# STABILIZATION OF A HIGH VOLTAGE ACDC POWER SYSTEM
# II. MULTIMACHINE SYSTEM SUBJECTED TO LARGE PERTURBATION

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Abstract - This article presents a stabilizing control strategy for a multimachine ACDC power system under large perturbation conditions. The controller is designed with local variables. The dynamics of the AC and DC systems are presented and a solution technique based on diakoptical methods is discussed. The response recorded for a multimachine system with a DC link shows that a sub-optimal proportional control of excitation voltage and firing angle of the converters provide an effective control of transients.

## 1. INTRODUCTION

Study of a simple ACDC power system in the companion paper [1] showed two interesting results: (a) the transient performance of the system can be improved by jointly controlling the excitation voltage of the generator and the firing angle of the converters, (b) a proportional control derived from "optimal" combination of velocity and acceleration of the generator is effective in transient control and has the potential of online application. In this article it will be demonstrated that a combination of exciter and converter controls helps to stabilize a multimachine system following large disturbances such as three phase faults.

Control studies for single or multimachine systems have traditionally been done on the basis of linear quadratic (LQ) designs [2]. Eigenvalue assignment technique from measurement of output variables [3] suffers the same drawback of model dependency as LQ design. It is well known that when power systems are subjected to large disturbances, the linearized model and hence the control thus obtained are not valid. This requires derivation of a control which takes the system nonlinearities into account. The other point of interest is that any control strategy should depend on variables "local" to the controllers. This paper gives a suboptimal feedback control strategy for a multimachine ACDC system which does not require linearization. The strategy is derived in terms of local variables.

Section 2 of this paper presents the dynamic model of the AC as well as DC system. Section 3 presents the solution of the steady state and transient equations of the combined ACDC system. The local control strategy is discussed in Section 4. The results and conclusion are presented in Section 5 and 6 respectively.

## 2. AC-DC SYSTEM TRANSIENT REPRESENTATION

### 2.1 AC System Representation

In the small perturbation analysis to evaluate the performance of various stabilizing control signals, a nine order dynamic model was used [1]. For multimachine system study, some of the non essential dynamics (such as those in a.c. lines) are ignored so as to keep the order of the system lower. However, the representations should be detailed enough to include the major nonlinearities in the dynamics. Each synchronous generator in the system is represented by a fourth order model given below [4].

$$\frac{d\omega}{dt} = \delta - \delta_0 \tag{1}$$

$$\frac{T_m}{\omega_0}\frac{d\omega}{dt} = T_i - T_0 \tag{2}$$

$$T'_{do}\frac{dE'_q}{dt} = E_{fd} - (x_d - x'_d)\,i_d - E'_q \tag{3}$$

$$T'_{qo}\frac{dE'_d}{dt} = (x_q - x'_q)\,i_q - E'_d \tag{4}$$

The excitation system is represented by a third order dynamic model as shown in Fig. A1 in Appendix I. The transmission network is assumed to be in quasi-steady state. Governor dynamics are not included in this study.

### 2.2 DC Link Representation

Although several control strategies are suggested for DC link operation, most of the DC systems that have been built or planned used the same control concept of constant extinction angle at the inverter with constant current control at the rectifier end. The current regulator at the rectifier varies the delay angle between the minimum and the maximum values to keep the direct current constant. At the same time the current regulator at the inverter maintains the direct voltage as high as possible. If the current drops below the setting, the inverter increases its angle of advance to control the current to a slightly lower value.

The steady state equations relating the voltages and currents in the HVDC link are given in Appendix II. These are obtained by equating the real power flow on both AC and DC sides of the converter and then combining them with the algebraic equations of the converter [1,5]. The HVDC line is represented by a single T-section as shown in Fig. 1.

