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Robust STATCOM Controller Design Using PSO Based Automatic Loop-Shaping Procedure

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***Abstract*—Static synchronous compensator (STATCOM) controls are known to enhance damping of a power system. The graphical loop-shaping procedure used in designing a robust STATCOM controller can be significantly improved by embedding some optimization procedure in it. In this article a particle swarm optimization (PSO) technique has been employed to find the fixed parameter robust controller parameters. PSO based robust control design greatly reduces the computational effort compared to the manual graphical techniques. Simulation studies on a simple power system indicate that the designed controller provides very good damping properties.**

I. Introduction

It is well established that flexible ac transmission system (FACTS) devices can improve both transient as well as dynamic performances of a power system. These devices dynamically control the power flow through a variable reactive admittance to the transmission network and, therefore, generally change the system admittance. Controllable synchronous voltage sources, known as static synchronous compensator (STATCOM) are a recent introduction in power systems for dynamic compensation and for real time control of power flow. The STATCOM provides shunt compensation in a similar way to the static var compensators but utilizes a voltage source converter rather than shunt capacitors and reactors [1, 2].

It has been reported that STATCOM can offer a number of performance advantages for reactive power control applications over the conventional approaches because of its greater reactive current output capability at depressed voltage, faster response, better control stability, lower harmonics and smaller size, etc. [3, 4].

Auxiliary signals in the STATCOM control circuit is known to improve the dynamic performance of a power system [3-5]. Such control strategies, which are robust in nature, have also been investigated [7-9]. A simple loop-shaping method, which yields a fixed parameter robust controller, has been investigated recently [7, 8]. The controller has been shown to provide very good damping characteristics. The graphical loop-shaping method involves a trial and error procedure. The success of the design depends, to a large extent, on the experience of the designer. For uncertain, unstable, and non-minimum phase plants, it is difficult to design a controller manually to satisfy all stability specifications. Also, designing a controller manually by loop-shaping technique for higher order systems like multi-machine power system can be quite complicated.

The convergence of the graphical loop-shaping method can be accelerated by embedding some optimization procedure in it. In this article, a particle swarm optimization (PSO) technique has been used for the design automation. Simulation studies carried out on a simple power system indicate that the PSO embedded optimum robust control design provides very good damping to the power system.

II. The Power System Model

A single machine infinite bus system with a STATCOM installed at the mid-point of the transmission line is shown in Fig. 1. The dynamics of the generator is expressed in terms of the second order electromechanical swing equation and the internal voltage equation. The IEEE type-ST is used for the voltage-regulator excitation system. These are,

$$\begin{aligned}
\dot{\delta} &= \omega - \omega_o \\
\dot{\omega} &= -\frac{1}{M}[P_m - P_e - D(\omega - \omega_o)] \\
\dot{e}_q' &= \frac{1}{T_{do}'}[E_{fd} - e_q] \\
\dot{E}_{fd} &= -\frac{1}{T_A}(E_{fd} - E_{fdo}) + \frac{K_A}{T_A}(V_{to} - V_t)
\end{aligned} \tag{1}$$

Here, ω and δ are the generator speed and rotor angle while e_q' and E_{fd} represent the generator internal voltage and field voltages, respectively. A list of symbols is included at the end of the article.

The STATCOM system consists of a step down transformer (SDT) with a leakage reactance X_{SDT}, a three to phase GTO-based voltage source converter, and a D.C. capacitor. The dynamic relation between the capacitor voltage (V_{dc}) and current (I_{dc}) in the STATCOM circuit are expressed as [3, 6],

$$\frac{dV_{dc}}{dt} = \frac{I_{dc}}{C_{dc}} = \frac{m}{C_{dc}}\left(I_{Lod}\cos\psi + I_{Loq}\sin\psi\right) \tag{2}$$

I_{Lod} and I_{Loq} are the direct and quadrature axes components of STATCOM current I_{Lo}. The output voltage phasor is

$$\bar{V}_o = mV_{dc}\angle\psi \tag{3}$$

Here, m is the modulation index and ψ is the phase angle.

