

# A Robust Design of a Static VAR Compensator Controller for Power System Stability Improvement

A.H.M.A. Rahim                      S.A. Al-Baiyat  
Department of Electrical Engineering  
King Fahd University of Petroleum & Minerals  
Dhahran, Saudi Arabia.  
E-mail: [ahrahim@kfupm.edu.sa](mailto:ahrahim@kfupm.edu.sa)

**Keywords:** Electrical power system, damping control, robust control, loop-shaping method.

## Abstract

A novel robust damping controller for a static var compensator (SVC) in a power system has been designed using a simple graphical technique. The graphical loop-shaping method starts by selection of a nominal plant function satisfying the robust stability and performance criterion. The variations in operating conditions from the nominal values are modeled as multiplicative structured uncertainty. The robust controller designed was tested for a number of operating conditions and disturbances. It is observed that the robust controller provides extremely good damping for a good range of operating points. The advantage of the proposed controller is that the design is based on a simple graphical method avoiding complex mathematical computations normally encountered in such designs.

## INTRODUCTION

Thyristor controlled reactors and capacitors, termed as static var compensators (SVC) are well known to improve power system properties such as steady state stability limits, voltage regulation and var compensation, dynamic over voltage and under voltage control, counteracting sub-synchronous resonance, and damp power oscillations [Hamad 1986; Hosseini and Mirshekhar 2001; So and Yu 2000]. Voltage controlled SVC, as such, does not provide any damping to the power system [Oliviera 1994; Zhou 1993]. However, it can be used to increase power system damping by introducing supplemental signals to the voltage set point [So and Yu 2000; Zhao and Jiang 1995].

A large volume of literature is available on SVC control design for the nonlinear power system problem. Controls for the nonlinear system are often realized through EL (exact linearization), LQR (linear quadratic regulator) theory, DFL (direct feedback linearization). DFL was employed by Tso and Wang [So et. al., 1997; Wang et. al., 1997] to generate SVC and other flexible AC transmission systems (FACTS) controllers. The method presents a complex nonlinear control

law derived through the solution of Matrix-Riccati equation. More complex methods of disturbances auto-rejection control (DARC) and variable structure adaptive fuzzy sliding mode control were presented by Zhang and Zhou [1998], and Ghazi et.al., [2001]. Optimum feedback control of SVC for stabilization of a power system was presented in [Rahim and Nassimi 1996].

One of the important goals of the control engineers is to design 'robust' fixed parameter controllers which will be effective for a large range of operation. Farasangi et. al., [2000], and Zhao and Ziang [1995] proposed a robust controller for SVC using  $H_\infty$ -based techniques. The designs require complicated minimization procedures restricting the realization of the controllers.

This article presents a simple and novel design technique of robust SVC susceptance control for damping power system oscillations. The variations of the operating conditions of the nonlinear power system have been taken into consideration by modeling them as multiplicative unstructured uncertainty. A loop-shaping technique [Doyle et. al., 1992] has been employed to design the controllers. Simulation results demonstrate that the controller designed effectively damps the electromechanical oscillations for a wide range of operating conditions.

## POWER SYSTEM DYNAMICS WITH SVC

The single machine power system configuration shown in Fig. 1 is considered for this study. The generator is connected to the load center termed as 'remote system bus' through a long transmission network. The SVC is placed at the middle of the transmission line which is generally considered to be the ideal site. The generator also has a local load connected at its terminal.

A 4th order nonlinear dynamic model for the power system including the second order electromechanical swing equation, the generator internal voltage equation, and an IEEE type-ST exciter model equation is considered. The static var compensator circuit, shown in Fig.2, contains the

voltage measuring and the voltage regulator circuit, output of which is fed to the thyristor firing control circuit. Normally, the susceptance of the SVC ( $B$ ) is varied to maintain the mid-bus voltage  $V_m$  within its pre-specified tolerance. The supplementary stabilizing signal is added to the output of the voltage regulator.

