with respect to time to yield

$$\ddot{n} = a_{41} p \, \Delta i_{fd} + a_{42} p \, \Delta i_d + a_{43} p \, \Delta i_q \tag{17}$$

If $p \, \Delta i_{fd}$, $p \, \Delta i_d$, and $p \, \Delta i_q$ are replaced by expressions obtained from (10), we obtain

$$\ddot{n} = L(\Delta i_{fd}, \Delta i_d, \Delta i_q, \Delta\delta) + [a_{41} b_1 + a_{42} b_2] \, \Delta E_{fd} \tag{18}$$

Since the perturbation variables are time-varying (18) can be written for purposes of computation as

$$\ddot{n} = L(t) + u_1(t) \tag{19}$$

The value of $L(t)$ is determined at each step of the iterative calculation in terms of the values of Δi_{fd}, Δi_d, Δi_q and $\Delta\delta$, which in turn are calculated at each step from (10). $|u_1(t)| \leqslant 1$.

Using this second-order model, we can obtain the optimal control strategy in terms of the states x_1 and x_2 (where $x_1 = n$ and $x_2 = \dot{n}$). Define

$$\gamma_p = x_1 - \frac{x_2^2}{2[L_p - \text{sgn}\{x_2\}]} \tag{20}$$

$$\Gamma_p = x_1 - \left[\frac{1}{2(L_p - \text{sgn}\{x_2\})} + \frac{2}{k}\right] x_2^2 \tag{21}$$

where L_p is the value of $L(t)$ at $t = t_p$.

Also define

$$\begin{bmatrix} S_1 = \{(x_1, x_2) : \gamma_p > 0,\ x_2 > 0 \text{ also } \Gamma_p > 0,\ x_2 < 0\} \\ S_2 = \{(x_1, x_2) : \Gamma_p < 0,\ x_2 > 0 \text{ also } \Gamma_p < 0,\ x_2 < 0\} \\ S_3 = \{(x_1, x_2) : \Gamma_p < 0 \text{ and } \gamma_p > 0 \text{ when } x_2 < 0\} \\ S_4 = \{(x_1, x_2) : \Gamma_p > 0 \text{ and } \gamma_p < 0 \text{ when } x_2 > 0\} \end{bmatrix} \tag{22}$$

Then the control sequence is $u_1^* = -1$ for all $(x_1, x_2) \in S_1$, $u_1^* = +1$ for all $(x_1, x_2) \in S_2$ and $u_1^* = 0$ for all $(x_1, x_2) \in S_3$ and S_4. This is shown in Figure 2. We shall refer to this control law as a quasi-optimal control law, since $L(\Delta i_{fd}, \Delta i_d, \Delta i_q, \Delta\delta)$ is approximated by a piecewise constant function of time and updated at several instants during the control interval.

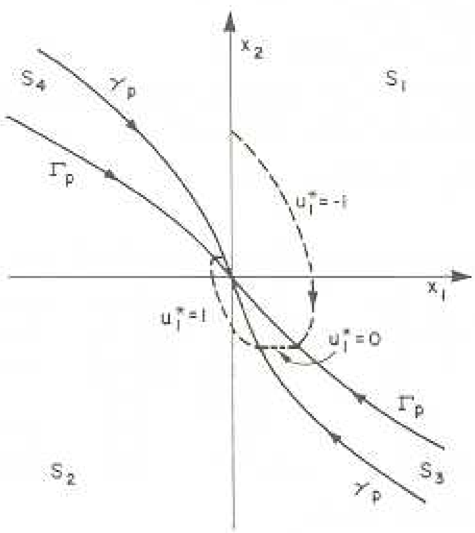

Figure 2. Switch curves for quasi-optimal control at $t = t_p$ in the $x_1 - x_2$ plane

A proportional control strategy

An examination of the quasi-optimal control strategy shows that the control at any instant of time is +1, or −1 or 0. This means that the field voltage has to change in a bang-bang fashion at the instants of switching. This control strategy has been obtained by assuming that the regulator time constant is negligible. That is, as a consequence of $T_r \simeq 0$, we set $u(t)$ in Figure 1(b) equal to $u_1(t)$ in (19). However, in practice the voltage E_{fd} does not change instantaneously. A proportional control strategy may be more realistic.

It is suggested that a control proportional to the switching function

$$u(t_p) = G\gamma_p \tag{23}$$

will drive the exciter voltages to the maximum values at the onset of a disturbance. This will override regulator action during the early portion of the transients but will restore automatic voltage regulator (AVR) action subsequently. The gain G is chosen in such a manner that normal (AVR) action can be restored once the oscillations have died down to a reasonable level after a disturbance.