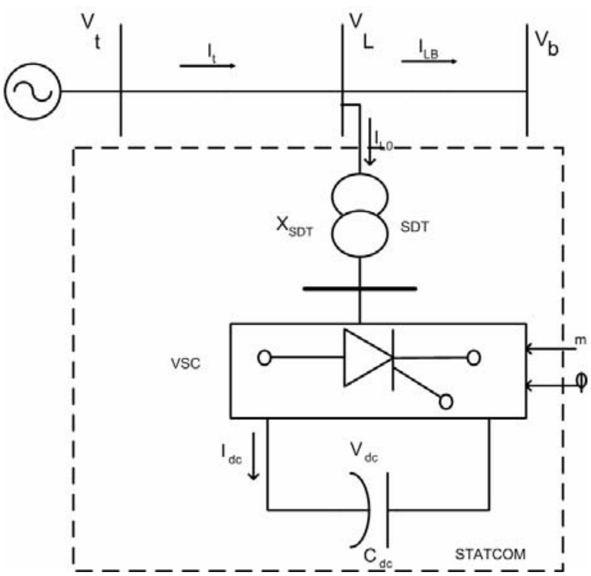

Fig. 1. Single machine infinite bus power system with STATCOM

For a choice of the state and control vectors as $\left[\Delta\delta\ \Delta\omega\ \Delta e_q'\ \Delta E_{fd}\ \Delta V_{dc}\right]^T$ and $\left[\Delta m\ \Delta\psi\right]^T$ the nonlinear state equations (1-2) for the system can be expressed as,

$$\dot{x} = f(x,u) \tag{4}$$

III. THE ROBUST CONTROLLER DESIGN USING LOOP-SHAPING

The robust control design for the generator-STATCOM system starts by linearizing the nonlinear set of equations (5) around a nominal operating point as,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Hx \end{aligned} \tag{5}$$

Here, control u is the modulation index m which is a measure of the STATCOM voltage magnitude. The nominal plant transfer function is,

$$P = H[sI - A]^{-1}B \tag{6}$$

Variations in the plant operating condition is included by a structured uncertainty model as,

$$\tilde{P} = (1 + DW_2)P \tag{7}$$

Here, W_2 is a fixed stable transfer function, the weight, and D is a variable transfer function satisfying $\|D\|_\infty < 1$. In the multiplicative uncertainty model (7), DW_2 is the normalized plant perturbation away from 1. If $\|D\|_\infty < 1$ then

$$\left|\frac{\tilde{P}(j\omega)}{P(j\omega)} - 1\right| \le \left|W_2(j\omega)\right|, \forall\omega \tag{8}$$

So, $\left|W_2(j\omega)\right|$ provides the uncertainty profile and in the frequency plane is the upper boundary of all the normalized plant transfer functions away from 1. For a control function C in cascade with the plant P, the robustness measures are,

a) The nominal performance measure is $\|W_1 S\|_\infty < 1$

b) C provides robust stability iff $\|W_2 T\|_\infty < 1$ (9)

c) Necessary and sufficient condition for robust nominal and robust performance is $\left\|\,|W_1 S| + |W_2 T|\,\right\|_\infty < 1$

