Electric ... wer Systems Research, 5 (1982) 207 - 218

A Weighted Least Squares Method for Determination of Power System Equivalents

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(Received [illegible]arch 15, 1982)

SUMMARY

The effect of several important factors, like the acceleration of convergence, selection of initial gradient, choice of parameter models, selection of output variables, etc., on the convergence [illegible] the parameters of a power system equivalent [illegible] investigated. It is observed that a fifth-order parameter model with two output variables provides little faster convergence near a minimum compared to the reduced-order models. The reduced-order models with fewer outputs are computationally [illegible] and produce reasonable values for the more important parameters. A check on the stability of the model has been proposed to save further computation time.

1. INTRODU[illegible]ION

With the increase in size of power systems the handling and computation of the vast amount of data required for system analysis is becoming increasingly difficult. For example, when new generating units are to be added to an existing power system, a large number of studies are required to find the dynamic interaction of the new plant with the existing system. Man[illegible] [illegible]f these studies require good dynamic m[illegible] of all the generators and their control devices, i.e. simulation of the entire system (the new and the existing ones) by means of hundreds of differential equations and algebraic relations.

For each study, these equations have to be solved over and over again. The question is, if we are interested in the dynamic interaction of the new plant with the existing power system, is it possible to replace the entire existing system [illegible]ith a simpler equivalent for simulation studies? This would then save a great deal of computation.

Various studies on the determination of power system equivalents have been reported in the literature [1 - 4]. The method of least squares has been used to estimate directly the unknown parameters of a synchronous machine [5]. Attempts have also been made to estimate the parameters of a synchronous machine equivalent of a grid system [6, 7]. The advantage of this method is that since the parameters to be estim[illegible] [illegible] qu[illegible], it [illegible] easier to make initial guesses.

A major difficulty with this nonlinear estimation process is the convergence of the algorithm. While convergence in isolated cases for a wide range of acceleration factors can be achieved [5, 6], in general, to make the method applicable to a real system study, a set of convergence criteria should be prespecified from simulation studies. Choice of output variables for good convergence, simplification in terms of calculations of state and output gradients, choice of parameter models and their possible reduction, stability of the estimated models, etc., require careful investigation. All these points are addressed in this article.

2. THE LEAST SQUARES METHOD AS APPLIED TO A POWER SYSTEM PROBLEM

Figure 1 shows a power system configuration in which the local system under study may contain at least one generator connected by a number of links to the grid system. It may be possible to represent both the local and grid system by one or more coherent

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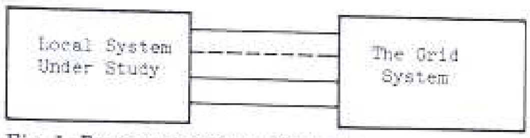


Fig. 1. Power system configuration.

groups of machines. After selecting the appropriate state variable vector X, the input vector U and the output Y can be related by the following standard equations:

$$\dot{X} = A(\alpha)X + BU \quad (1)$$

$$Y = C(\alpha)X \quad (2)$$

where α is a vector of parameters by which the grid system is characterized. The input U and the coefficient matrix B are quantities of the study system and hence are independent of α. The value of α is chosen to minimize

$$J = \frac{1}{2}\int_{t_0}^{t_f} (Y_r - Y)^T R (Y_r - Y)\, dt \quad (3)$$

Here, Y_r represents the measured outputs of the local system obtained from field measurements or from simulation of the entire local or grid system. This could be a vector of terminal voltage variation ΔV_t, real and reactive power changes ΔP and ΔQ, frequency variation $\Delta\omega$, etc., of the local system. Expanding the output Y in a Taylor series and retaining only the first-order variations, the partial derivative equation $\partial J/\partial \Delta\alpha = 0$ gives the following set of algebraic relationships [5, 7]:

$$D\Delta\alpha = N \quad (4)$$

where

$$D = \int_{t_0}^{t_f} \left[\frac{\partial Y}{\partial \alpha}\right]^T_{\alpha=\alpha_0} R \left[\frac{\partial Y}{\partial \alpha}\right]_{\alpha=\alpha_0} dt \quad (5)$$

$$N = \int_{t_0}^{t_f} \left[\frac{\partial Y}{\partial \alpha}\right]^T_{\alpha=\alpha_0} R[Y_r - Y(\alpha_0)]\, dt \quad (6)$$

The variables $\partial Y/\partial\alpha$ are obtained from the following equations:

$$\left.\frac{\partial Y}{\partial \alpha}\right|_{\alpha=\alpha_0} = C(\alpha_0)\left.\frac{\partial X}{\partial \alpha}\right|_{\alpha=\alpha_0} + \left.\frac{\partial C(\alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0} X \quad (7)$$

$$\frac{d}{dt}\left[\frac{\partial X}{\partial \alpha}\right]_{\alpha=\alpha_0} = A(\alpha_0)\left.\frac{\partial X}{\partial \alpha}\right|_{\alpha=\alpha_0} + \left.\frac{\partial A}{\partial \alpha}\right|_{\alpha=\alpha_0} X(\alpha_0) \quad (8)$$

The recursion formula for computing successive estimates of α is

$$\alpha^i = \alpha^{i-1} + K\Delta\alpha^i \quad (9)$$

i is the iteration count and K is a matrix of acceleration factors. For simplicity, K will be considered to be a diagonal matrix.

3. FACTORS AFFECTING CONVERGENCE

A few of the important factors which affect the rate of convergence of the unknown parameters to the desired values are (i) the initial guess of the values of α, (ii) proper selection of the acceleration factor matrix K, (iii) the choice of the initial state gradient $\partial X/\partial\alpha$, (iv) the order of the parameter model. For the nonlinear problem stated, convergence of the parameters to the desired values is never guaranteed. They may converge if the initial guesses are reasonably close to the desired values. Again, the algorithm may be very sensitive to the variation of certain parameters (e.g., the transient reactance of the equivalent model) compared to others. In general, good engineering judgement is required in selecting the initial values.