SIMULATION RESULTS

The quasi-optimal control strategy and the proportional control law derived in the preceding section using a second-order model were tested on a sixth-order model by computer simulations. A number of disturbances, large and small, were simulated and the system response recorded in each case. The results obtained with these two control strategies were compared with those obtained by means of a linear regulator formulation. The linear regulator control was derived using the sixth-order model. The derivation of the linear regulator control is well known.[14,15]

The following numerical values were used.

Rating of generator: 46MVA; 13·8kV

Per unit parameter values (nomenclature used is taken from reference 13):

X_{ff} 0·693, X_q 0·4363, r 0·00435, r_{tie} 0·07/section

X_f 0·0562, H 3·0, T_r 0·0125, X_t 0·1

X_d 0·6803, r_f 0·000508, X_{tie} 0·7/section, K_r 200

In order to linearize the system model about a nominal operating point, the state variables were assumed to have the nominal values (per unit):

$i_{fd0} = 1{\cdot}957$, $i_{d0} = 0{\cdot}137$, $i_{q0} = 0{\cdot}412$, $n_0 = 0$

$\delta_0 = 30°$, $E_{fd0} = 1{\cdot}1$

The linearized system matrix **A** has the following numerical values:

$$\begin{bmatrix} -0{\cdot}40013 & -22{\cdot}233 & 369{\cdot}7 & 152{\cdot}57 & -258{\cdot}97 & 0{\cdot}7119 \\ -0{\cdot}15191 & -27{\cdot}376 & 455{\cdot}22 & 187{\cdot}86 & -318{\cdot}88 & 0{\cdot}2703 \\ 171{\cdot}38 & -451{\cdot}41 & -22{\cdot}672 & 273{\cdot}44 & 152{\cdot}47 & 0{\cdot}0 \\ -0{\cdot}0937 & 0{\cdot}01678 & -0{\cdot}181 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 377{\cdot}0 & 0{\cdot}0 \\ -5725{\cdot}6 & 2422{\cdot}7 & -301{\cdot}17 & 0{\cdot}0 & 1889{\cdot}0 & -84{\cdot}25 \end{bmatrix}$$

The vector **b** has the numerical values

$$\mathbf{b} = [0, 0, 0, 0, 0, 16000]^T$$

The non-zero element in the **b** vector is the ratio of the gain to time constant of the voltage regulator-exciter, K_t/T_t. A typical value for T_t for a solid-state exciter system is around 0·0125 s. Excitation system gains ranging from 100 to 500 have been reported in the literature. Although higher gains are required for better response, they give rise to dynamic instability. PSS are presently being used to eliminate the instability caused by high-gain, fast-acting, excitation systems.

Several cases of disturbances were considered. In each case three control strategies were used: (a) linear regulator control, (b) quasi-optimum control and (c) proportional control derived using (b).

The weighting matrix R was set equal to 1 in each case, and the following diagonal matrix **Q** was used

$$Q = \text{diag}[0\cdot 1, 0\cdot 1, 0\cdot 1, 4, 4, 0\cdot 1]$$

The gain matrix $-R^{-1}\mathbf{B}^T\mathbf{K}$ for the linear regulator formulation turned out to be

$$[-2\cdot 593, 2\cdot 2626, 0\cdot 0196, 57\cdot 1269, -1\cdot 7278, -0\cdot 3112]$$

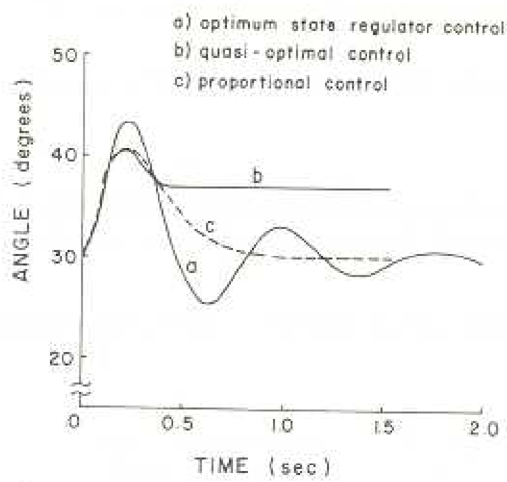

Figure 3. Case i: Angle–time characteristics due to a 60 per cent input torque pulse for 3 cycles