In the above, W_1 is a real, rational, stable and minimum phase function. T is the input-output transfer function, complement of the sensitivity function S, and is given as

$$T = 1 - S = \frac{1}{1+L} = \frac{1}{1+PC} \tag{10}$$

Loop shaping is a graphical procedure to design a proper controller C satisfying robust stability and performance criteria given in (9). The basic idea of the method is to construct the loop transfer function, L = PC to satisfy the robust performance criterion approximately, and then to obtain the controller from the relationship C = L/P. Internal stability of the plants and properness of C constitute the constraints of the method. Condition on L is such that PC should not have any pole zero cancellation. A necessary condition for robustness is that either or both $|W_1|$, $|W_2|$ must be less than 1 [8]. For a monotonically decreasing function W_1, it can be shown that at low frequency the open-loop transfer function L should satisfy

$$|L| > \frac{|W_1|}{1 - |W_2|} \tag{11}$$

while, for high frequency,

$$|L| < \frac{1 - |W_1|}{|W_2|} \approx \frac{1}{|W_2|} \tag{12}$$

At high frequency |L| should roll off at least as quickly as |P| does. This ensures properness of C. The general features of open loop transfer function are that the gain at low frequency should be large enough, and |L| should not drop-off too quickly near the crossover frequency to avoid internal instability.

Steps in the controller design include: determination of dB-magnitude plots for P and $\tilde{P}$, finding W_2 from (8), choosing L subject to (11-12), check for the robustness criteria, constructing C from L/P and checking internal stability. The process is repeated until satisfactory L and C are obtained.

IV. THE PARTICLE SWARM OPTIMIZATION

A. Introduction to PSO

The particle swarm optimization is an evolutionary computation technique developed by Eberhart and Kennedy inspired by the social behavior of bird flocking and fish schooling [10]. PSO is a population based optimization tool. Population is formed by a predetermined number of particles; each particle is a candidate solution to the problem. In a PSO system, particles fly around in a multi-dimensional search space until relatively unchanging positions have been encountered or until computational limits are exceeded. During the flight, each particle adjusts its position according to its own experience and experience of its neighboring particles [10-12]. Compared to other evolutionary algorithms the merit of PSO is that, it has memory i.e., every particle remembers its best solution (local best, 'J_{pbest}') as well as the group best solution (global best, 'J_{gbest}'). The PSO algorithm has been found to reach the global optimum at each run, and appears to approximate the results reported for elementary genetic algorithms for some applications[10]. The algorithm is simple, fast and can be programmed in a few steps.

In PSO each particle adjusts its flight according to its own and its companion's flying experience. The best position in the course of flight of each particle(s) is called X_{pbest}, and the solution associated with it is denoted by J_{pbest}. Initially J_{gbest} (global best) is set to J_{pbest} and the particle(s) associated with it is denoted by X_{gbest}. Later on as the particle(s) is updated, J_{gbest} represents the best solution attained by the whole population and X_{gbest} denotes the corresponding best position. Every particle(s) updates itself through the above mentioned best positions. The particle(s) updates its own velocity and position according to the following equations [11, 12],

$$V_i = QV_i + K_1 rand_1(\,)(X_{pbest} - X_i) + K_2 rand_2(\,)(X_{gbest} - X_i) \quad (13)$$

$$X = X_i + V_i \quad (14)$$

where K_1 and K_2 are two positive constants, $rand_1$ ( ) and $rand_2$ ( ) are random numbers in the range [0, 1], and Q is the inertia weight. X_i represents position of the ith particle and V_i is its velocity. The first term in (14) is the former velocity of the particle(s), the second is the cognition modal, which expresses the thought of the particle itself, and the third represents the social model. The three parts together determines the space searching ability. The first part has the ability to search for local minimum. The second part causes the swarm to have a strong ability to search for global minimum and avoid local minimum. The third part reflects the information sharing among the particles. Under the influence of the three parts, the particle can reach the best position.

B. PSO Algorithm

The PSO algorithm used in this paper can be briefly discussed by the following steps.