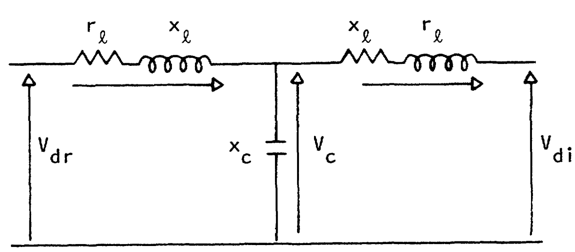


Fig. 1. DC line representation.

The transmission line dynamics is represented by the following equations.

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$$V_{dr} = r_\ell I_{dr} + \frac{X_\ell}{\omega} p I_{dr} + V_c \quad (5)$$

$$V_{di} = -r_\ell I_{di} - \frac{X_\ell}{\omega} p I_{di} + V_c \quad (6)$$

$$pV_c = X_c (I_{dr} - I_{di}) \quad (7)$$

where

$r_\ell$ = one half of total DC line resistance

$X_\ell = \omega L_d$

$L_d$ = one half of total inductance including DC reactors

$$X_c = \frac{1}{2\Pi f C_{dc}}$$

$C_{dc}$ = DC line capacitance

This representation of the HVDC link is integrated with the AC system using a direct diakoptical approach.

## 3. SOLUTION TECHNIQUES OF THE ACDC EQUATIONS

An integrated AC-HVDC system can be described by the following sets of algebraic and differential equations

$$g(X,Y) = 0 \quad (8)$$

$$pY = f(X, Y) \quad (9)$$

where the set of algebraic equations (8) describe both the AC network and the steady state operation of the HVDC link. The differential equations (9) describe the dynamics of the synchronous machines and their excitation system controls, the dynamics of the converters (current controllers) as well as those of the HVDC line given in equations (5) - (7).

Two methods of solution of equations (8) and (9) are known. "A block iterative solution" solves the algebraic and differential equations one after the other separately. Obviously there is a time lag introduced. The alternative is a simultaneous solution where the differential equations are transformed into an algebraic form and the two groups of equations are solved iteratively. The conversion of the system differential equations into an algebraic form also enables them to be used in the direct diakoptical solution load flow method [5]. The simultaneous solution method is faster over the block iterative solution and hence is used in this study.

The implicit trapezoidal rule is used to transform the differential equations into algebraic forms. To demonstrate how this is done consider the line equation (5) which can be expressed as

$$pI_{dr} = [G(V_{dr} - V_c)/r_\ell - I_{dr}]/T_{dc} \quad (10)$$

where $T_{dc}$ is the HVDC line time constant and G = 1. The algebraic form of equation (10) is

$$I_{dr} = C + Mx \quad (11)$$

where

$C = (1 - 2B) I_{dro} + Bx$
$M = BG$
$B = h_d/(2T_{dc} + h_d)$
$x = (V_{dr} - V_c)/r_\ell$
$h_d$ = dc integration step
$I_{dro}$ = nominal direct current

Similarly, all other differential equations are converted to their algebraic forms. Note that solution of differential equation (10) as given by (11) is not in closed form.

The set of nonlinear algebraic equations (8) are solved by a fast decoupled Newton-Raphson method. The D.C. link is directly integrated into the A.C. solution through a direct diakoptical method. The direct diakoptical solution of ACDC networks have evolved from the solution of asymmetrical changes in an AC network. The presence of an HVDC link in an AC system may be viewed as an asymmetrical change in a previously symmetrical system and can be treated then as the addition of a branch to the AC system. The technique obtains a new solution for the asymmetrical network from the old solution of the symmetrical network without reinverting the new network matrix.

Once the differential equations relating the machine dynamics, the HVDC link current regulators and the DC line are converted to algebraic equations, these in addition to the algebraic equations (8) are solved through an iterative scheme. The solution is started by an initial estimate of the variables, the network injected currents being calculated from the machine steady state relationships. The busbar voltages are calculated using the AC network injected currents and the DC link contribution as obtained through the diakoptical method. The estimates for machine, current regulator and DC link state variables are then corrected. The procedure is repeated until the convergence criteria are met for a particular step size.