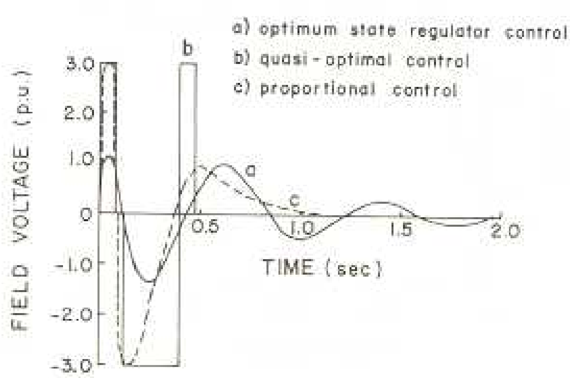

Figure 4. Case i: field voltage variation due to a 60 per cent input torque pulse for 3 cycles

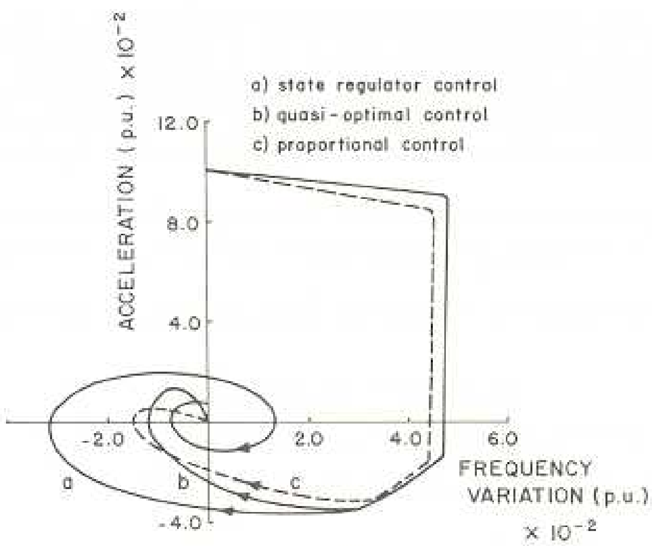

Figure 5. Case i: variation of acceleration against frequency for a 60 per cent input torque pulse of 3 cycles duration

In the case of control strategies (b) and (c), the value of the constant k in the cost functional (12) was set equal to 1.

Case i

The system was subjected to a 60 per cent input torque pulse of 0·05 s or 3 cycle duration. The responses obtained with the three control strategies are shown in Figures 3, 4 and 5.

Case ii

The system was subjected to a 30 per cent step change in input torque. The results are shown in Figures 6, 7 and 8.

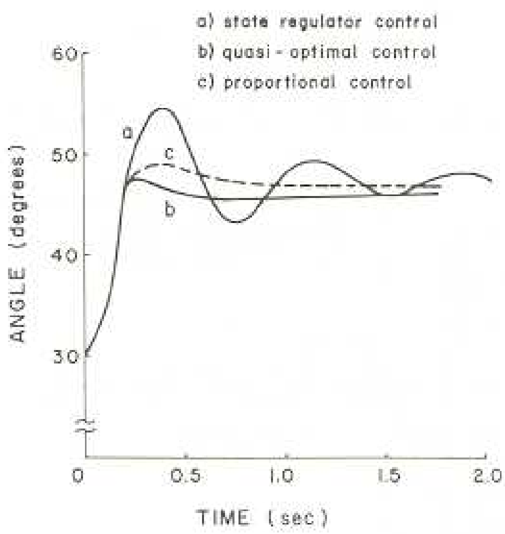

Figure 6. Case ii: Angle–time characteristics for a 30 per cent input torque step change

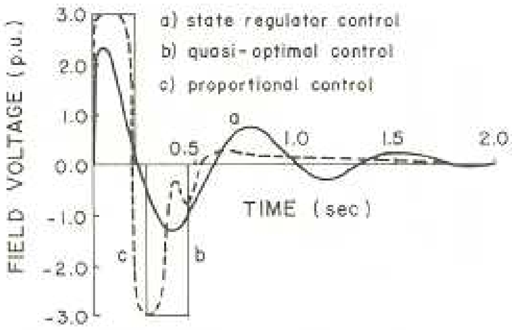

Figure 7. Case ii: variation in field voltage due to a 30 per cent input torque step change

