

# A Robust STATCOM Damping Controller for a Multi-machine Power System

S.F.Faisal

A.H.M.A.Rahim

Department of Electrical Engineering  
K.F.University of Petroleum and Minerals  
Dhahran, Saudi Arabia.

**Abstract:** This paper presents a robust STATCOM controller design for damping transients in a multi-machine power system. The method of multiplicative uncertainty is employed to model the variations in the power system operation. A graphical loop shaping technique incorporating a particle swarm optimization procedure has been used to realize the fixed parameter controller. A balanced realization based model reduction technique was used for reducing the order of the system. The robust design was implemented on a 4-machine system considering various loading and disturbance conditions. It is observed that the robust STATCOM controller provides very good damping profile for a wide range of operation

*Index Terms* - Damping control, loop-shaping method, particle swarm optimization, robust control, STATCOM.

## 1. INTRODUCTION

Synchronous static compensators (STATCOM) are known to offer a number of performance advantages for reactive power control applications over the conventional approaches because of its greater reactive current output capability at depressed voltage, faster response, better control stability, lower harmonics, and smaller size, etc. [1,2]. The STATCOM provides shunt compensation in a similar way to static var compensation but utilizes a voltage source converter rather than shunt capacitors and reactors [3,4].

Many control strategies have been suggested in the literature in recent years on STATCOM control for damping improvement. Most of these controllers are designed through linear models making them operating point dependent [1, 2, 5]. Robust control studies on simple single machine system have shown good damping characteristics [6,7]. Investigation of the damping aspect of STATCOM control for multi-machine power systems, particularly robust designs, is very limited. Fuzzy logic based controller STATCOM controllers have been reported in recent publications [8-10]. Though robust in performance, implementation of these controllers is not yet feasible.

Damping enhancement of a 4-machine power system through robust STATCOM control is considered in this study. A particle swarm optimization technique has been embedded in the graphical-loop shaping procedure to accelerate the construction of the controller. The STATCOM control has been tested over a range of operation considering a number of disturbances.

## 2. THE POWER SYSTEM MODEL

The configuration of a 4-machine power system is shown in Fig.1. STATCOMs are installed at the middle of the transmission lines connecting generators to the rest of the grid.

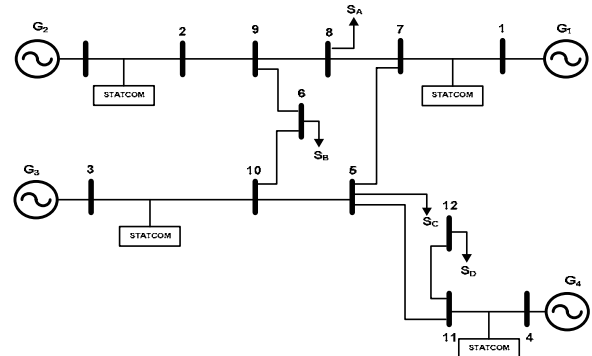


Fig. 1 Multi-machine power system with STATCOM

The synchronous generators are represented by a two-axis model for the internal voltages and the swing equations and the exciter by the first order IEEE type-ST model. The STATCOM is represented by a first order differential equation relating the STATCOM DC capacitor voltage.

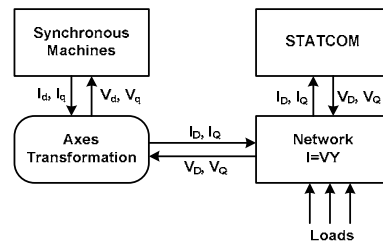


Fig. 2 Block diagram showing transformation of variables

The dynamic models for *i*-th machine including the exciter and the STATCOM can be given by the following,

$$\begin{aligned}
\dot{e}'_{di} &= \left[ -e'_{di} + (x_{qi} - x'_{di}) I_{qi} \right] \frac{1}{T'_{qoi}} \\
\dot{e}'_{qi} &= \left[ E_{fdi} - e'_{qi} - (x_{di} - x'_{di}) I_{di} \right] \frac{1}{T'_{doi}} \\
\dot{\omega}_i &= -\frac{1}{2H_i} [P_{mi} - P_{ei} - K_{Di} \omega_i] \\
\dot{\delta}_i &= \omega_o \omega_i \\
\dot{E}_{fdi} &= -\frac{1}{T_{Ai}} E_{fdi} - \frac{K_{Ai}}{T_{Ai}} (V_{toi} - V_{ti}) \\
\dot{V}_{DCi} &= \frac{m_i}{C_{DCi}} [I_{sdi} \cos \psi_i + I_{sqi} \sin \psi_i]
\end{aligned} \tag{1}$$