## 4. STABILIZING CONTROL STRATEGY FROM LOCAL VARIABLES

The various stabilizing controls investigated in the accompanying paper for a single machine infinite bus sytem clearly show that the proportional control is a more suitable alternative. For nonlinear system, obviously, the linear state and output regulator strategies cannot be used as such. When the disturbance is large (such as 3-$\phi$ faults, line switchings etc.), the rotor angle excursion is large, the optimization problem in such a case can be reformulated as:

Given the nonlinear system

$$X = f(X, u) \quad (12)$$

find the control u subject to the constraint

$$|u_i(t)| \le 1 \qquad 1 = 1,2,3 \quad (13)$$

So that

$$n(t_f) = 0 \quad (14)$$

$$pn(t_f) = 0$$

$$0 \le \delta(t_f) \le \Pi/2$$

minimizing the cost index

$$J = \int_{t_o}^{t_f} dt \quad (15)$$

Here n, pn and $\delta$ are frequency, acceleration and rotor angle variations respectively. The control is obtained from the following analysis. Consider the torque equation

$$T_m p^2(\delta/\omega_0) = T_i - T_0 \quad (16)$$

where

$$T_0 = V_d i_d + V_q i_q \quad (17)$$

Differentiating (16) with respect to time and substituting the other derivatives we arrive at the following equation

$$p^3(\delta/\omega_0) = \frac{V_d E_q}{x_d' T_{do}' T_m} + \frac{V_d e_d}{x_q' T_{qo}' T_m} + \frac{2 i_d e_d}{T_m T_{qo}'} - \frac{V_d}{x_d' T_{do}' T_m} E_{fd} \quad (18)$$

Derivation of equation (18) is given in Reference [6] and so is not repeated here. Note this equation is valid for a synchronous generator irrespective of the external connection. Let us now try to construct the strategy when a generator is connected to the system through AC/DC line. For simplicity we can neglect $T'_{q0}$ and hence the direct axis induced voltage $E_d$, this will modify (18) to the form

$$p^3(\delta/\omega_0) = \frac{V_d E_q}{x_d' T_{d0}' T_m} - \frac{V_d}{x_d' T_{d0}' T_m} E_{fd} \quad (19)$$

where

$$E_q = E_q' + (x_d - x_d')\, i_d \quad (20)$$

$i_d$ is the component of total generator current i along the direct axis of the ACDC system shown in Fig. A2 in Appendix I.

The total a.c. current is

$$i = I_L + I_{ac\ line} + V_r T_r^2 Y_r - V_{sr} T_r Y_r \quad (21)$$

where

$I_L$ is the local load current.

The direct axis component of current is

$$I_d = (I_L + I_{ac\ line})_d + |V_r T_r^2 Y_r| \cos(\Theta'_r + \gamma') - |V_{sr} T_r Y_r| \cos(\Psi'_r + \gamma') \quad (22)$$

where $\Theta'_r$, $\gamma'$, $\Psi'_r$ are the angles of $V_r$, $Y_r$ and $V_{sr}$ respectively w.r.t. the q axis. Recognizing that

$$|V_{sr}| = V_{dr} \sec\alpha + \frac{\Pi}{6} x_{cr} I_{dc} \sec\alpha \quad (23)$$

Substitution of equations (20), (22) and (23) sequentially in equation (19) results into the following equation

$$p^3(\delta/\omega_0) = L(x) + b_1(x)\, E_{fd} + b_2(x) \sec\alpha \quad (24)$$

where

$$L(x) = \frac{V_d E_q'}{X_d' T_{d0}' T_m} + \frac{V_d (X_d - X_d')}{X_d' T_{d0}' T_m} [(I_L + I_{ac\ line})_d + |V_r T_r^2 Y_r| \cos(\Theta'_r + \gamma')]$$

$$b_1(x) = -\frac{V_d}{x'_d T'_{d0} T_m}$$

$$b_2(x) = \frac{V_d}{X_d' T_{d0}' T_m} [T_r Y_r V_{dr} \cos(\Psi'_r + \gamma') + \frac{\Pi}{6} T_r Y_r x_{cr} I_{dc} \cos(\Psi'_r + \gamma')]$$

Note that inclusion of $T'_{q0}$ term in (18) will contribute an extra term in L(x) and $b_2(x)$ each. Following the same line as in Reference [1], the minimum time quasi-optimal control $E_{fd}$ and $\alpha$ can be obtained from the following steps.