As mentioned, for simulation studies a set of acceleration factors (K_{ii}) may be arrived at easily where the desired values of parameters are known *a priori*. One difficulty with nonlinear problems is that a local minimum or a point of inflexion may be approached when variation of α would seem very small and the algorithm may be thought to have converged. In order to avoid a wrong assumption, simultaneous evaluation of $\Delta\alpha$ and J has to be carried out and acceleration factors should be chosen to reflect changes in both. In the following section such a scheme of acceleration factors has been proposed.

In order to determine the gradients in eqns. (7) and (8), the initial values of $\partial X/\partial\alpha$ are required. Reference 5 shows how to calculate some of the initial state gradients from steady state relationships. If the equivalencing is done through a number of machine models the analysis will become very complicated, especially for large systems. Initial state gradients are zero for the study system because the states are independent of parameter variation. It has been observed that if all the initial guesses for the state gradients are considered to be zero, very little error is introduced in terms of the estimation process. The initial values of all the state gradients were assumed to be zero in the calculations of the following section.

The normal procedure in this type of iterative study is to set an iteration count. If the number of iterations exceeds this number, then the procedure is restarted with a new guess. Instead of carrying out a maximum number of iterations, if a stability check is performed on the model at each iteration and corrective action taken then, a great deal of computer time can be saved. In this study, an eigenvalue check was performed between each iteration and the location of poles of the estimated system checked before the next iteration.

Higher order parameter models, in addition to increasing the dimensions of eqns. (7) and (8), also increase the order of the acceleration matrix K in eqn. (9). The elements of the K matrix are normally chosen through trial and error. Since with a lower order parameter model fewer computations are required and a smaller number of acceleration factors is to be controlled, the more important parameters can be estimated approximately from a reduced-order model. If further accuracy in the power system model is then desired, a higher order parameter model can be solved starting from the lower order model estimates. One such reduction is reported in the following section. Note that this is different from the usual model reduction from system eigenvalue considerations. Care should also be taken that the parameter model is not over-reduced. In such a case, the large error between the measured output from the power system and the reduced-model output will cause the nonlinear estimation algorithm to diverge.

4. AN EXAMPLE

A simple power system model with one equivalent generator in the study system and an equivalent synchronous motor for the grid system given in Fig. A1 in the Appendix is considered. Combining the steady state current voltage relations shown in Fig. A2 (Appendix), the dynamic relations of the system are given in the block diagram of Fig. A3 (Appendix). Two parameter models are considered:

(a) A five-parameter model containing H, D, T'_{d0}, X, and X' for the third-order synchronous machine equivalent of the grid system. (The symbols stand for inertia constant (s), damping coefficient, open-circuit field time constant, synchronous reactance and transient reactance of the equivalent motor, respectively.)

(b) A fourth-order parameter model containing H, T'_{d0}, X and X' for the same synchronous motor equivalent.

Output records were taken from ΔV_t and $\Delta\omega$ and the convergence characteristics were compared with that from ΔV_t of the local generator alone. The general estimation algorithm is shown in Fig. 2. From a large number of case studies of varying loading conditions, power factors, interties, etc., it has been found that the use of a variable acceleration factor dependent on both $\Delta\alpha$ and the normalized cost index $\hat{J}\ (= J/J_{yr})$ provides a smooth convergence of the parameter when the initial guesses of α are in the range 0.6 - 3.5 per unit. The strategy of updating the parameters as shown in Fig. 2 is as follows:

(a) if $|\Delta\alpha_i| > ZT_i$ or $|\Delta\alpha_i| < ZT_i$ but $\hat{J} > \sigma_1$, then

$$\alpha_{\mathrm{new}\,(i)} = \alpha_{\mathrm{old}\,(i)} \pm 0.3\alpha_{\mathrm{old}\,(i)}$$

± depending on the sign of $\Delta\alpha_i$
$i = 1, 2, \ldots$

(b) if $|\Delta\alpha_i| < ZT_i$ and $\sigma_2 < \hat{J} < \sigma_1$,

$$\alpha_{\mathrm{new}\,(i)} = \alpha_{\mathrm{old}\,(i)} + ACF_{2(i)} \cdot \alpha_{\mathrm{old}\,(i)} \cdot \Delta\alpha_i$$

(c) if $|\Delta\alpha_i| < ZT_i$ and $\sigma_3 < \hat{J} < \sigma_2$,

$$\alpha_{\mathrm{new}\,(i)} = \alpha_{\mathrm{old}\,(i)} - ACF_{3(i)} \cdot \alpha_{\mathrm{old}\,(i)} \cdot |\Delta\alpha_i|$$

(d) if, however, $\hat{J} < \sigma_3$,

$$\alpha_{\mathrm{new}\,(i)} = \alpha_{\mathrm{old}\,(i)} + ACF_{3(i)} \cdot \alpha_{\mathrm{old}\,(i)} \cdot |\Delta\alpha_i|$$

The vectors ZT, ACF_2, ACF_3 for the five-parameter model are:

$$ZT = [0.137\ 5.2\ 5.2\ 8.0\ 5.2]^T \times 10^{-2}$$

$$ACF_2 = [60\ 2.0\ 2.0\ 2.0\ 2.0]^T$$

$$ACF_3 = [6.0\ 0.2\ 0.2\ 0.3\ 0]^T$$

$$\sigma_1 = 18.3 \times 10^{-4},\ \sigma_2 = 5.6 \times 10^{-5}$$

$$\sigma_3 = 1.5 \times 10^{-5}$$

For two outputs, the numbers σ_1 and σ_2 are 15×10^{-4} and 2.9×10^{-5}, respectively, and the third acceleration factor is not necessary.