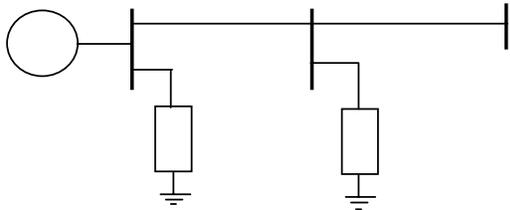


Figure 1. Power system configuration

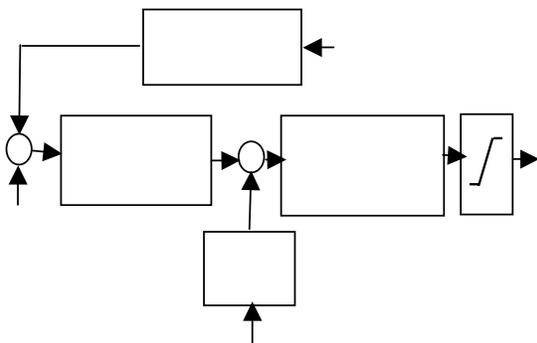


Figure 2. The SVC controller block diagram

The variation of the susceptance ( $B$ ) can be related through the differential equation,

$$\Delta \dot{B} = [-\Delta B + B_o + K_c u] / T_c \quad (1)$$

$K_c$  and  $T_c$  are the gain and time constants, respectively of the SVC firing angle control circuits; and  $u$  is the extra stabilizing signal. Combining (1) with the fourth order generator model, the dynamic equations of the generator-SVC system are expressed as,

$$\dot{x} = f[x, u] \quad (2)$$

Here, vector  $x$  comprises of the states  $[\delta, \omega, e_q, E_{fd}, B]$ . Linearizing the nonlinear equations around a nominal

operating condition, the input-state-output equations are written as,

$$\Delta \dot{x} = A \Delta x + Bu \quad (3)$$

$$y = Hx \quad (4)$$

The output  $y$  is the angular frequency deviation which can be measured remotely, or synthesized from the line flow.

## ROBUST CONTROL DESIGN

The changes in operating conditions of a power system which can be viewed as changes in the coefficient matrix  $A$  in (3) are considered as model uncertainties [Zhao and Jiang 1995]. In this article these perturbations are modeled as multiplicative uncertainties and robust design procedure is applied to the perturbed linear systems. This section gives a brief theory of the uncertainty model, the robust stability criterion, a graphical design technique termed loop shaping, which is employed to design the robust controller. Finally, the algorithm for the control design is presented.

### Uncertainty Modeling

Suppose that a plant having a nominal transfer function  $P = H[sI-A]^{-1}B$  belongs to a bounded set of transfer functions  $\mathcal{P}$ . Consider that the perturbed transfer function resulting from the variations in operating conditions, can be expressed in the form,

$$\tilde{P} = (1 + \Omega W_2)P \quad (5)$$

Here,  $W_2$  is a fixed stable transfer function, also called the weight, and  $\Omega$  is a variable transfer function satisfying  $\|\Omega\|_\infty < 1$ . The infinity norm ( $\infty$ -norm) of a function is the least upper bound of its absolute value, also written as  $\|\Omega\|_\infty = \sup_\omega |\Omega(j\omega)|$ , is the largest value of gain on a Bode magnitude plot.  $|W_2(j\omega)|$  provides the uncertainty profile, and in the frequency plane is the upper boundary of all the normalized plant transfer functions away from 1 [Doyle et. al., 1992].

### Robust Stability and Performance

Consider a multi-input control system given in Fig 3. A controller  $C$  provides stability if it provides internal stability for every plant in the uncertainty set  $\mathcal{P}$ . If  $L$  denotes the open-loop transfer function ( $L=PC$ ), then the sensitivity function  $S$  is written as,

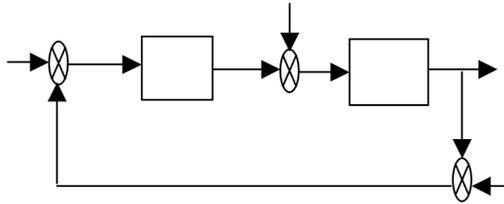


Figure 3. Unity feedback plant with controller.

$$S = \frac{1}{1+L} \quad (6)$$

The complimentary sensitivity function, or the input-output transfer function is,

$$T = 1 - S = \frac{PC}{1+PC} \quad (7)$$

For a multiplicative perturbation model, robust stability condition is met if and only if  $\|W_2 T\|_\infty < 1$  [Doyle et. al., 1992; Chan and Athans 1996]. This implies that,

$$\left| \frac{W_2(j\omega)L(j\omega)}{1+L(j\omega)} \right| < 1, \text{ for all } \omega \quad (8)$$