1: Initialize a population of n particles with random positions within the lower and upper bound of the problem space. Similarly initialize randomly n velocities associated with the particles.
2: Evaluate the optimization fitness functions J for the initial population.
3: Find the minimum fitness value for fitness functions J in step 2 and call it J_{pbest} and let the particle associated with it be X_p.
4: Initially set J_{gbest} equal to J_{pbest}.
5: Update the weight Q using the following equation

$$Q = Q_{\max} - \left(\frac{Q_{\max} - Q_{\min}}{iter_{\max}} \right) iter \quad (15)$$

'iter' is the iteration count
6: Update the velocity of each particle using (13).
7: Check V for the range [V_{max}, V_{min}]. If not, set it to the limiting values.
8: Update the position of each particle using (14) which gives the new population.
9: Repeat 7 for the new population.
10: Evaluate the optimization fitness functions J for new population.
11: Obtain J_{pbest} for fitness functions J in step 10.
12: Compare the J_{pbest} obtained in step 11 with J_{gbest}. If J_{pbest} is better than J_{gbest} then set J_{gbest} to J_{pbest}.
13: Stop if convergence criteria are met, otherwise go to step 5. The stopping criteria are, good fitness value, reaching maximum number of iterations, or no further improvement in fitness.

C. The Robust Design Using PSO

In the proposed approach the controller structure is pre-selected and is given by,

$$C(s) = \frac{b_m s^m + \cdots\cdots + b_1 s + b_o}{a_n s^n + \cdots\cdots + a_1 s + a_o} \quad (16)$$

The open loop function L is then constructed from (16) as,

$$L(s) = P(s)C(s) \quad (17)$$

The performance index J in steps 2 and 10 of the PSO algorithm for the STATCOM controller design is chosen to include the robust performance and stability criterion (9), the constraints on L given in (11-12), etc. The performance index is expressed as,

$$J = \sum_{i=1}^{N} r_i J_{Bi} + r_o J_S \quad (18)$$

where, J_{Bi} are the robust stability indices and J_S is the stability index. r_i and r_o are the penalties associated with the respective indices and N are the number of frequency points in Bode plot of $L(j\omega)$.

In general, the robust stability bounds are very complicated and are non convex. It is difficult to give analytical expressions of the robust stability bounds. In this paper robust stability bounds are obtained from graphical method using Bode plots. At each frequency ω_i, the magnitude of open-loop transmission $L(j\omega_i)$ is calculated and then checked to see whether or not the robust stability bound is satisfied at that frequency. A robust stability index is defined by,

$$J_{Bi} = \begin{cases} 0 \text{ if bound at } \omega_i \text{ is satisfied} \\ 1 \text{ otherwise} \end{cases} \tag{19}$$

$$i = 1,2,3,\ldots\ldots N$$

The stability of the closed loop nominal system is simply tested by solving the roots of characteristic polynomial and then checking whether all the roots lie in the left side of the complex plane. The stability index J_S is defined as,

$$J_s = \begin{cases} 0 \text{ if stable} \\ 1 \text{ otherwise} \end{cases} \tag{20}$$

The coefficients $b_m, \ldots..b_1$ and $a_n, \ldots..,a_1$ are searched by the PSO algorithm to satisfy the constraint equations. a_n can be set to 1.

The flow chart for the proposed automatic loop-shaping is shown in Fig. 2.

V. IMPLEMENTATION OF THE ROBUST CONTROLLERS

A. Graphical Procedure

The generator angular speed deviation $\Delta\omega$ is selected as the plant output, while the input is STATCOM voltage modulation index m. The nominal plant transfer function for selected operating point is,

$$P = \frac{0.2104 s^2 (s+100.827)(s-0.234)}{(s+99.17)(s+1.10)(s+0.05)(s^2+0.68s+21.63)} \tag{21}$$