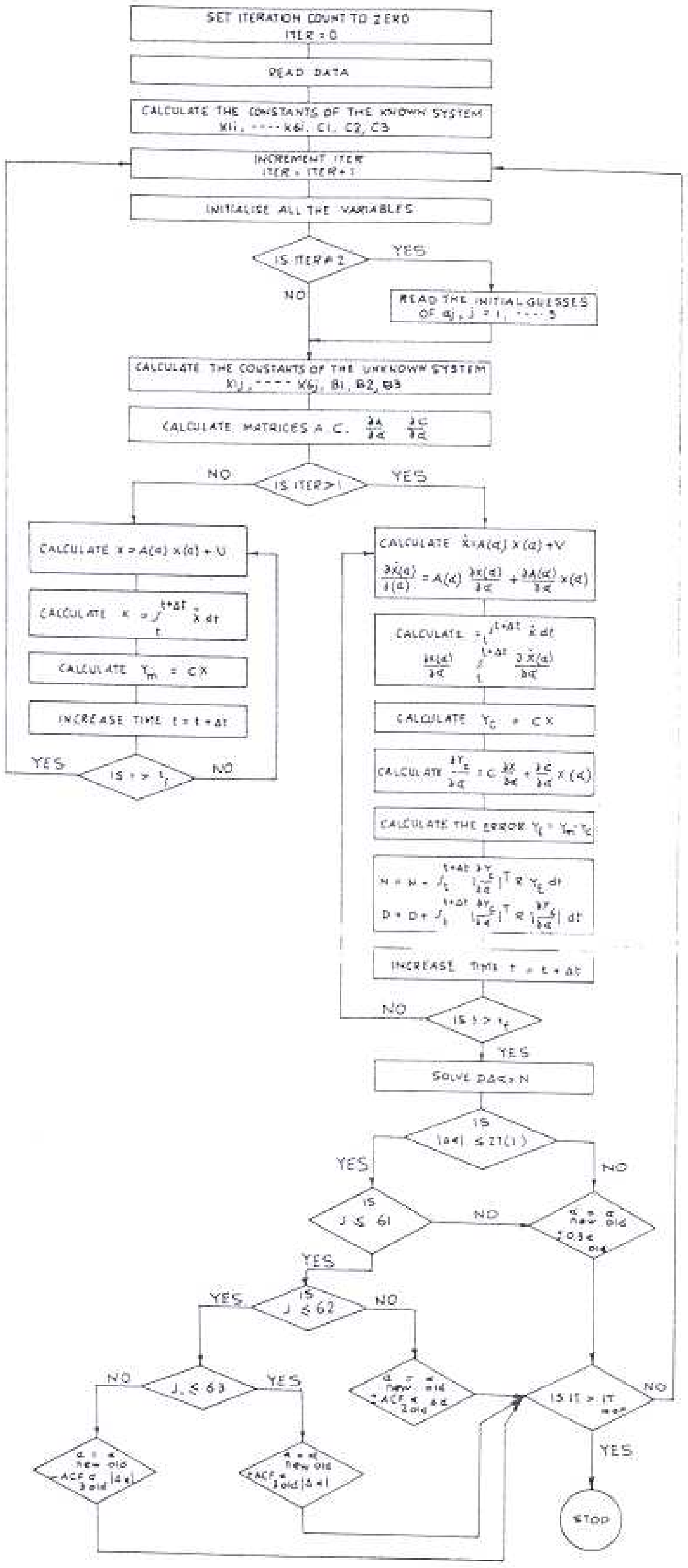


Fig. 2. An improved algor
parameter estimation.

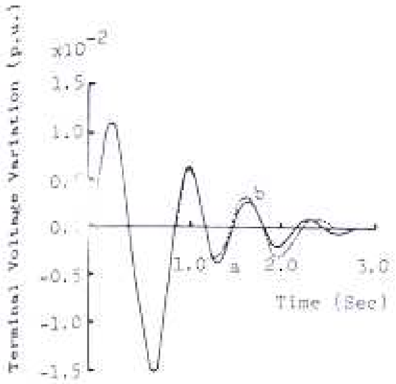


Fig. 3. The terminal voltage variation V_{1r} for a pulsed mechanical input considering (a) a 5th-order parameter model and (b) a 4th-order parameter model.

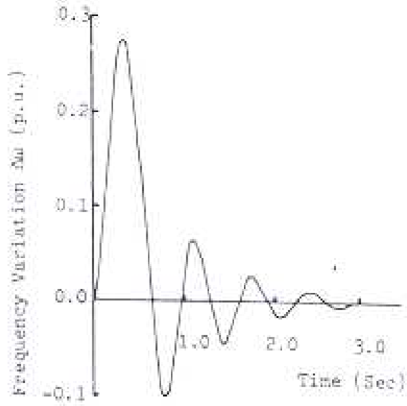


Fig. 4. The frequency variation $\Delta\omega_r$ of the local generator for a pulsed mechanical input (five-parameter model).