The block diagram in Fig. 2 shows the conversion of the variable from the synchronously rotating reference frame [D-Q] to individual machine frames [d-q]. The [D-Q] quantities relate to [d-q] variables by the transformation matrix  $T_r = \text{diag}[e^{j(\delta-\pi/2)}]$ . In this article, the network quantities were transformed to the generator frames for relative ease in controller design. The loads are represented as constant impedances and the load buses eliminated. The non-state variables in (1) are expressed in terms of the state variables using the reduced network equations. The dynamic equations for the multi-machine system are then expressed as,

$$\dot{x} = f[x, u] \tag{2}$$

where, the state  $x$  is a vector of  $[e'_d, e'_q, \omega, \delta, E_{fd}, V_{DC}]^T$  for each machine, and  $u$  is the control vector.

### 3. THE ROBUST CONTROLLER DESIGN USING LOOP-SHAPING

The robust control design for the synchronous generator-STATCOM system starts by linearizing the nonlinear set of equations (2) around a nominal operating point as,

$$\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Hx
\end{aligned} \tag{3}$$

The nominal plant transfer function between the input  $u$  and selected output variable  $y$  is written as,

$$P = H[sI - A]^{-1}B \tag{4}$$

Variations in the plant operating condition is included by a structured uncertainty model as,

$$\tilde{P} = (1 + DW_2)P \tag{5}$$

$W_2$  is a fixed stable transfer function, the weight, and  $D$  is a variable transfer function satisfying  $\|D\|_\infty < 1$ . In the multiplicative uncertainty model (5),  $DW_2$  is the normalized plant perturbation away from 1. If  $\|D\|_\infty < 1$  then

$$\left| \frac{\tilde{P}(j\omega)}{P(j\omega)} - 1 \right| \leq |W_2(j\omega)|, \forall \omega \tag{6}$$

So,  $|W_2(j\omega)|$  provides the uncertainty profile and in the frequency plane is the upper boundary of all the normalized plant transfer functions away from 1. For a control function  $C$

in cascade with the plant  $P$ , the robustness performance and stability measures are satisfied if,

$$\| |W_1 S| + |W_2 T| \|_\infty < 1 \tag{7}$$

In the above,  $W_1$  is a real, rational, stable and minimum phase function.  $T$  is the input-output transfer function, complement of the sensitivity function  $S$ .

The basic idea of the graphical loop-shaping method is to construct the loop transfer function  $L = PC$  to satisfy the robust performance criterion approximately, and then to obtain the controller from the relationship  $C = L/P$ . For a monotonically decreasing function  $W_1$ , it can be shown that at low frequency the open-loop transfer function  $L$  should satisfy,

$$|L| > \frac{|W_1|}{1 - |W_2|} \tag{8}$$

while, for high frequency,

$$|L| < \frac{1 - |W_1|}{|W_2|} \approx \frac{1}{|W_2|} \tag{9}$$

At high frequency  $|L|$  should roll off at least as quickly as  $|P|$  does. This ensures properness of  $C$ . The general features of open loop transfer function are that the gain at low frequency should be large enough, and  $|L|$  should not drop-off too quickly near the crossover frequency to avoid internal instability. Steps in the controller design include: determination of dB-magnitude plots for  $P$  and  $\tilde{P}$ , finding  $W_2$  from (6), choosing  $L$  subject to (7-9), check for the robustness criteria, constructing  $C$  from  $L/P$  and checking internal stability. The process is repeated until satisfactory  $L$  and  $C$  are obtained [6-7]. The iterative determination of controller  $C$  from the choice of open-loop function  $L$ , subject to the constraints, can be accelerated by incorporating an optimization procedure in the graphical loop-shaping method. In this article a particle swarm optimization (PSO) technique has been included.