1. Compute the maximum and minimum values of r.h.s. of equation (24). Call them $u_{max}$ and $u_{min}$ respectively.

2. Calculate

$$\Sigma = n - \frac{(pn)^2}{2\beta}$$

where

$$\beta = u_{max} \text{ if } pn < 0$$
$$= u_{min} \text{ if } pn > 0$$

3. If $\Sigma \geq 0$ set $\sigma = u_{max}$, otherwise $\sigma = u_{min}$.

4. Determine

$$\Sigma_1 = \frac{\delta}{\omega_0} - \frac{\Pi}{2\omega_0} - \frac{n.pn}{\sigma^2} + \frac{1}{3\sigma^2}(pn)^3$$

$$\Sigma_2 = \Sigma_1 + \frac{\Pi}{2\omega_0}$$

5. If $\Sigma_1 \leq 0$ and $\Sigma_2 \geq 0$

$$u^*(t) = u_{min} \text{ if } \Sigma > 0$$
$$= u_{max} \text{ if } \Sigma < 0$$

6. If $\Sigma_1 > 0$ $\qquad u^*(t) = u_{min}$

$\Sigma_2 < 0$ $\qquad u^*(t) = u_{max}$

Steps 5 and 6 give the time optimal strategies on any integration step and control $E_{fd}$ and $\alpha$ can be worked back from (24). Since the regulator time constants are not zeroes, the controls cannot be bang bang and suboptimal proportional control

$$u_1(t) = K_1 \gamma$$

$$u_2(t) = K_2 \gamma$$

are proposed. $\gamma$ is either $\Sigma_1$, $\Sigma_2$, or $\Sigma$ as the case may be.

Note the proportional scheme does not provide any information on the inverter control. However, a control in the inverter similar to the rectifier one has been found to improve the transient response.

## 5. RESULTS

The Blue Nile Power System [7] as given in Fig. 2 was

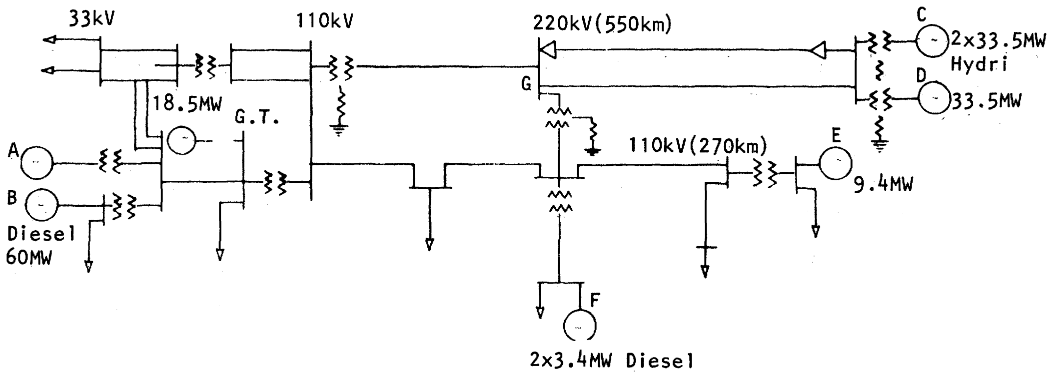


Fig. 2. Power system configuration.