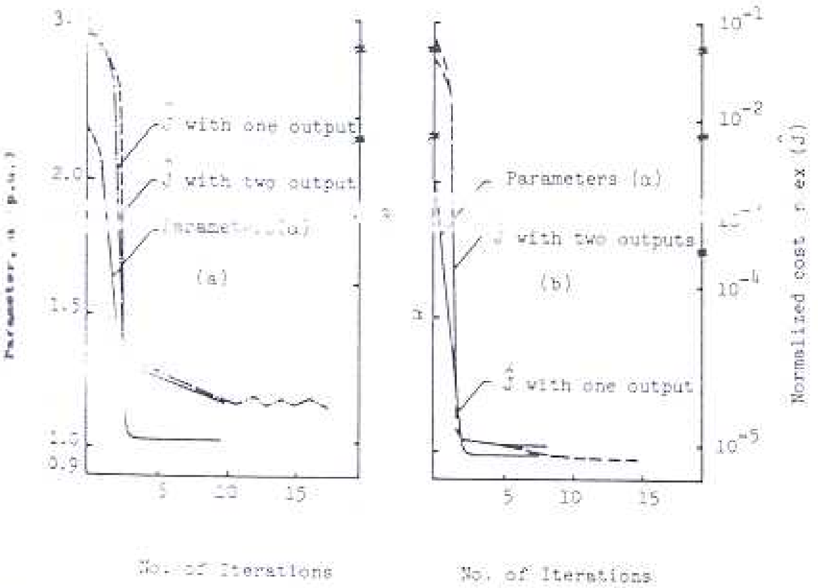


Fig. 5. Parameter variations and comparison of cost indices for the two output records with an initial guess of (a) 2.5 p.u. and (b) 2 p.u. Local system P_1 = 0.9 p.u. at 0.9 p.f. lag, local load G = 0.35 p.u. at 0.9 lag; R = 0.04 and X = 0.5 p.u. Five-parameter model, parameter variation from two outputs.

These numbers are a compromise for all of the few hundred cases studied. Figures 3 and 4 show the reference outputs Y_r; the dashed curve is the terminal voltage output when damping is not considered in the parameter model. Some plots of parameter variation with two outputs are given in Figs. 5 - 12. They also show a comparison of the values of J when the estimation is performed on the basis of one output. Note that, for particular cases, faster convergence could be obtained by varying some of the acceleration factors, but the objective was to arrive at unique numbers for the particular power system under study.

As can be seen from Figs. 5 and 6, it is possible to bring the parameters to the desired values in two or three iterations when the initial estimates are all regular numbers like 2.0, 2.5 p.u., etc. However, when the guess is random, the convergence is slower (Figs. 7, 8). In real applications, however, the desired values are not known and any initial guess of α will amount to a random choice in simulation studies. As can be seen from all these plots, inclusion of frequency variation does

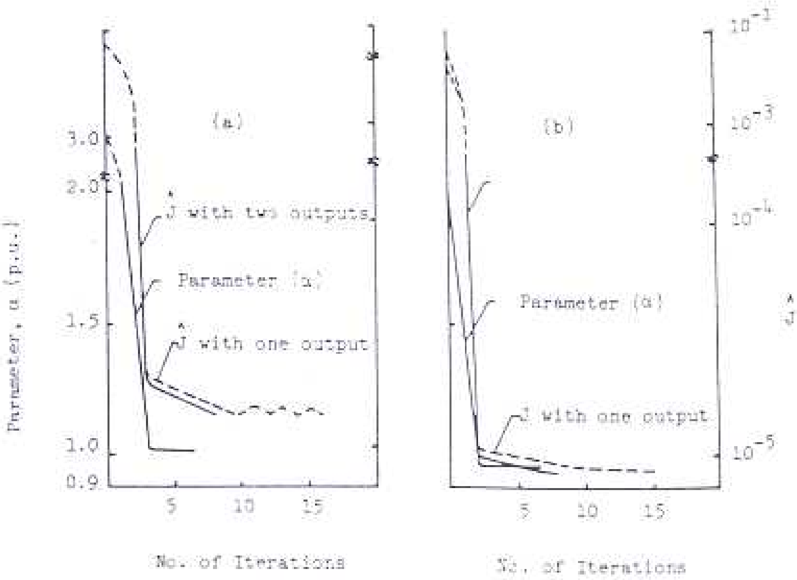


Fig. 6. As for Fig. 5, but (a) α_0 = 3.0 p.u. and (b) α_0 = 2.0 p.u. Local load G = 0.25 at 0.9 lag.

not reduce $\hat{J}$ significantly. In these studies, equal weights on terminal voltage and frequency variation were provided. Since the possibility of noise terms in the frequency measurement is greater than that in the terminal voltage, normally its weight in the cost index will be less, thus bringing the $\hat{J}$ values for the two cases closer.

The damping term in the electromech swing equation is normally very smal hence can be ignored in many power s studies. The effect of dropping the da term from the estin ... algorithm was tigated. As can be s... 3, the diff in the response with or without damp very small in the early part of the transi