### 4. THE PARTICLE SWARM OPTIMIZATION

The particle swarm optimization is a population-based optimization tool developed by Eberhart and Kennedy [11]. PSO technique conducts search using a population of particles where each particle is a candidate solution. Particles change their positions by flying around in a multidimensional search space until either computational limits are exceeded or relatively unchanging positions have been encountered. During the flight each particle adjusts its position according to its own experience and experience of the neighboring particle [11-13]. The advantages of PSO over other evolutionary computational algorithms are the ease of programming and fast convergence of the algorithm.

In the PSO algorithm, each particle updates its velocity and position by the relationship,

$$\begin{aligned}
V_i(k+1) &= Q(k)V_i(k) + K_1 \text{rand}_1(\cdot)(X_{pbest}(k) - X_i(k)) \\
&\quad + K_2 \text{rand}_2(\cdot)(X_{gbest} - X_i(k))
\end{aligned} \tag{10}$$

$$X_i(k+1) = X_i(k) + V_i(k) \tag{11}$$

where,  $K_1$  and  $K_2$  are two positive constants,  $\text{rand}_1(\cdot)$  and  $\text{rand}_2(\cdot)$  are random numbers in the range [0, 1], and  $Q$  is the inertia weight.  $X_i$  represents position of the  $i$ th particle and  $V_i$  is its velocity. The

first term in (10) depends on the former velocity of the particle(s), the second is the cognition modal, which includes particles' own thinking and memory, and the third part represents the socio-psychological adaptation knowledge of the particles. The three parts together determine the space searching ability. The first part has the ability to search for local minimum. The second part causes the swarm to have a strong ability to search for global minimum and avoid local minimum. The third part reflects the information sharing among the particles. Under the influence of the three parts, the particle can reach the best position.

The PSO algorithm starts by initializing the velocity and position of the population of particles randomly. In the first iteration the global best (gbest) and the local best (pbest) are set equal. The process is iterated for a certain maximum number of iterations, or until some acceptable solution is reached.

## 5. DESIGN USING PSO-BASED LOOP-SHAPING

In the proposed approach the robust controller structure is pre-selected as,

$$C(s) = \frac{b_m s^m + \dots + b_1 s + b_0}{a_n s^n + \dots + a_1 s + a_0} \quad (12)$$

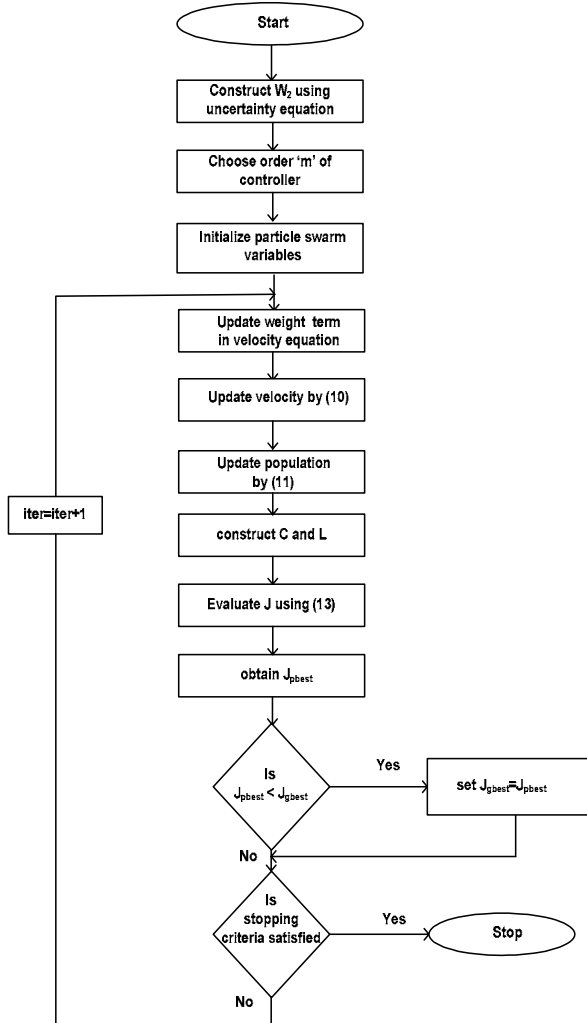


Fig. 3 Flow chart for the proposed PSO based loop-shaping.