considered for this study. One of the lines between generators CD and busbar G was converted to a DC line for this study. Fault studies were made for various fault locations and also different fault durations. Figures 3 through 8 show the frequency variation, rotor angle field voltage, DC power, terminal voltage and rectifier firing angle variations respectively for a 0.1 sec duration fault on the high tension bus of generator CD. The response recorded were with

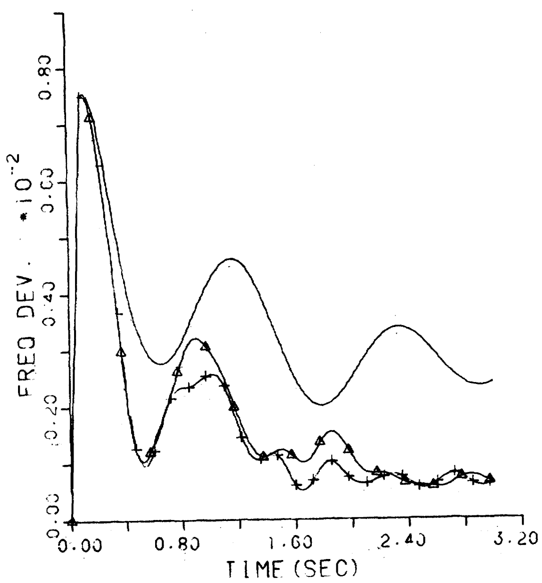


Fig. 3. Normalized frequency deviation characteristics for a three phase fault of 0.1 sec duration on generator bus CD. Responses are with (a) No control (no symbol), (b) with excitation control alone (Δ) and (c) with excitation and rectifier firing control (+).

i) No control. This means no extra stabilizing control other than the internal stabilizing feedback in the excitation system. The responses in the different figures are shown without any symbols. It can be seen that the system is stable but the transient response is poor.

ii) Stabilizing control in the excitation system only. The response of the system was investigated by putting $u_2(t) = u_3(t) = 0$ (notations used as in Reference 1). It can be seen

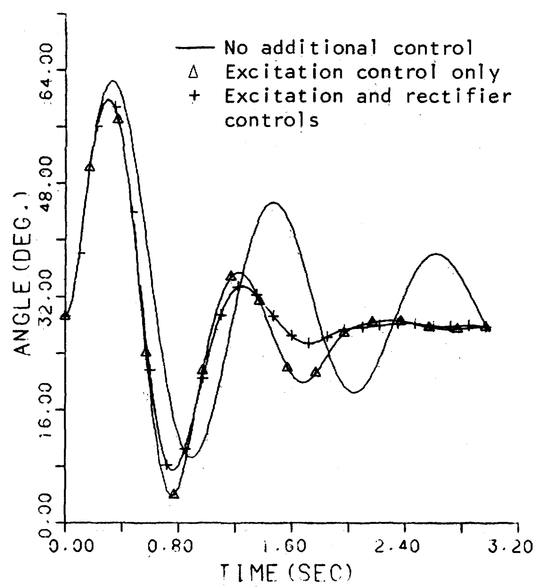


Fig. 4. Rotor angle variation of generator CD (lumped) for 0.1 sec duration fault corresponding to Fig. 3.

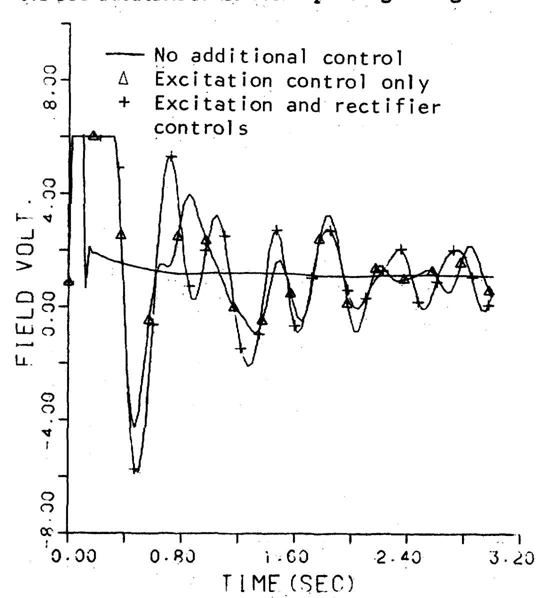


Fig. 5. Field voltage variation of generator CD corresponding to fault conditions of Fig. 3.