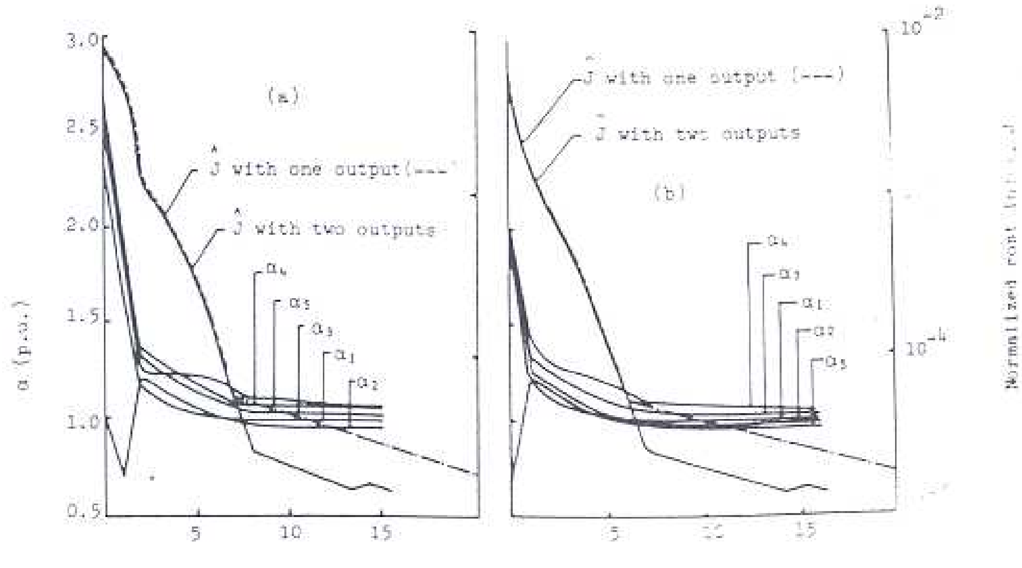
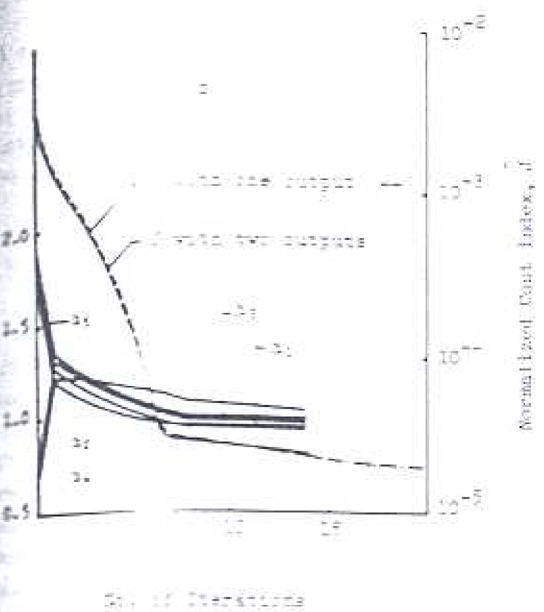


Fig. 9. (*For legend please see facing page.*)



[illegible]. 7. Parameter variations and cost indices against iteration counts for three random guesses of initial [illegible] of α. All operating conditions are the same as [illegible] 6, except [illegible] lag.

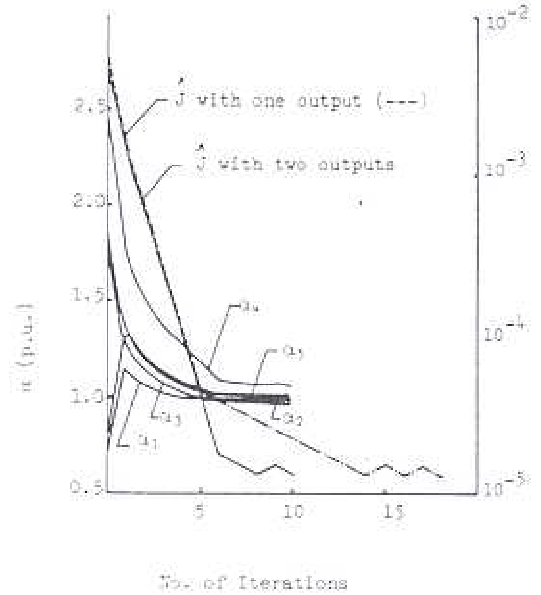


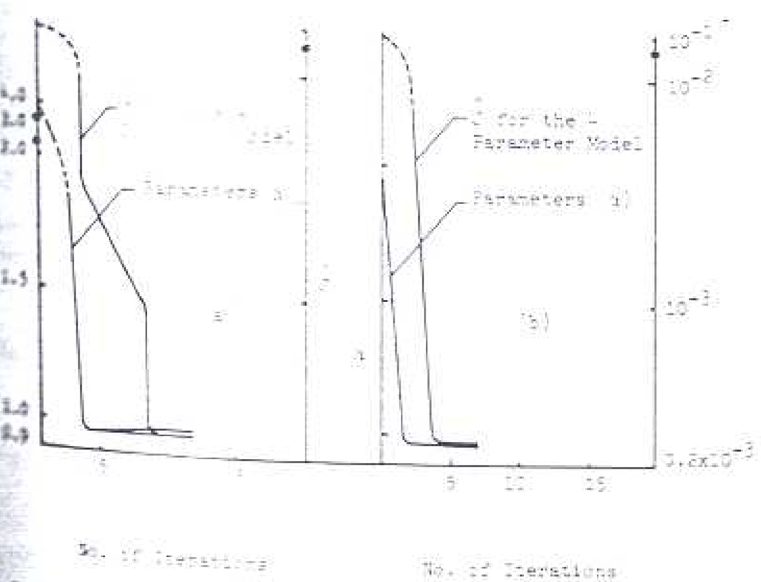
Fig. 8. As for Fig. 7 for a different random choice of initial α. R = 0.06 and X = 0.5 p.u.

Figures 9 - 12 show the parameter variations and normalized cost indices for the four-parameter model [illegible] damping is kept constant at a small [illegible] value. It is clear that the other parameters can be estimated reasonably well when keeping the damping at a constant value. From a study of a large number of cases, especially those with random initial guesses for the parameters, the following set of acceleration factors has been found to provide rapid convergence:

$$ZT = [1.2 \;\; 0.49 \;\; 0.49 \;\; 1.2]^{T}$$

$$ACF_2 = [0.06 \;\; 0.06 \;\; 0.1 \;\; 0.06]^{T}$$

$$\sigma_1 = 20 \times 10^{-3}, \sigma_2 = 3.4 \times 10^{-4}$$



[illegible] 9. Parameter variations and cost indices for a four-parameter model with (a) α_0 = 4.0 p.u. and (b) α_0 = 2.0 [illegible]. Operating quantities are P_i = 0.9 at 0.9 lag, G = 0.5 at 0.9 lagging p.f. R = 0.04 and X = 0.5.