Off-nominal power outputs between 0.4 and 1.4 pu and power factor form 0.8 lagging to 0.8 leading were considered. The quantity, $\left|\tilde{P}(j\omega)/P(j\omega)-1\right|$ for each perturbed plant is constructed and the uncertainty profile is fitted to the following function,

$$W_2(s) = \frac{0.9s^2 + 15s + 27}{s^2 + 5s + 31} \tag{22}$$

A Butterworth filter satisfies all the properties for $W_1(s)$ and is written as

$$W_1(s) = \frac{K_d f_c^2}{s^3 + 2s^2 f_c + 2sf_c^2 + f_c^3} \tag{23}$$

For $K_d = 0.01$ and $f_c = 0.1$, and for a choice of open-loop transfer function L as,

$$L = \frac{5.173s^2 (s+100.827)(s-0.234)(s+1)(s+1.094)}{(s+99.17)(s+1.10)(s+0.05)(s^2+0.68s+21.63)} \times \frac{(s+0.0476)(s+0.001)(s+99.174)}{(s+10)(s+0.1)(s+0.01)(s+0.03)(s+0.05)} \tag{24}$$

the desired controller transfer function is given by,

$$C = \frac{24.583(s+1)(s+1.094)(s+0.0476)(s+0.001)(s+99.174)}{(s+10)(s+0.1)(s+0.01)(s+0.03)(s+0.05)} \tag{25}$$

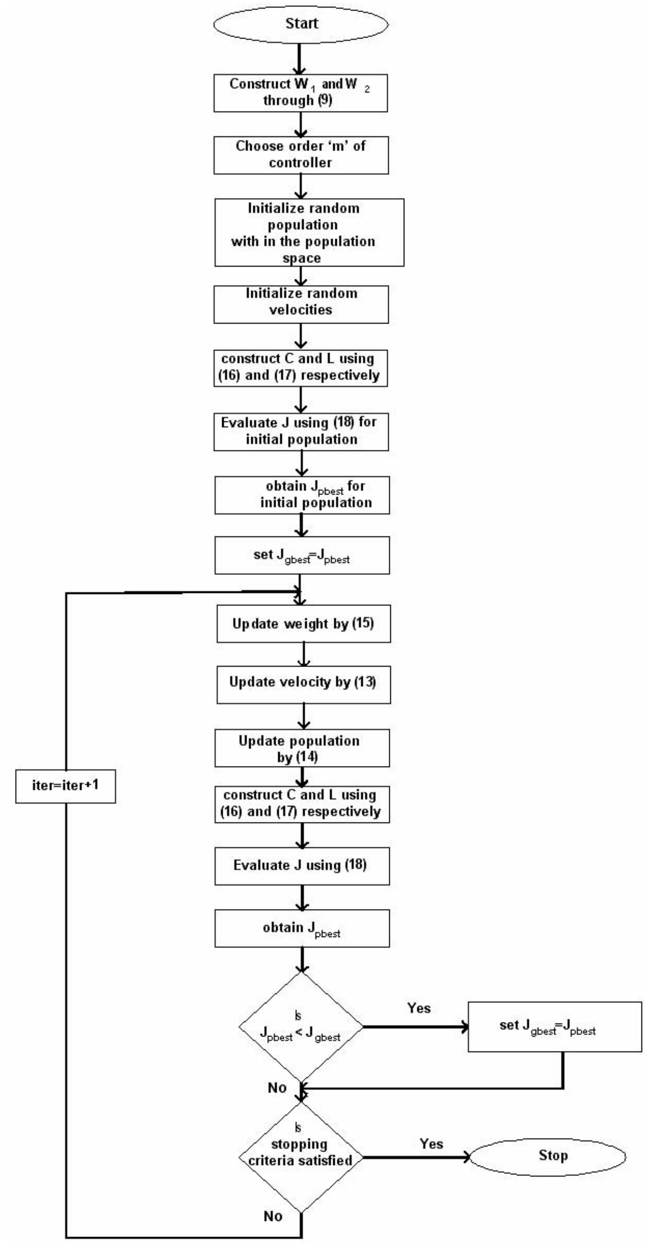

Fig. 2. Flow chart for the proposed automatic loop-shaping.