The open loop function  $L$  is then constructed from  $L(s) = P(s)C(s)$ . The performance index  $J$  is chosen to include the robust performance and stability criterion (7), the constraints on  $L$  given in (8-9), etc. and is expressed as,

$$J = \sum_{i=1}^N r_i J_{Bi} + r_o J_S \quad (13)$$

where,  $J_{Bi}$  are the robust stability indices and  $J_S$  is the stability index of the over-all closed loop system.  $r_i$  and  $r_o$  are the penalties associated with the respective indices and  $N$  is the number of frequency points in Bode plot of  $L(j\omega)$ . It is, generally, difficult to give analytical expressions for the robust stability bounds. In this paper robust stability bounds are obtained from graphical method using Bode plots. At each frequency  $\omega_i$ , the magnitude of open-loop transmission  $L(j\omega_i)$  is calculated and then checked to see whether or not the robust stability bound is satisfied at that frequency. The robust stability indices are defined by,

$$J_{Bi} = \begin{cases} 0, & \text{if bound at } \omega_i \text{ is satisfied} \\ 1, & \text{otherwise} \end{cases} \quad (14)$$

$i = 1, 2, 3, \dots, N$

The stability of the closed loop nominal system is tested by solving the roots of characteristic polynomial and then checking whether all the roots lie in the left side of the complex plane. The stability index  $J_S$  is defined as,

$$J_S = \begin{cases} 0, & \text{if stable} \\ 1, & \text{otherwise} \end{cases} \quad (15)$$

The coefficients  $b_m, \dots, b_1$  and  $a_n, \dots, a_1$  are searched by the PSO algorithm to satisfy the constraint equations;  $a_n$  being set to 1. The flow chart for the proposed PSO based loop-shaping algorithm is shown in Fig. 3.

## 6. IMPLEMENTATION OF THE ROBUST STRATEGY

The robust control strategy was implemented on the multi-machine power system given in Fig.1. Considering only one STATCOM, the detailed dynamic model is expressed in terms of 20 first order equations. Designing a controller by placing the poles and zeroes of  $L$  is a complicated process. Using a balanced realization technique, a reduced order model was obtained considering the angular speed deviation ( $\Delta\omega_2$ ) of the generator 2 as the plant output and voltage modulation index  $m$  of the STATCOM as the input. The nominal plant function for the 6<sup>th</sup> order reduced system is,

$$P = \frac{-100s(s - 40.93)(s + 15.27)(s^2 + 0.66s + 17.22)}{s(s + 30)(s + 4.44)(s + 0.33)(s^2 + 0.62s + 31.19)} \quad (16)$$

Off-nominal outputs between 0.4 and 1.4 pu and power factor form 0.8 lagging to 0.8 leading were considered for the different generators. The quantity,  $|\tilde{P}(j\omega)/P(j\omega) - 1|$  for each perturbed plant is constructed and the uncertainty profile is fitted to the following function,

$$W_2(s) = \frac{0.191(s+20.6)(s+0.86)}{(s^2+10.01s+25.57)} \quad (17)$$

A Butterworth filter satisfies all the properties for  $W_1(s)$  and is written as,

$$W_1(s) = \frac{K_d f_c^2}{s^3 + 2s^2 f_c + 2s f_c^2 + f_c^3} \quad (18)$$

For  $K_d = 0.0001$  and  $f_c = 1$ , and for a choice of open-loop transfer function  $L$  as,

$$L(s) = \frac{-1.68 \times 10^3 s(s-40.93)(s+15.27)(s^2+0.66s+17.22)}{s(s+30)(s+4.44)(s+0.33)(s^2+0.62s+31.19)} \times \frac{(s+2.63)(s+0.71)(s^2+0.63s+0.39)(s^2+0.46s+0.37)}{(s^2+11.41s+79.21)(s^2+1.68s+1.64)(s^2+0.66s+0.67)} \quad (19)$$

The desired controller transfer function is given by,

$$C(s) = \frac{16.88(s+2.63)(s+0.71)(s^2+0.63s+0.39)(s^2+0.46s+0.37)}{(s^2+11.41s+79.21)(s^2+1.68s+1.64)(s^2+0.66s+0.67)} \quad (20)$$