The nominal performance condition for an internally stable system is given as  $\|W_1 S\|_\infty < 1$ , where  $W_1$  is a real-rational, stable, minimum phase transfer function, also called a weighting function. It can be shown that a necessary and a sufficient condition for robust performance is,

$$\|W_1 S\| + \|W_2 T\|_\infty < 1 \quad (9)$$

### The Loop Shaping Technique

Loop shaping is a graphical procedure to design a proper controller  $C$  satisfying robust stability and performance criteria given above. The basic idea of the method is to construct the loop transfer function  $L$  to satisfy the robust performance criterion approximately, and then to obtain the controller from the relationship  $C=L/P$ . Internal stability of the plants and properness of  $C$  constitute the constraints of the method. Condition on  $L$  is such that  $PC$  should not have any pole zero cancellation.

A necessary condition for robustness is that either or both  $|W_1|, |W_2|$  must be less than 1 [Chan and Athans 1996; Dahleh 1996]. If we select a monotonically decreasing  $W_1$  satisfying the other constraints on it, it can be shown that at low frequency the open-loop transfer function  $L$  should satisfy,

$$|L| > \frac{|W_1|}{1-|W_2|} \quad (10)$$

while, for high frequency

$$|L| < \frac{1-|W_1|}{|W_2|} \approx \frac{1}{|W_2|} \quad (11)$$

At high frequency  $|L|$  should roll-off at least as quickly as  $|P|$  does. This ensures properness of  $C$ . The general features of open loop transfer function is that the gain at low frequency should be large enough, and  $|L|$  should not drop-off too quickly near the crossover frequency resulting into internal instability.

### The Algorithm

The algorithm to generate a control transfer function  $C$  involves the following steps.

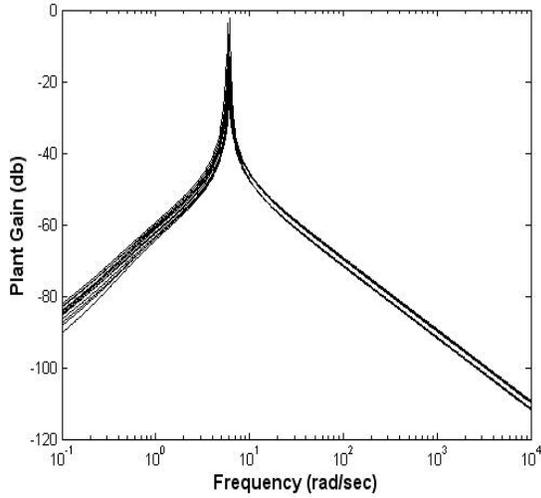
1. Obtain the db-magnitude plot for the nominal as well as perturbed plant transfer functions.
2. Construct  $W_2$  satisfying constraint (5).
3. Select  $W_1$  as a monotonically decreasing, real, rational and stable function.
4. Choose  $L$  such that it satisfies conditions (10) and (11). The transition at crossover frequency should not be at slope steeper than  $-20\text{db/decade}$
5. Check for the nominal and robust performance criteria (8) and (9).
6. Test for internal stability by direct simulation of the closed loop transfer function for pre-selected disturbance or input.
7. Repeat steps 4 through 6 until satisfactory  $L$  and  $C$  are obtained.

### RESULTS

For nominal generator power output of 0.8pu at 0.85 lagging power factor the plant transfer function  $P$ , considering the power flow as the plant output, is obtained as,

$$P = \frac{0.0339s(s+32.7)(s+0.74)}{(s+0.548)(s+33.03)(s^2+0.1282s+35.97)} \quad (12)$$

Off-nominal power output between the range of 0.3-1.3 pu and power factor of up to 0.8 lag/lead which gave steady state stable situations were considered in the robust design. The log-magnitude vs. frequency plots for the nominal and perturbed plants are shown in Fig.4. Data for the generator-SVC system has been taken from [Rahim and Nassimi 1996].