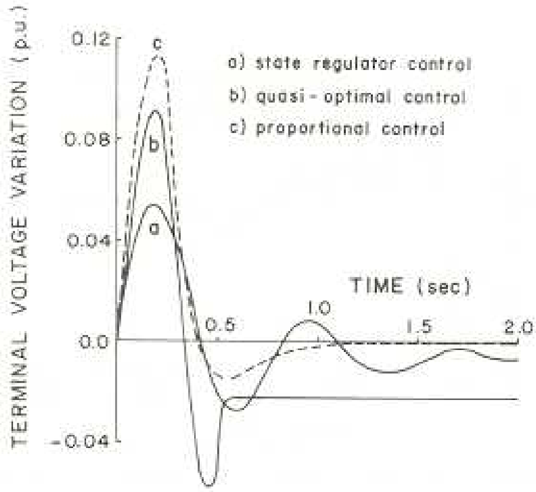

Figure 8. Case ii: variation in terminal voltage due to a 30 per cent input torque step change

Case iii

In this case a 100 per cent voltage pulse was applied to the exciter for 0·2 s. Only the proportional control and state regulator control strategies were tried. (The results are shown in Figures 9 and 10.)

Case iv

Here, it is assumed that a momentary fault occurs on the sectionalizing bus, resulting in momentary loss of half of the transmission line. Two control strategies, the state regulator control and proportional control, were tried. Since the disturbance was relatively large, the non-linear system model (equations (1)–(6)) was used in the simulation studies. The linear regulator control however was determined on the basis of the linearized post-disturbance **A** matrix. The proportional control was derived as in other cases using the reduced-order linear

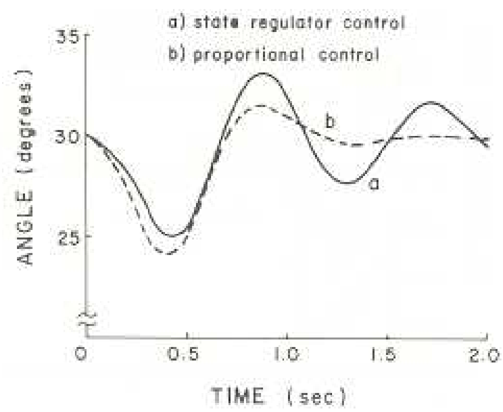

Figure 9. Case iii: variation in rotor angle due to a 100 per cent pulse input to exciter for 0·2 s

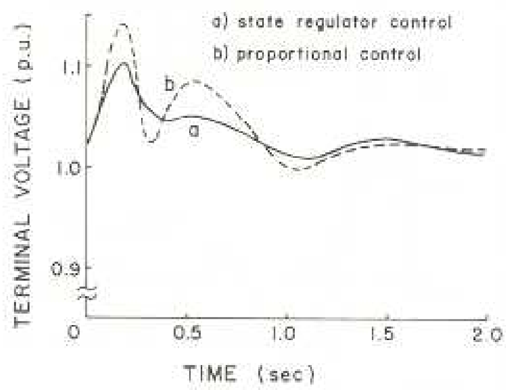

Figure 10. Case iii: variation in terminal voltage due to a 100 per cent pulse input to exciter for 0·2 s

model. The post-disturbance **A** matrix was

$$\begin{bmatrix} -0\cdot35 & -25\cdot638 & 343\cdot9 & 91\cdot06 & -153\cdot81 & 0\cdot6228 \\ -0\cdot0902 & -31\cdot56 & 423\cdot46 & 112\cdot18 & -189\cdot39 & 0\cdot1603 \\ 109\cdot42 & -424\cdot51 & -28\cdot105 & 176\cdot06 & 97\cdot35 & 0\cdot0 \\ -0\cdot09366 & 0\cdot01077 & -0\cdot1823 & 0\cdot0 & 0\cdot0 & 0\cdot0 \\ 0\cdot0 & 0\cdot0 & 0\cdot0 & 0\cdot0 & 377\cdot0 & 0\cdot0 \\ -6920\cdot9 & 2938\cdot8 & -215\cdot63 & 0\cdot0 & 1343\cdot2 & -84\cdot37 \end{bmatrix}$$

The gain matrix $-R^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{K}$ for this case turned out to be

$$[-4\cdot2742, 3\cdot7261, 0\cdot0145, 208\cdot8341, -1\cdot1635, -0\cdot3114]$$

The responses due to the two control strategies are shown in Figures 11 and 12.