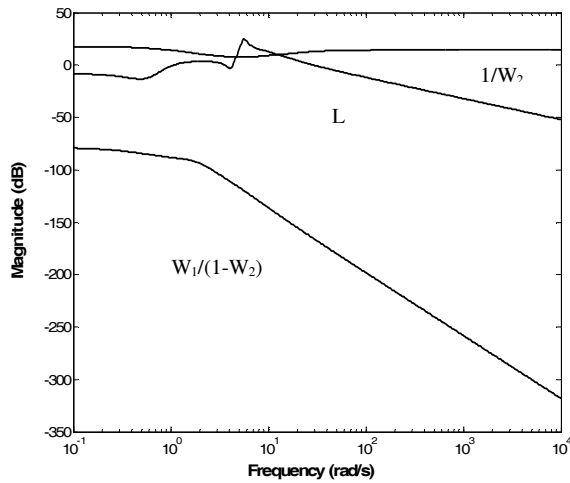


Fig. 4 Loop-shaping plots relating  $W_1$ ,  $W_2$  and  $L$ .

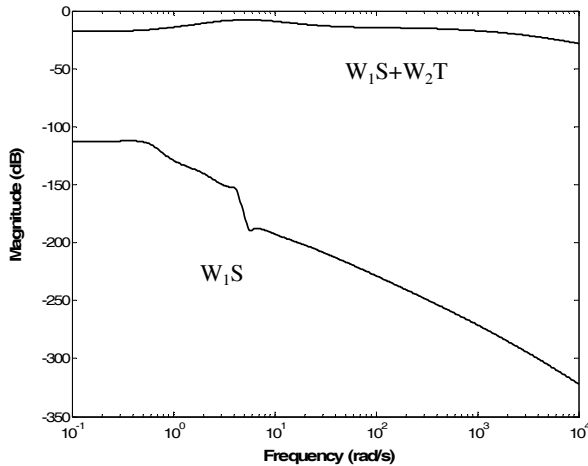


Fig. 5 The nominal and robust performance measures.

The dB-magnitude plots relating  $W_1$ ,  $W_2$  and  $L$ , which were employed to arrive at this controller, are shown in Fig. 4. The open-loop function  $L$  is selected to fit the bounds set by (8-9).

The plots for the nominal and robust performance measures are shown in Fig. 5. It can be observed that the nominal as well as the robust stability criteria are well satisfied.

### Loop-shaping with PSO

For the nominal plant transfer function (16), the PSO starts with  $W_1$  and  $W_2$  arrived at through the graphical procedure given above. In stead of selecting  $L$ , the controller function is chosen as,

$$C(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} \quad (21)$$

The PSO algorithm converged to give the following robust controller function,

$$C(s) = \frac{12.82(s+7.33)(s+0.23)}{s^2 + 3.6768s + 77.455} \quad (22)$$

The open-loop function  $L(s)$  obtained from  $L(s) = P(s) C(s)$  is constructed as,

$$L(s) = \frac{-1.281 \times 10^3 s(s-40.93)(s+15.27)(s+7.33)(s+0.22)(s^2+0.66s+17.22)}{s(s+30)(s+4.44)(s+0.33)(s^2+3.68s+77.46)(s^2+0.62s+31.19)} \quad (23)$$

The parameters used in the PSO algorithm are:  $K_1=K_2=2$ ; maximum and minimum weights ( $Q$ ) 1.2 and 0.1, respectively; population size 20; maximum iterations 1500.

The dB magnitude vs. frequency plot relating  $L(s)$ ,  $W_1(s)$  and  $W_2(s)$  obtained through the PSO based procedure is shown in Fig. 6. It can be seen from the figure that the loop-shaping requirements on  $L(s)$  are satisfied at all frequencies. Fig.7 shows the nominal and robust performance measures.

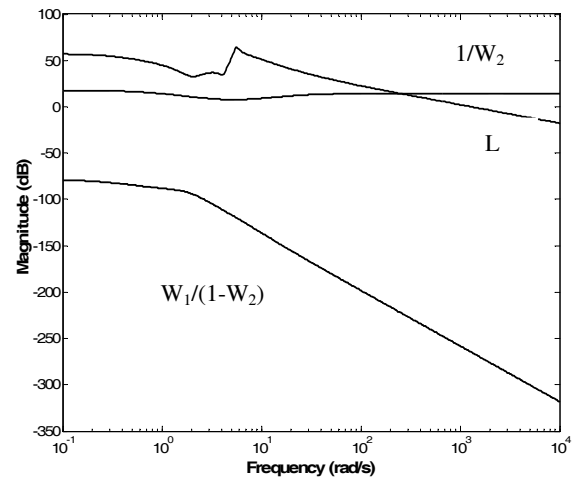


Fig. 6. PSO based loop-shaping plots relating  $W_1$ ,  $W_2$  and  $L$ .