**Figure 4.** Bode magnitude diagram of nominal and perturbed plant functions

The quantity  $\left| \frac{\tilde{P}(j\omega)}{P_{nom}(j\omega)} - 1 \right|$  is constructed for each perturbed plant  $\tilde{P}(j\omega)$  and the upper envelope in the frequency plane is fitted to the function,

$$W_2(s) = \frac{0.2428s^3 + 3.8506s^2 + 14.1633s + 10.5625}{s^3 + 1.8s^2 + 42.9s + 21.125} \quad (13)$$

A Butterworth filter, which satisfies the properties of  $W_1(s)$ , is selected as,

$$W_1(s) = \frac{K_c f_c^2}{s^3 + 2s^2 f_c + 2s f_c^2 + f_c^3} \quad (14)$$

Values of  $K_c=0.1$  and  $f_c=0.05$  were observed to satisfy the requirement on the open loop transfer function  $L$ . For  $W_1$  and  $W_2$  selected above, and for the following choice of the open-loop function,

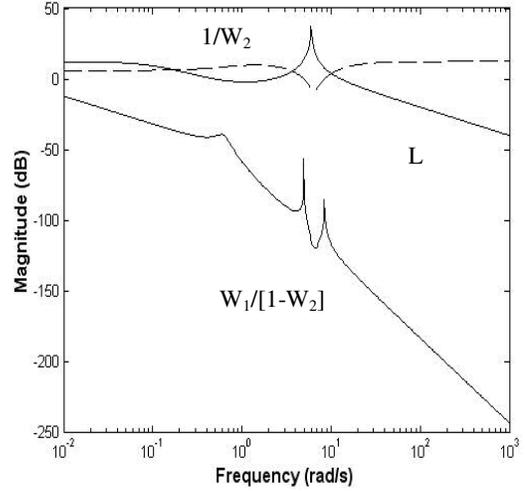
$$L = \frac{10(s + 32.7)(s + 0.74)(s + 2)}{(s + 0.1)(s + 33.03)(s^2 + 80.42s + 1677.3)} \quad (15)$$

the controller transfer function obtained through the relation  $C=L/P$  is,

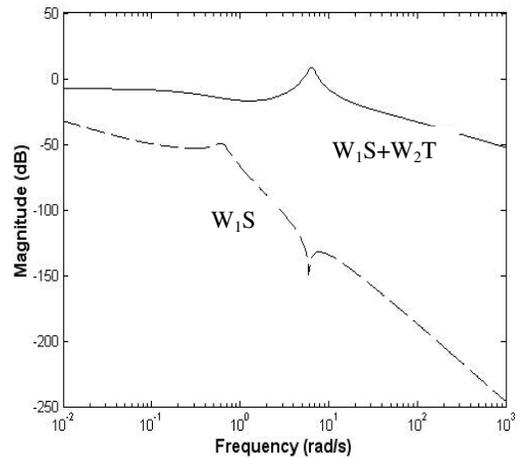
$$C = \frac{294.98(s + 0.5)(s + 2)}{s(s + 0.1)} \quad (16)$$

The log-magnitude plot relating  $W_1$ ,  $W_2$  for the upper and lower bounds of  $L$  given in (10) and (11) are shown in Fig.5.

The robust and nominal performance measures for the robust designs are shown in Fig.6. It can be observed that the nominal performance measure is very small relative to 0 db. The robust stability measure is marginally violated at the corner frequency. This is for a worst-case design in the absence of damping term in the electromechanical swing equation.



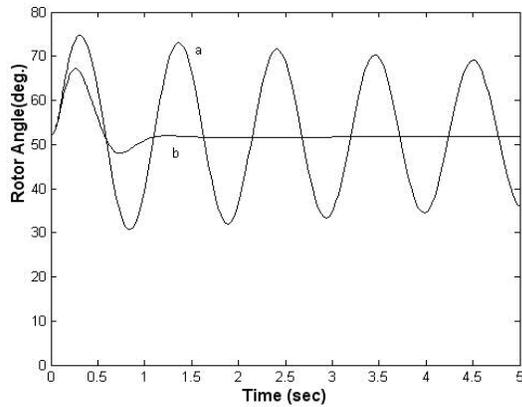
**Figure 5.** The loop-shaping boundary plots



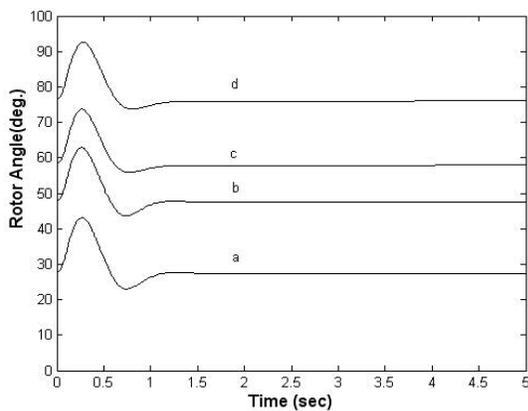
**Figure 6.** Plot of nominal and robust performance indices