The dB-magnitude plots relating W_1, W_2 and L, which were employed to arrive at this controller, is shown in Fig. 3. The open-loop function L is selected to fit the bounds set by (11-12). The plots for the nominal and robust performance criteria are shown in Fig. 4. While the nominal performance measure W_1S is well-satisfied, the combined robust stability and performance measure peaks slightly.

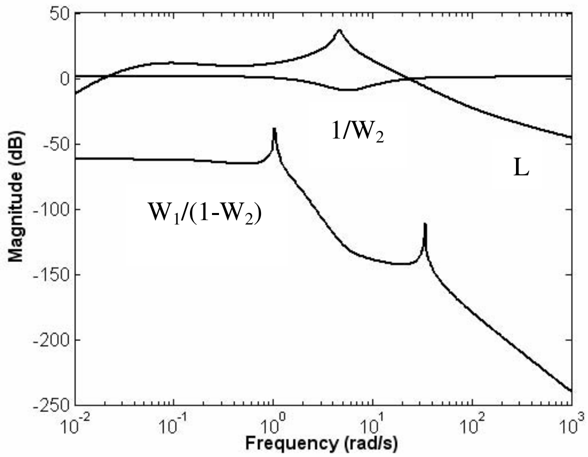

Fig. 3. Loop-shaping plots relating W_1, W_2 and L.

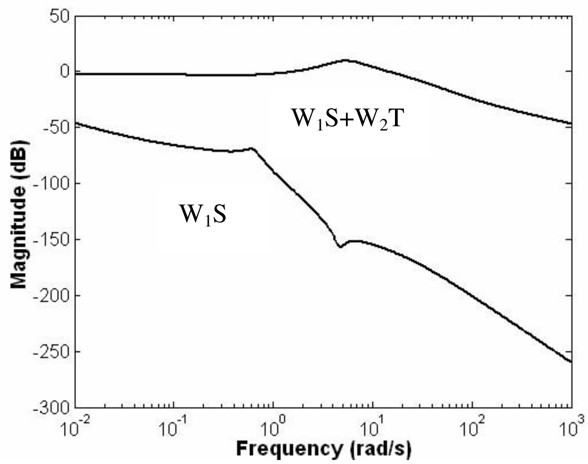

Fig. 4. The nominal and robust performance criteria.

B. Automatic Loop-shaping with PSO

For the nominal plant transfer function (21), the PSO starts with W_1 and W_2 arrived at through the graphical procedure given in section V-A. A second order controller function is chosen as,

$$C(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_2 s^2 + a_1 s + a_0} \tag{26}$$

where, the coefficients a's and b's are to be determined. The control parameters for the PSO algorithm chosen are given in Table 1.

The PSO algorithm converged to give the following robust controller function,

$$C(s) = \frac{25(s+3999.8)(s+0.22)}{s^2 + 0.07454s + 2.797} \tag{27}$$

The open-loop function L(s) obtained from L(s) = P(s) C(s) is,

$$L(s) = \frac{5.26s^2(s+100.827)(s-0.234)}{(s+99.17)(s+1.10)(s+0.05)(s^2+0.68s+21.63)} \times \frac{(s+3999.8)(s+0.22)}{s^2+0.07454s+2.797} \tag{28}$$

TABLE I
PSO parameters

Parameters	Values
Maximum iteration	1000
Population size	20
Value of K_1	2.0
Value of K_2	2.0
Maximum Weight	0.90
Minimum Weight	0.40

The dB magnitude vs. frequency plot relating L(s), $W_1(s)$ and $W_2(s)$ obtained through the automatic procedure is shown in Fig. 5. It can be seen from the figure that the loop-shaping requirements on L(s) are satisfied at all frequencies. Fig.6. shows the convergence profiles for the automatic robust design.