A comparison of the simulation results of the original graphical loop-shaping and PSO based loop-shaping methods are given in Figs. 8-9. Fig.8 shows the comparison of the relative rotor angles for a 50% torque pulse on generator 2 for 0.1 sec at nominal loading. Fig. 9 shows the angle variations when a three phase fault for 0.1 sec is applied at bus 2. It can be observed that both the graphical and PSO based loop-shaping techniques produce controller functions that give almost identically good transient control.

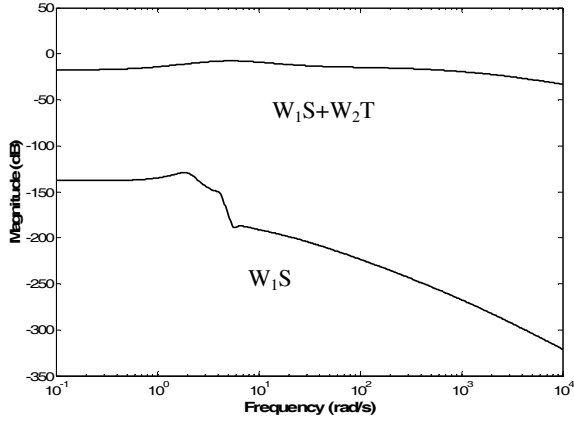


Fig. 7 Robust and nominal performance measures with PSO based loop-shaping.

The robust design was tested for a number of other operating conditions. Figs. 10 and 11 show the variation of relative rotor angles for a disturbance of 50% input torque pulse for 0.1 seconds on generator 2 and a three phase fault at bus 2 for 0.1 sec, respectively. The different loading conditions are given in Tables I and II. It can be observed from the figures robust STATCOM controller damps the oscillations very quickly.

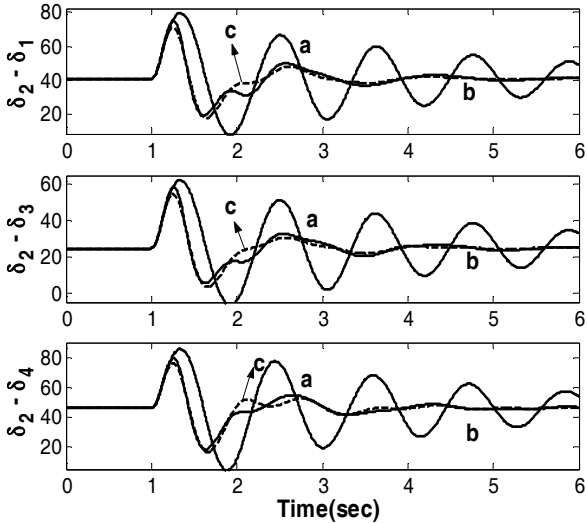


Fig. 8 Comparison of relative angles for a 50% input torque pulse on generator 2 for 0.1 sec. with (a) no control, (b) PSO based loop-shaping, and (c) graphical loop shaping.

Gen	Case a		Case b		Case c		Case d	
G <sub>1</sub>	232	119	307	226	68	19	123	31
G <sub>2</sub>	700	244	725	366	535	78	330	49
G <sub>3</sub>	300	193	575	336	165	74	165	105
G <sub>4</sub>	450	266	675	461	245	111	157	94

Table I. Generation: P (MW) and Q (MVAR) for various test cases

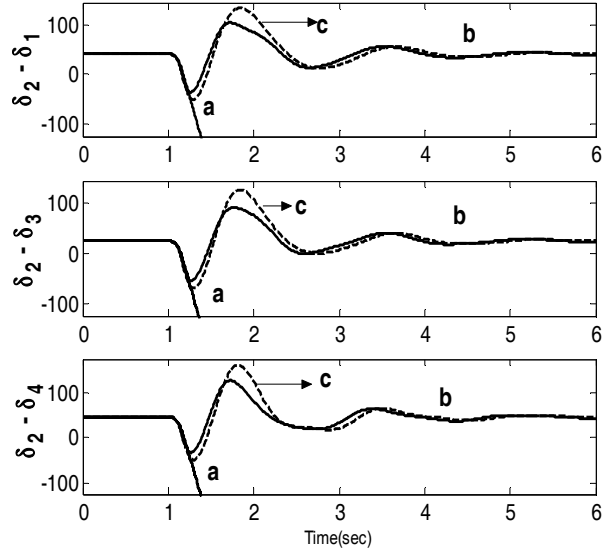


Fig. 9 Comparison of relative angles following a 3 phase fault at bus 2 for 0.1 sec with (a) no control, (b) PSO based loop-shaping, and (c) graphical loop shaping.