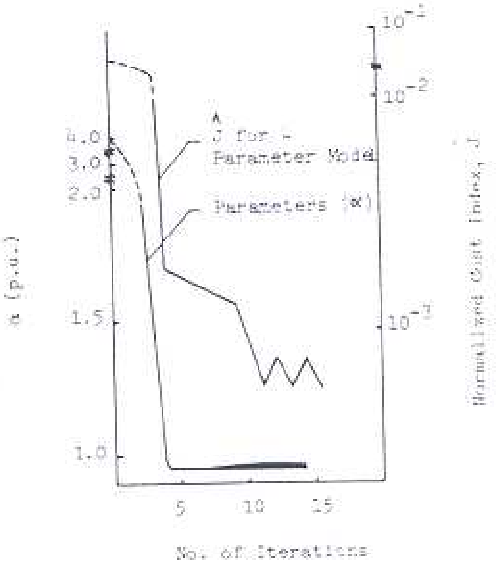


Fig. 10. Similar to Fig. 9 but with initial guess of parameter α = 4.0 p.u. G = 0.25 p.u. at 0.9 lag.

Quantities ACF_3 and σ_3 are not required since J does not decrease any further because of the steady state error between the two model outputs. The parameters reach the normal tolerance band faster than in the five-parameter model. Also, as expected, computation time per iteration is reduced. On an IBM 3033 N system, it was observed that the CPU time for the four-parameter model per iteration was 2.3 s, while it was about 3 s for the five-parameter model. The four-parameter model is simpler, more efficient in terms of computation, and model checking in between iterations is easier.

5. CONCLUSIONS

This paper investigates the convergence of the least squares algorithm applied to a power system problem. For the algorithm to be reliable and hence applicable to off-line computation, it must be assured that the parameters continue to converge for a certain set of acceleration factors and for a reasonable initial estimate of the parameters. Checking the stability of the model in between iterations is suggested. This will save computation time in the long-run in that the algorithm can be stopped or corrective action taken before the maximum number of iterations is reached. This study also shows that with more measurable outputs and a [illegible] order parameter model, improved convergence characteristics can be obtained. However, the amount of improvement may not be justified in terms of computation time and the possibility of introduction of more noise with more measurable outputs. The unknown parameters of the

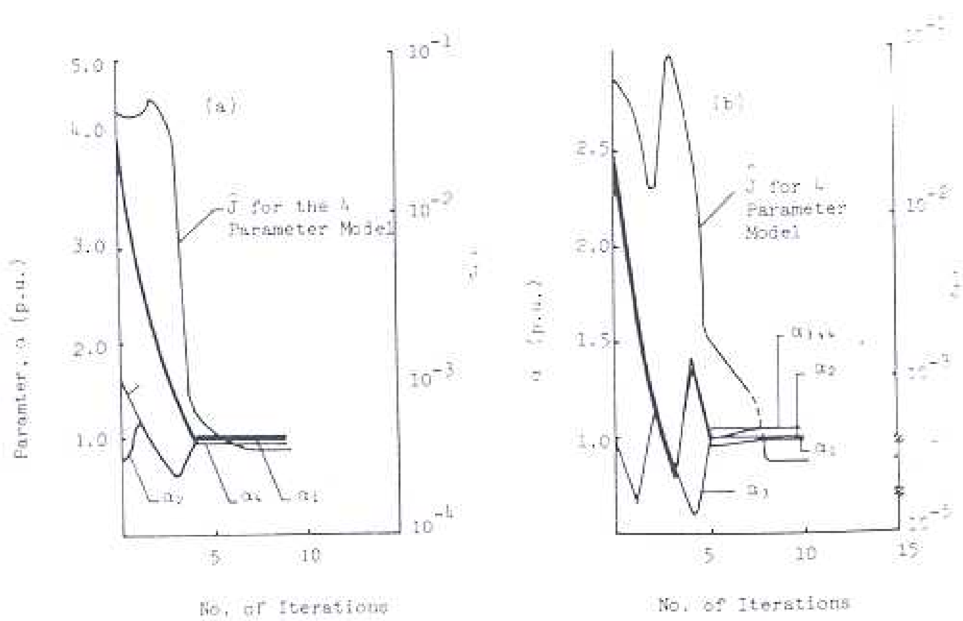


Fig. 11. Parameter variations and cost indices with various random guesses of initial α (four-parameter model). (a) Operating quantities as in Fig. 9, (b) P_1 = 0.9 at 0.9 load p.f.