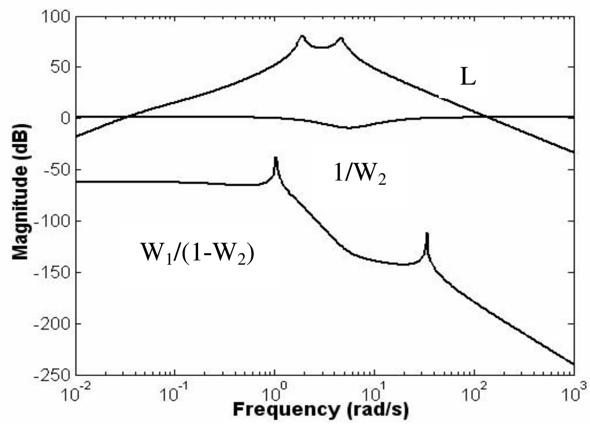

Fig. 5. Automatic loop-shaping plots relating W_1, W_2 and L

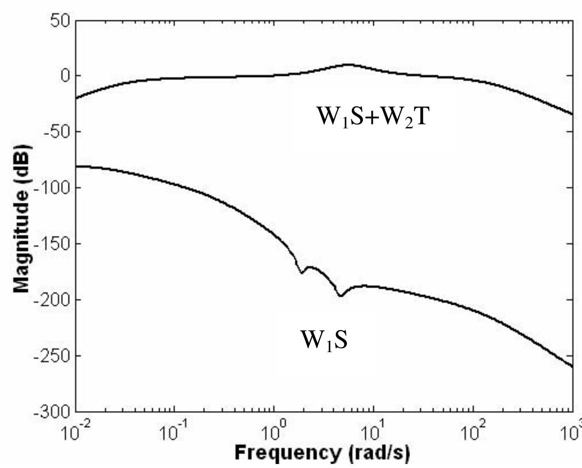

Fig. 6. Robust and nominal performance criteria with automatic loop-shaping

A comparison of the simulation results with the graphical and PSO based automatic loop-shaping methods are given in Figs. 7 and 8.

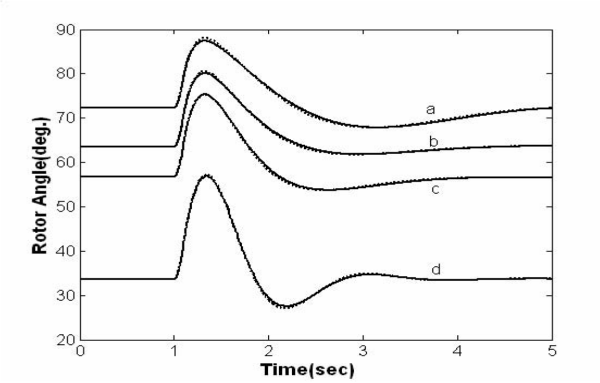

Fig. 7. Comparison of generator rotor angle variations following a 50% input torque pulse (solid line is for graphical method and dotted line for automatic loop-shaping).

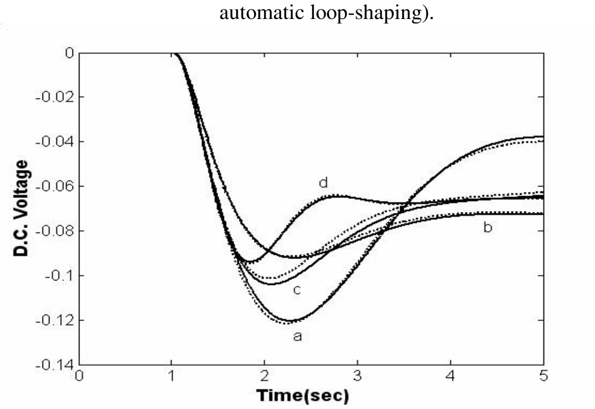

Fig. 8. D.C. capacitor voltage variations of the STATCOM corresponding to Fig.7.