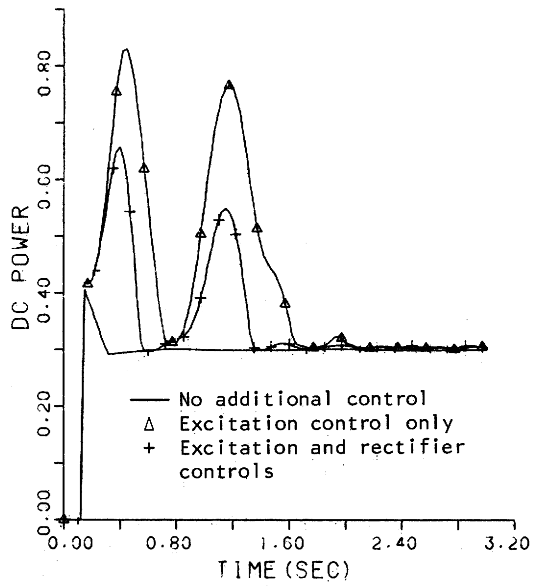


Fig. 6. Transient power flow on the DC line for a 0.1 sec duration fault corresponding to Fig. 3.

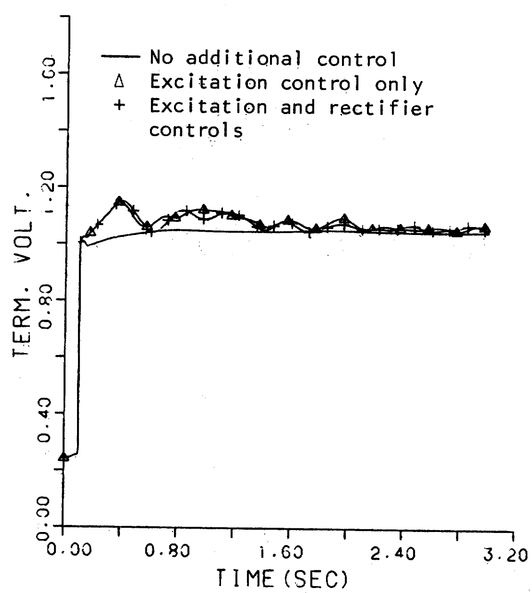


Fig. 7. Terminal voltage variation of generator CD for a 0.1 sec duration fault corresponding to Fig. 3.

that excitation control alone is very effective in damping out the transients. The gains used with the switching functions to make the control proportional were -25 when the states were such that $\Sigma_1 < 0$ and $\Sigma_2 > 0$. Outside these switching boundaries, gains used were -50 each.

iii) A combination of exciter, rectifier and inverter control. A combination of exciter and converter controls as given by the control strategy of section 4 provided the best transient performance as shown in the figures (labelled with + symbol). An inverter control of same magnitude as the rectifier one was seen to improve the transients only slightly. It was observed that with only converter controls, the response was slightly better than with no control. The excitation control in addition to rectifier control proved to eliminate the transients satisfactorily. The gain used in the rectifier stabilizing loop was -25.