Loads	Case a		Case b		Case c		Case d	
S <sub>A</sub>	350	195	460	275	275	135	200	150
S <sub>B</sub>	350	195	460	275	175	135	125	175
S <sub>C</sub>	650	375	900	475	410	250	350	250
S <sub>D</sub>	325	155	450	220	150	100	125	75

Table II Load: P (MW) and Q (MVAR) for various test cases

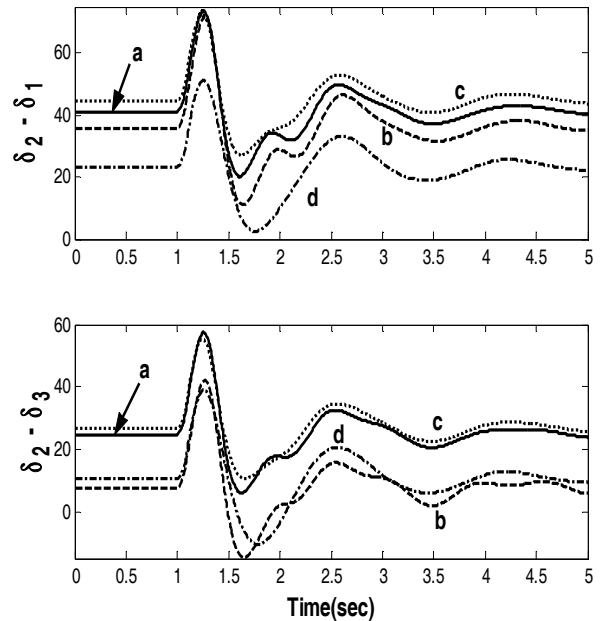


Fig. 10 Relative rotor angle deviations for four different cases for a disturbance of 50% torque pulse on shaft of generator 2 for 0.1 sec.

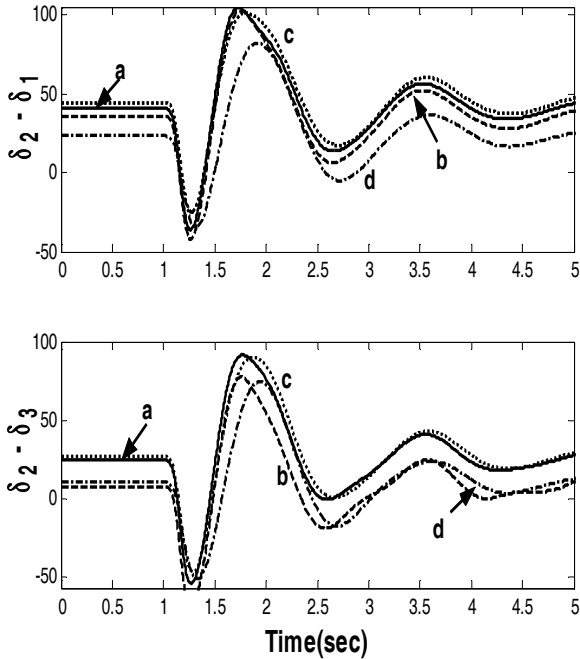


Fig. 11 Relative rotor angle deviations for four different cases for a 3 phase fault at bus 2 for 0.1 sec.

## 7. CONCLUSIONS

Fixed parameter robust STATCOM controllers are designed for multi-machine power system by a graphical loop-shaping procedure. The trial and error part of the loop-shaping procedure is eliminated by incorporating a particle swarm optimization procedure in the design process. The advantage of using the PSO is that the order of the controller can be chosen a-priori in the computation process. Since the graphical loop-shaping procedure is not computationally very efficient for controller design of high order systems, a reduced order model was obtained through a balanced realization technique. The designs are implemented and verified through the full order detailed multi-machine dynamic model. The loop-shaping based STATCOM controller with PSO embedded algorithm is computationally efficient and has been observed to provide very good damping profile over a range of operation.