While selecting the open-loop transfer function, the internal stability of the plant in addition to the design criterion (8)-(11) had to be checked. A disturbance of 50% input torque pulse for 0.1 second on the generator shaft was simulated for this purpose. The rotor angle variations of the

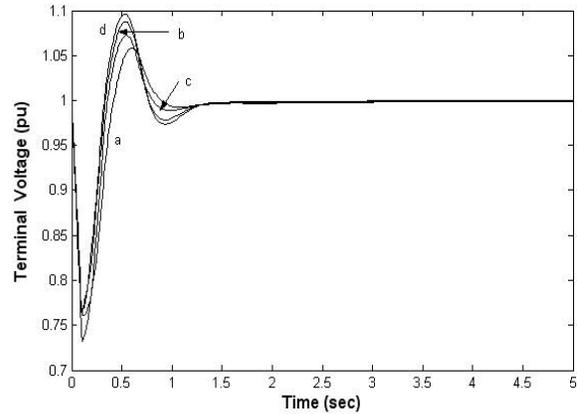
generator for the nominal operating point with and without the robust controller are plotted in Fig. 7. The controller designed was then tested for operation on a number of other operating conditions in the power output range of 0.3-1.3 pu. The rotor angle characteristics for 4 operating conditions are given in Fig.8 considering the same 50% torque pulse disturbance. As can be observed, the robust controller provides extremely good damping to the electromechanical oscillations. The terminal voltage characteristics are also extremely good as depicted in Fig. 9.



**Figure 7.** Generator rotor angle variation following a 50% input torque pulse for 0.1sec with, (a) no control, (b) proposed robust damping controller.

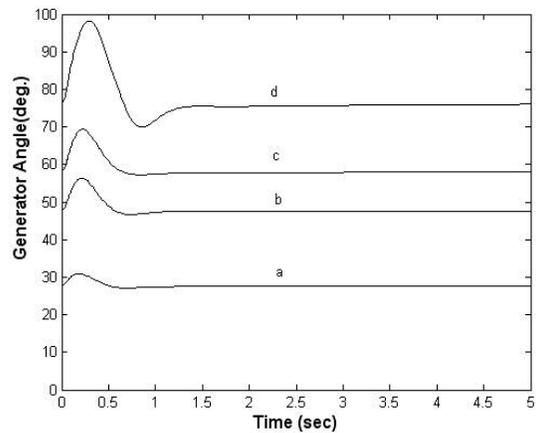


**Figure 8.** Rotor angle characteristics with the robust damping controller for (a) 0.5 pu output at 0.85 lagging pf, (b) 0.8 pu power at 0.9 pf lag, (c) 1pu power at 0.85 pf, and (d) 1.3 pu power at 0.95 pf lagging.



**Figure 9.** Terminal voltage variations of the generator corresponding to Fig. 8

For a 3-phase fault of 0.05sec duration on the remote bus, the rotor angle variations for the 4 operating conditions are plotted in Fig.10. The generator terminal voltage variations are shown in Fig. 11. Without the control the response is extremely oscillatory to first swing unstable depending on the loading conditions. As can be observed the robust controller is able to stabilize the system in a very short period for a wide range of operating conditions. While the controller could be designed to provide even more damping, it would have to be compromised with terminal voltage transients.



**Figure 10.** Generator rotor angle variations with the robust controller following a three-phase fault for 0.05 sec on the remote bus. The loading conditions are, a) 0.5 pu power output at 0.85 pf lag, b)0.8 pu at 0.9 pf lag, c)1pu at 0.85 pf lag and, d)1.3 pu at 0.95 lagging pf.