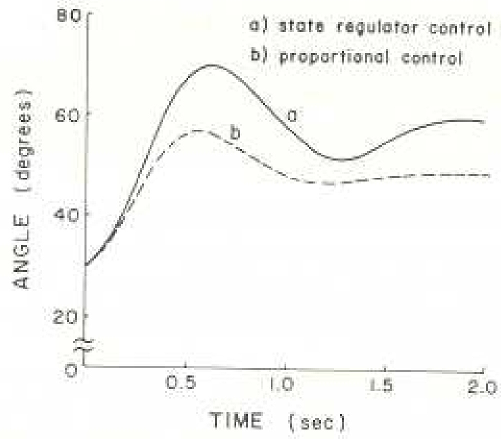

Figure 11. Case iv: variations of angle against time of generator due to loss of half of the transmission line following a momentary fault

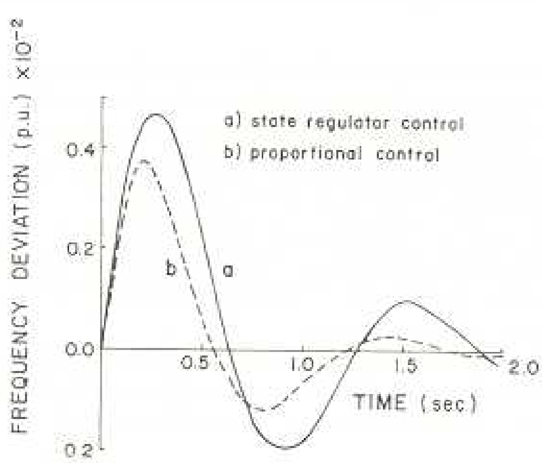

Figure 12. Case iv: per unit frequency deviation as a function of time due to loss of half of the transmission line following a momentary fault

ANALYSIS OF SIMULATION STUDY RESULTS

Case i: 60 per cent input torque pulse for 0·05 s

The quasi-optimal control strategy based on minimzing a linear combination of response time and control effort seems to provide the best response in the sense that the transients are damped out quickly (Figure 3). The swing in the rotor angle and the frequency deviation go to zero in about 0·4 s. An obvious disadvantage of this strategy is that the field voltage has to be driven to its maximum value almost instantaneously. Even the proportional control performs better than state regulator control.

Case ii: 30 per cent input torque step change

The quasi-optimal control provides a quicker transient control. The state regulator control provides better field voltage and terminal voltage characteristics (Figures 6, 7 and 8).

Case iii: 100 per cent voltage pulse applied to exciter for 0·2 s

Only the proportional control and state regulator control responses are shown in Figures 9 and 10. Since the proportional control depends on variations in the frequency and acceleration of the generator, it is not effective in the early stages of the transient response. Even for a 100 per cent pulse, these variations are fairly small until about 0·5 s.

Case iv: transmission line loss due to momentary fault (Figures 11 and 12)

The results indicate that proportional control provides better transient characteristics compared to the state regulator control. As mentioned earlier, since this is a relatively large disturbance, the linear controls were tested on the non-linear model. Use of the linear model would have led to erroneous results.

From the various case studies, it would appear that the quasi-optimal control law provides a much better transient response compared to the response obtained with the state regulator control. Even the suboptimal proportional control, which is easy to implement, provides a better response than the state regulator control strategy. The salient comparative features of the state regulator control strategy and the suboptimal proportional control strategy are summarized below.

(a) The optimum state regulator control provides smoother terminal voltage and field voltage characteristics, whereas the frequency characteristics are better with the proportional control. The transient frequency, acceleration angle, and other characteristics are inferior for the state regulator control because of the inherent inadequacy that it is derived from the steady-state solution of the matrix Riccati equation.

(b) Application of the control derived from the state regulator formulation is handicapped by the fact that the desired final values of states and controls, if they are different from the initial values, and also the final system matrix must be known *a priori*. Otherwise, the control action is not so effective.

(c) The proportional control does not require prior knowledge of the final states. The formulation can take into consideration the non-linearities of the system dynamics, and hence it is suitable for small as well as large disturbances.

CONCLUSIONS

A quasi-optimum feedback stabilzing control strategy derived by minimizing a linear combination of response time and control effort has been compared with a linear regulator control. This formulation appears to be more effective in controlling power system transients than the linear regulator strategy.

The proportional control derived from the minimum control effort-time scheme is simple to realize and requires only an optimal combination of frequency and acceleration. This control is also more effective than the state regulator control. Machine acceleration/deceleration is proportional to the difference between the mechanical shaft power and the electrical power. Under steady-state conditions, the power flowing out of the machine is equal to the mechanical shaft power minus losses. If the machine power flow is recorded, the deviations from its nominal value represent acceleration/deceleration. This measurement can be done easily.

The state regulator control was determined on the basis of maximum shift of the eigenvalues and hence employed the same principle of the design as the 'power system stabilizers'. Since the proposed proportional control is superior to the state regulator control and enhances both dynamic and transient stability, it may provide a suitable alternative to the present-day power system stabilizers.

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