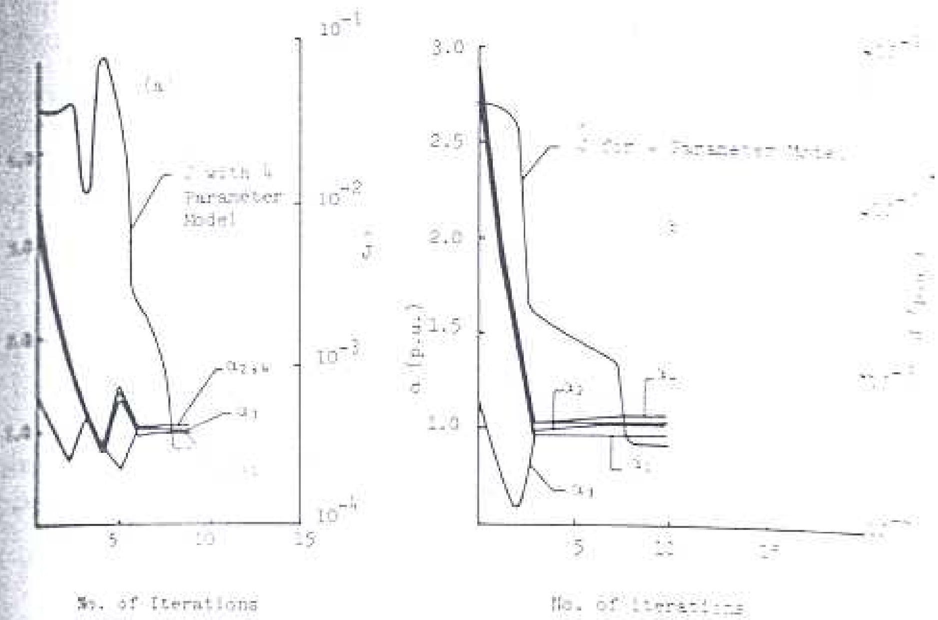


Fig. 12. α and $\bar{J}$ plots for random choice of initial α. (a) Operating quantities as in Fig. 11(b) with a rand[illegible] ... and (b) a different [illegible]

...uivalent system can be estimated reasonably ...ll from a reduced four-parameter model — ...e amount of com[illegible]tion is less and a lower ...mber of accel[illegible]actors has to be con-...lled.

ACKNOWLEDGEMENT

The authors wish to acknowledge the facilities provided by the University of Petroleum and Minerals towards this work.

...PENDIX

The local study system and the equivalent grid system are each considered to be represented ...y one synchronous machine, as in ref. 5. The same nominal parameter values were considered. ...he terminology, symbols, etc., are maintained for easy reference. Note that the expressions are ...ifferent in most cases. The system configuration and phasor diagram are given in Figs. A1 and ...2, respectively.

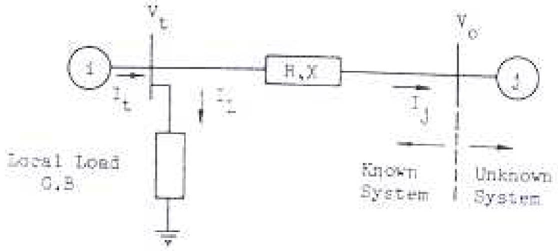
Using the nomenclature of Fig. A1, the terminal equations for the local generator are

$$\ldots = (1 + ZY)V_t - V_0$$

...king components of I, V_t and V_0, using the symbols in the phasor diagram (Fig. A2) and ...earizing about nominal values designated by subscript 0, the following equations for the study ...stem (i) can be obtained:

$$\begin{bmatrix} \Delta I_{di} \\ \Delta I_{qi} \end{bmatrix} = \begin{bmatrix} Y_{di} \\ Y_{qi} \end{bmatrix} \Delta E'_{qi} + \frac{V_0}{Z_i^2} \begin{bmatrix} X_1 & -R_2 \\ R_1 & X_2 \end{bmatrix} \begin{bmatrix} \sin \delta_{i0} \\ \cos \delta_{i0} \end{bmatrix} \Delta\delta_i - \frac{1}{Z_i^2} \begin{bmatrix} R_2 & X_1 \\ -X_2 & R_1 \end{bmatrix} \begin{bmatrix} \sin \delta_{i0} \\ \cos \delta_{i0} \end{bmatrix} \Delta V_0$$

$$= \begin{bmatrix} Y_{di} \\ Y_{qi} \end{bmatrix} \Delta E'_{qi} + \begin{bmatrix} F_{di} \\ F_{qi} \end{bmatrix} \Delta\delta_i + \begin{bmatrix} G_{di} \\ G_{qi} \end{bmatrix} \Delta V_0 \qquad \text{(A1)}$$



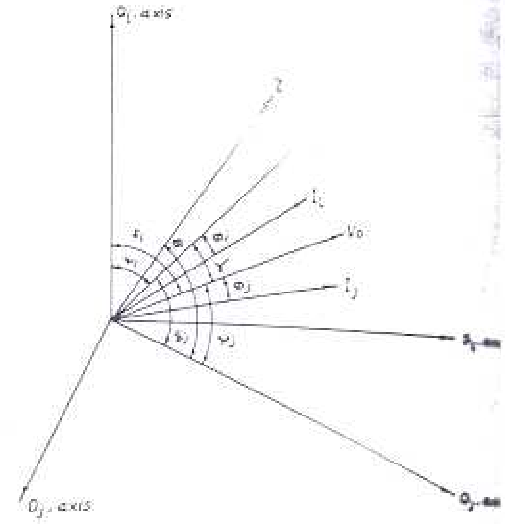


Fig. A1. Equivalent power system configuration.

Fig. A2. Phasor diagram showing voltage, current, etc., of the study and grid equivalent systems.

where

$Y_{di} = (C_1'X_1 - C_2'R_2)/Z_i^2$ $\quad Y_{qi} = (C_1'R_1 + C_2'X_2)/Z_i^2$

$1 + ZY = C_1' + jC_2'$ $\quad R_1 = R - C_2'X_{di}'$ $\quad R_2 = R -$

$X_1 = X + C_1'x_{qi}$ $\quad X_2 = X + C_1'x_{di}'$ $\quad Z_i^2 = R_1R_2 + X_1X_2$