## 8. ACKNOWLEDGEMENT

The authors wish to acknowledge the facilities provided at the King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia.

## REFERENCES

- [1] H.Wang and F.Li, "Multivariable sampled regulators for the coordinated control of STATCOM AC and DC voltage", *IEE Proc.-Gen.Transm. Distrib.*, vol.147,no.2, pp.93-98, March 2000.
- [2] C.Li, Q.Jiang, Z.Wang and D.Rezmann, "Design of a rule based controller for STATCOM", *Proc. 24<sup>th</sup> Annual Conf. of IEEE Ind.Electronic Society, IECOn'98*, vol.1,pp 467-472, 1998.

- [3] L.Gyugi, "Dynamic compensation of AC transmission lines by solid state synchronous voltage sources", *IEEE Trans. Power Delivery*, Vol.9, no.2, pp. 904-911, April 1994.
- [4] J.Machowski, "Power System Dynamics and Stability", John Wiley and Sons, 1997.
- [5] H.F.Wang and F.Li, "Design of STATCOM multivariable sampled regulator", *Int. Conf. Electric Utility Deregulation and Power Tech. 2000*, City University, London, April 2000.
- [6] A.H.M.A.Rahim, F.M.Kandlawala, "Robust STATCOM voltage controller design using loop-Shaping technique", *Electric Power System Research*, Vol.68, pp.61-74, 2004.
- [7] A.H.M.A.Rahim, S.A.Al-Baiyat, H.M. Al-Maghrabi, "Robust Damping Controller Design for a Static Compensator", *IEE Proc.- Generation, Transmission and Distribution*, Vol. 149(4), July 2002, pp. 491-496.
- [8] Mohagheghi, S., Harley, R.G., Venayagamoorthy, G.K., "Modified Takagi-Sugeno fuzzy logic based controllers for a static compensator in a multimachine power system", *Industry Applications Conference, 2004. 39th IAS Annual Meeting*, Vol. 4, 3-7 Oct. 2004, pp 2637 – 2642.
- [9] S.Morris, P.K.Dash, K.P.Basu, "A fuzzy variable structure controller for STATCOM", *Electric power system research* 65, 2003, pp 23-34.
- [10] L.O.Malik , Y.X.Ni, C.M.Shen, "STATCOM with fuzzy controllers for interconnected power systems", *Electric Power System Research*, 55, pp 87-95, 2000.
- [11] Kennedy J. and Eberhart R, " Particle swarm optimization", *IEEE International Conference on Neural Networks*, 4, pp 1942-1948, 1995.
- [12] Yuhui Shi, Russell C Eberhart, "Empirical study of PSO", *IEEE, Proceedings of the 1999 Congress on Evolutionary Computation*, vol 3, pp 6-9, September 1999.
- [13] Kennedy J, "The particle swarm optimization: social adaptation of knowledge", *International Conference of Evolutionary Computation*, Indianapolis, April 1997, pp 303-308.



**S.F. Faisal** was born in Hyderabad, India in 1978. He received his B.Tech Electrical Engineering degree from JNT University; Hyderabad, India in 2001 and M. Sc. from the King Fahd University of Petroleum and Minerals (KFUPM) in 2005. He is working as Research Assistant in the Electrical Engineering Department of KFUPM since Sep. 2002. His major area of interest is power system analysis, FACTS devices & control, artificial intelligence and optimization.



**Abu H. M. A. Rahim** did his B.Sc. in Electrical Engineering from the Bangladesh University of Engineering and Technology (BUET), Dhaka in 1966 and Ph.D. from the University of Alberta, Edmonton, Canada in 1972. After a brief post-doctoral work at the University of Alberta, he rejoined the Faculty in BUET, Dhaka. Dr. Rahim was a Visiting Fellow at the University of Strathclyde, Glasgow (U.K.) in 1978. Since then he worked at the University of Bahrain, University of Calgary and at the King Fahd University of Petroleum and Minerals, where he is now a Professor.

Dr. Rahim's main fields of interest are Power System Stability, Control and application of artificial intelligence to power systems. Dr. Rahim is a senior member of the IEEE and Fellow of the Institute of Engineers, Bangladesh.