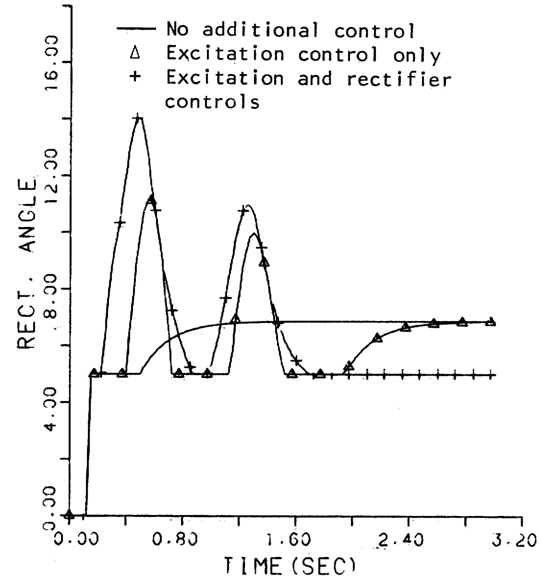


Fig. 8. Rectifier firing angle variation for a three phase fault corresponding to Fig. 3.

## 6. CONCLUSION

This paper demonstrated the application of a sub-optimal feedback control of excitation voltage and converter firing angles to a multimachine ACDC power system. The control is determined in terms of local variables which makes it easy for implementation. It was observed that when the excitation as well as converter firing angle controls are applied, the best transient response is obtained. The excitation control or converter control independently are not so effective. The control scheme suggested retains nonlinearities in the system dynamics and hence it is valid for any type of perturbation in the system, large or small.

## 7. ACKNOWLEDGEMENT

The work reported in this paper was performed under UPM project EE/POWER/28 titled, "Control of HVDC Links in Power System,". The authors acknowledge the support of the University of Petroleum and Minerals, Dhahran, Saudi Arabia towards this research.

## 8. LIST OF SYMBOLS

$x'_d$ direct axis transient reactance
$x'_q$ quadrature axis transient reactance
$E'_q$ transient voltage behind quadrature axes
$E'_d$ transient voltage behind direct axis
$I_{dr}$ rectifier d.c. current
$I_{di}$ inverter dc current
$V_c$ voltage across capacitance in T representation
$T_{dc}$ d.c. line time constant
$C_{dc}$ dc line capacitance
$T$ time constant of transfer function
$G$ forward gain of the transfer function
$I_{ds}$ d.c. current setting

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## 11. APPENDIX II

$$v_{dr} - k_{1r} v_r \cos\alpha + k_2 x_{cr} i_{dr} = 0 \qquad (A2.1)$$

$$v_{di} - k_{1i} v_i \cos\gamma + k_2 x_{ci} i_{di} = 0 \qquad (A2.2)$$

$$v_{dr} - k_{1r} v_{sr} \cos(\Theta_r' - \phi_r) = 0 \qquad (A2.3)$$

$$v_{di} - k_{1i} v_{si} \cos(\Theta_r' - \phi_i) = 0 \qquad (A2.4)$$

$$i_{dr} \cos\phi_r - b_r(t_r v_{sr} \sin\psi'_r - v_r \sin\Theta_r') = 0 \qquad (A2.5)$$

$$i_{dr} \sin\phi_r + b_r(t_r v_{sr} \cos\psi_r' - v_r \cos\Theta'_r) = 0 \qquad (A2.6)$$

$$i_{di} \cos\phi_i + b_i(t_i v_{si} \sin\psi'_i - v_i \sin\Theta'_i) = 0 \qquad (A2.7)$$

$$i_{di} \sin\phi_i - b_i(t_i v_{si} \cos\psi'_{di} - v_i \cos\Theta'_i) = 0 \qquad (A2.8)$$

$$v_{dr} - v_{di} - i_d r_\ell = 0 \qquad (A2.9)$$

Small letters indicate per unit system.