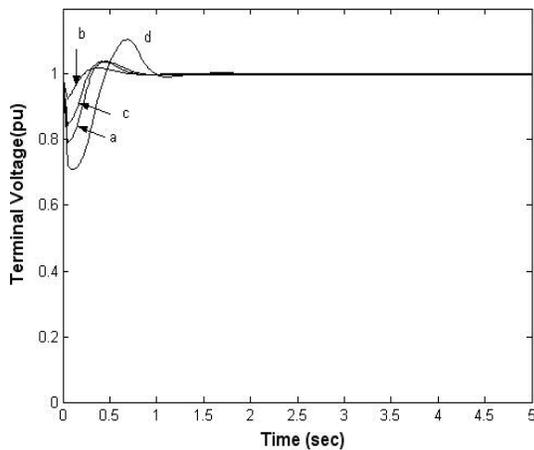


Figure 11. Terminal voltage variations corresponding to Fig. 10

## CONCLUSIONS

A novel method of designing a robust damping controller for a static var system is proposed. A graphical loop-shaping technique has been employed to select the open loop transfer function subject to satisfaction of robust stability and performance criteria. The controller designed was tested for a number of disturbance conditions including symmetrical three-phase faults. The robust design has been found to be very effective for damping control over a wide range of operating conditions of the power system. The operating conditions for which the controller provides good performance depends on the spectrum of perturbed plants selected in the design process. The graphical loop-shaping method utilized to determine the controller function is simple and is straightforward to implement.

## ACKNOWLEDGEMENT

The authors wish to acknowledge the facilities provided by the King Fahd University of Petroleum and Minerals towards this research. This work is funded by KFUPM under Project #EE/ROBUST/252.

## REFERENCES

- Chao, A., and Athans, M. 1996, "Stability robustness to unstructured uncertainty for linear time invariant systems", *The Control Handbook (Editor, W.S. Levine)*, CRC Press and IEEE Press.
- Dahleh, M.A. 1996. " $l_1$  Robust Control: Theory, Computation and Design", *The Control Handbook (Editor, W.S. Levine)*, CRC Press and IEEE Press.
- Doyle, J.C., Francis, B.A., and Tannenbaum, A.R. 1992, *Feedback Control Theory*, MacMillan Publishing Co, New York.

- Farasangi, M.M., Song, Y.H., and Sun, Y.Z. 2000. "Supplementary Control Design of SVC and STATCOM Using  $H_\infty$  Optimal Robust Control", *Proc. Intl. Conf. on Electric Utility Deregulation 2000*, City University London, 355-360.
- Ghazi, R., Azemi, A., and Badakhshan, K.P. 2001. "Adaptive Fuzzy Sliding Mode Control of SVC and TCSC for Improving the Dynamic Performance of a Power System", *Proc. 7th Intl. Conf. on AC-DC Power Transmission*, 333-337.
- Hamad, A.E., 1986. "Analysis of power system stability enhancement by static var compensators", *IEEE Trans. on Power Systems*, Vol. 1, no. 4, 222-227.
- Hosseini, S.H. and Mirshekhar, O. 2001. "Optimal Control of SVC for Subsynchronous Resonance Stability in Typical Power System", *Proc. ISIE*, Vol. 2, 916-921.
- Oliveria, S.E.M. 1994. "Synchronizing and damping torque coefficients and power system steady state stability as affected by static var compensators", *IEEE Trans. on Power Systems*, Vol. 9, no. 1, 109 - 116.
- Rahim, A.H.M.A., and Nassimi, S.G.A. 1996. "Synchronous Generator Damping Enhancement through Coordinated Control of Exciter and SVC", *IEE Proc-Gener. Transm. Distrib.*, Vol. 143, no. 2, 221-218.
- So, P.L., and Yu, T. 2000. "Coordination of TCSC and SVC for Inter area Stability Enhancement", *Proc. POWERCON 2000*, Vol. 1, 553-558.
- Tso, S.K., Liang, J., Zeng, Q.Y., Lo, K.L., and Zhou, X.X. 1997. "Coordination of TCSC and SVC for Stability Improvement of Power Systems", *Proc. 4th Intl. Conf. on Advances in power System Control, Operation and Management, APSCOM-97*, Hong Kong, 371-376.
- Wang, Y., Tan, Y., and Guo, G. 1997. "Robust Non-linear Coordinated Excitation and SVC Control for Power Systems", *Proc. 4th APSCOM-97*, Hong Kong, 415-420.
- Zhang, C., and Zhou, X.X. 1998. "Coordinated Non-linear Control of TCSC and SVC in Power Systems", *Proc. POWERCON '98*, Vol. 1, 338-343.
- Zhao, Q and Jiang J. 1995. "Robust SVC Controller Design for Improving Power System Damping", *IEEE Trans. Power System*, Vol. 10, no. 4, 1927-1932.
- Zhao, Q., and Jiang, J. 1995. "Robust SVC Controller Design for Improving Power System damping", *IEEE Trans. Power System*, Vol. 10, no. 4, 1927-1932.
- Zhou, E.Z. 1993. "Application of static var compensators to increase power system damping", *IEEE Trans. on Power Systems*, Vol. 8, no. 2, 655- 661.