Similarly the equations for the equivalent grid system (j) are

$$\begin{bmatrix}\Delta I_{dj}\\ \Delta I_{qj}\end{bmatrix} = \begin{bmatrix}Y_{dj}\\ Y_{qj}\end{bmatrix}\Delta E_{qj}' + V_{t0}\begin{bmatrix}[illegible]\sin\delta_{j0} & Y_{qj}\cos\delta_{j0}\\ Y_{qj}\sin\delta_{j0} & -\frac{X_e'}{Z_j^2}\cos\delta_{j0}\end{bmatrix}\Delta\delta_j + \begin{bmatrix}Y_{qj}\sin\delta_{j0} & -Y_{qj}\cos\delta_{j0}\\ \frac{X_e'}{Z_j^2}\sin\delta_{j0} & -Y_{qj}\cos\delta_{j0}\end{bmatrix}\Delta I$$

$$= \begin{bmatrix}Y_{dj}\\ Y_{qj}\end{bmatrix}\Delta E_{qj}' + \begin{bmatrix}F_{dj}\\ F_{qj}\end{bmatrix}\Delta\delta_j + \begin{bmatrix}G_{dj}\\ G_{qj}\end{bmatrix}\Delta V_t$$

where

$Y_{dj} = -X_e/Z_j^2$ $\quad Y_{qj} = Z\sin\theta/Z_j^2$ $\quad X_e = x_j + Z\cos\theta$

$Z_j^2 = (Z\sin\theta)^2 + (Z\cos\theta + x_j)(Z\cos\theta + x_j')$

and θ is the angle between vector Z and the quadrature axis of the unknown system. Substit the expressions for ΔI_{di} and ΔI_{qi} into the expression for ΔT_{ei} we get,

$$\Delta T_{ei} = K_{1i}\,\Delta\delta_i + K_{2i}\,\Delta E_{qi}' + C_1\,\Delta V_0$$

$$\Delta E_{qi}' = \frac{1}{K_{3i} + pT_i'}\,\Delta E_{fdi} - K_{4i}\,\Delta\delta_i - C_2\,\Delta V_0$$

$$\Delta V_t = K_{5i}\,\Delta\delta_i + K_{6i}\,\Delta E_{qi}' + C_3\,\Delta V_0$$

where

$$\begin{bmatrix} K_{1i} \\ K_{2i} \\ C_1 \end{bmatrix} = \begin{bmatrix} F_{di} & F_{qi} \\ Y_{di} & Y_{qi} \\ G_{di} & G_{qi} \end{bmatrix} \begin{bmatrix} (x_q - x'_d) I_{qi0} \\ E'_{qi0} + (x_q - x'_d) I_{di0} \end{bmatrix} + \begin{bmatrix} 0 \\ I_{qi0} \\ 0 \end{bmatrix} \tag{A6}$$

$$K_{3i} = 1 + (x_d - x'_d) Y_{di} \qquad K_{4i} = (x_d - x'_d) F_{di} \qquad C_2 = (x_d - x'_d) G_{di} \tag{A7}$$

$$\begin{bmatrix} K_{5i} \\ K_{6i} \\ C_3 \end{bmatrix} = \begin{bmatrix} F_{di} & F_{qi} \\ Y_{di} & Y_{qi} \\ G_{di} & G_{qi} \end{bmatrix} \begin{bmatrix} -x'_{di} & V_{qi0}/V_{t0} \\ x_{qi} & V_{di0}/V_{t0} \end{bmatrix} + \begin{bmatrix} 0 \\ V_{qi0}/V_{t0} \\ 0 \end{bmatrix} \tag{A8}$$

Corresponding equations for the synchronous motor are

$$\Delta T_{ej} = K_{1j} \Delta\delta_j + K_{2j}\, \Delta E'_{qj} + B_1\, \Delta V_t \tag{A9}$$

$$\Delta E'_{qj} = \frac{1}{pT' + K_{3j}} K_{4j}\, \Delta\delta_j + B_2\, \Delta V_t \tag{A10}$$

$$\Delta V_0 = K_{5j}\, \Delta\delta_j + K_{6j}\, \Delta E'_{qj} + B_3\, \Delta V_t \tag{A11}$$

Expressions for K_{1j}, K_{2j} and B_1 are similar to those of K_{1i}, K_{2i} and C_1, respectively, but the current directions are reversed. K_{4j} and B_2 for the jth machine are similar to K_{4i} and C_2 for the ith: $K_{3j} = 1 - (x_j$ [illegible] and B_3 are similar to C_3, K_{6i} and C_3, respectively, but with a reversal of polarity for the voltages. Hence the diagram in Fig. A3 follows.

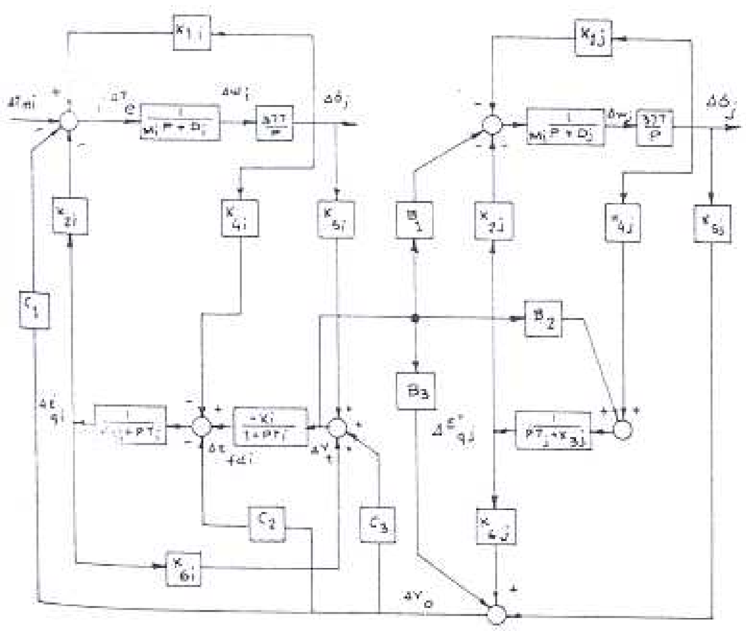


Fig. A3. Block diagram showing the combined study and grid equivalent systems.

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