## 10. APPENDIX I

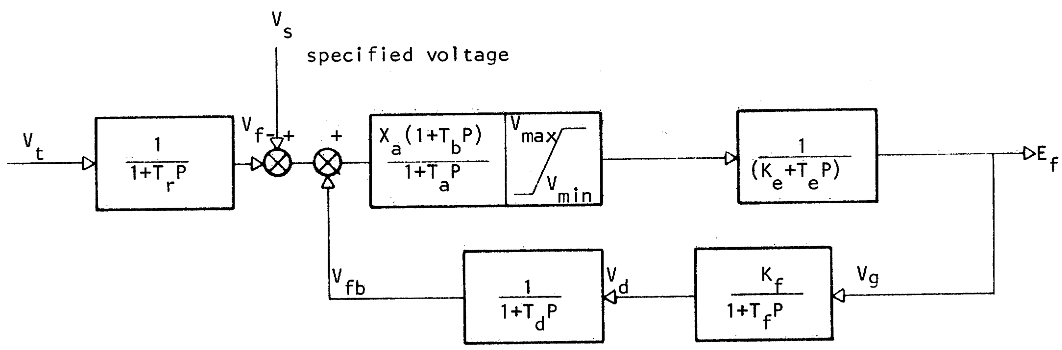


Fig. A1. AVR block diagram.

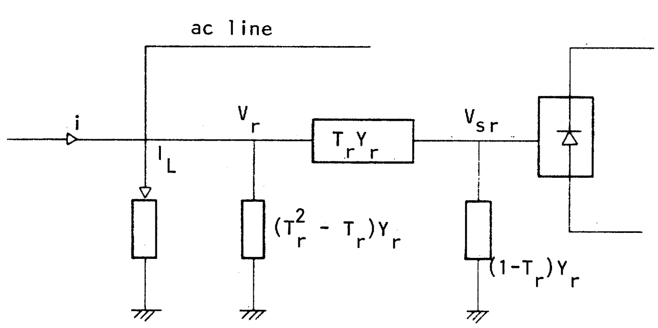


Fig. A2. ACDC rectifier transformer end configuration.

## Discussion

**S. Lefebvre** (Institut de recherche d'Hydro-Quebec, Varennes, Quebec, Canada): The authors must be complimented for their interesting large signal approach in showing that the stability of an AC/dc power system may be improved by appropriately designed stabilizers. I have some specific concerns on the model of the dc controls that was used. Figure 6 shows a very fast dc recovery time which would not normally be expected. I would like to know the assumed rating of the dc link since the power overshoot in Fig. 6 is very large and could lead to commutation failures at the inverters.

Concerning the suitability of decentralized output feedback regulators such as in [3] for handling large perturbations, it is well understood that the selected objective function optimized at one operating point will not be optimal at all operating conditions. However I do believe that well designed output feedback controllers based on small signal variables can often lead to improvement of transient stability in many practical cases. The implementation of such regulators requires washout filters but it is relatively straightforward. Studies such as in [A,B] a test that it is feasible to base the design on the linearized models with of course some additional work. Could the authors kindly benefit us of their experience in this area?

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**I. M. El-Amin** and **A. H. M. A. Rahim:** The authors wish to thank the discusser for his comments. It is true that dc recovery time is fast but no commutation failures were experienced at the inverter end. This may be due to fact that the d.c. link is switched off completely during the fault period. The power system used in the study "The BLUE Nile power system of Sudan- is a small power system characterized by long transmission line (approx. 550 Kms) carrying approximately 65 percent of the power from the end of generators C&D towards the load center. For the purpose of this study one of the a.c. lines was converted to a d.c. line. The transmitted d.c. power is about 30 MW, reaching about 80 MW transiently, and this is equal to the a.c. power that was carried by the line when it was operating as an a.c. line. This conversin enables the authors to test the suitability of the proposed control.

The discusser rightly pointed out that output feedback regulators will not give optimal response for all operating points. And if such controllers have to be implemented it requires some extra work of finding out the optimal gain matrix for all possible operating conditions. Which, in turn, require continuous monitoring of large number of variables to determine the elements of coeffcient matrices. The propoprtional controller suggested requires the various measurements but once the circuits are available, the control is found directly in terms of these variables. Besides, the proportional control has been found to be superior to those obtained by the two optimal output regulator formulations as reported in the companion paper[1].

### REFERENCE


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Manuscript received April 1, 1985