Fig.7. shows the rotor angle variations following a 50% torque pulse on the generator shaft with the robust control for the following 4 loading conditions: (a) 1.2-pu generator power output, 0.98-pf lead, (b) 1-pu output, 0.95 pf lag, (c) 0.9-pu output, unity pf and (d) 0.5-pu output, 0.95-pf lag. Fig.8. shows the comparison of the dc capacitor voltage variations of the STATCOM. It can be observed that both manual and PSO based automatic techniques produce controller functions that give almost identically good transient control.

CONCLUSIONS

A particle swarm optimization procedure has been employed to design a loop-shaping based robust STATCOM controller. The automatic loop-shaping using PSO minimizes the trial and error procedure involved in the graphical loop-shaping construction procedure. The controller structure can also be pre-selected reducing the overall computational burden. Simulation studies indicate that the PSO based robust loop-shaping design provides almost equally effective power system damping control compared with the manual graphical method. The PSO method, however, is computational superior.

LIST OF SYMBOLS

P_m, Pe	input and output power
M,D	inertia and damping coefficients
V_t,V_{to}	generator terminal voltage and reference voltage
P_m, P_e	generator input and output power
C_{dc}	d.c. capacitance
ψ	phase angle of the mid-bus voltage
e_q	internal voltage behind synchronous reactance
E_{fd}	generator field voltage
δ	generator rotor angle
T'_{do}	open circuit field time constant
K_A,T_A	exciter gain and time constants
x_d,x_d	direct axis synchronous and transient reactance
ω_o	base radian frequency
V_b, V_L	infinite bus and STATCOM bus voltages

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REFERENCES

[1] L.Gyugi, "Dynamic compensation of AC transmission lines by solid state synchronous voltage sources", *IEEE Trans. Power Delivery*, Vol.9, no.2, pp. 904-911, April 1994.

[2] J.Machowski, "*Power System Dynamics and Stability*",John Wiley and Sons, 1997.

[3] H.Wang and F.Li, "Multivariable sampled regulators for the coordinated control of STATCOM AC and DC voltage", *IEE Proc.-Gen.Transm. Distrib.*, vol.147,no.2, pp.93-98, March 2000.

[4] C.Li, Q.Jiang, Z.Wang and D.Rezmann, "Design of a rule based controller for STATCOM", *Proc. 24th Annual Conf. of IEEE Ind.Electronic Society, IECon'98,* vol.1,pp 467-472, 1998.

[5] H.F.Wang and F.Li, "Design of STATCOM multivariable sampled regulator", *Int. Conf. Electric Utility Deregulation and Power Tech. 2000*, City University, London, April 2000.

[6] H.F.Wang, "Philips-Hefron model of power systems installed with STATCOM and applications", *IEE Proc. Gen. Trans. and Distr.,* vol.146, no.5, pp.521-527, September 1999.

[7] A.H.M.A.Rahim, S.A.Al-Baiyat, F.M.Kandlawala, "A robust STATCOM controller for power system dynamic performance enhancement" *IEEE PES Summer Meeting*, Vancouver, pp.887-892, July 2001.

[8] A.H.M.A.Rahim, F.M.Kandlawala, "Robust STATCOM voltage controller design using loop-Shaping technique", *Electric Power System Research,* Vol.68, pp.61-74, 2004.

[9] M.M.Farasangi, Y.H.Song, Y.Z.Sun, "Supplementary control design of SVC and STATCOM using H_∞ otimal robust control*", Proceeding International Conference on Electric Utility Deregulation 2000*, City University London, April, 2000, pp.355-360.

[10] Kennedy J. and Eberhart R, " Particle swarm optimization", *IEEE International Conference on Neural Networks,* vol 4, pp 1942-1948, 1995.

[11] Yuhui Shi, Russell C Eberhart, "Empirical study of PSO", *IEEE, Proceedings of the 1999 Congress on Evolutionary Computation*, vol 3, pp 6-9, September 1999.

[12] Kennedy J, "The particle swarm optimization: social adaptation of knowledge", *International Conference of Evolutionary Computation,* Indianapolis, April 1997, pp 303-308.