Rotor Dynamics and Simplified Transient Stability Studies

In order to simplify transient stability studies, the following assumptions are made:

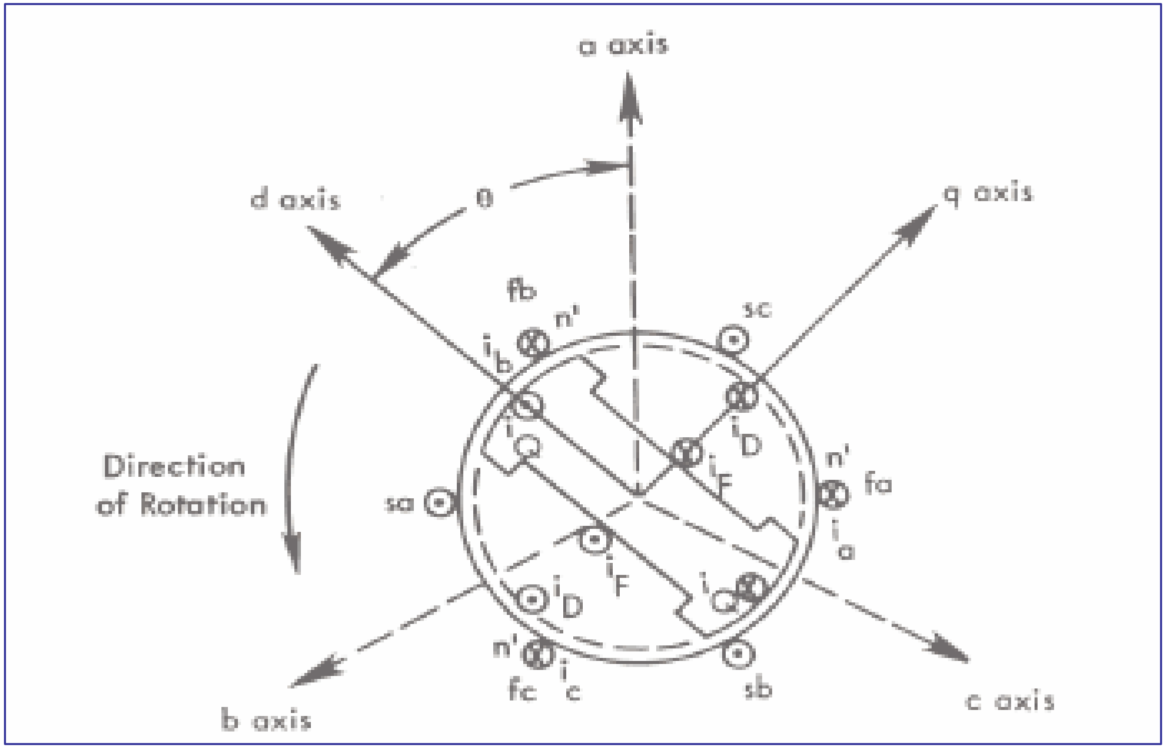
1. Only balanced 3-φ systems and balanced disturbances are considered. Therefore, only positive sequence networks are employed

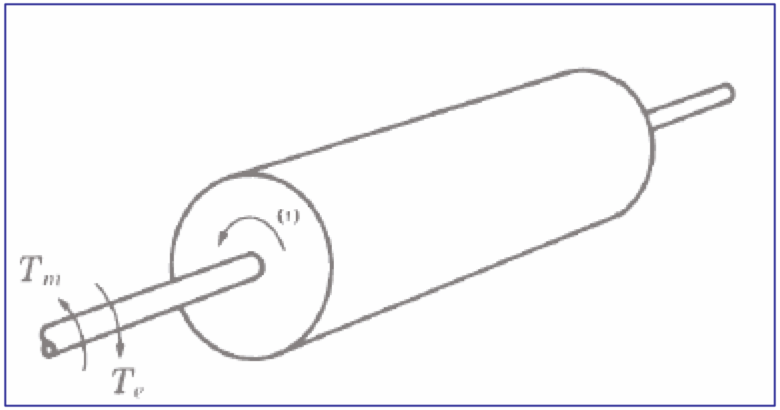
2. Deviation of machine frequencies from synchronous frequency is small, and dc offset currents and harmonics are neglected. Therefore, the network of transmission lines, transformers and impedance loads are essentially in steady state; and voltage, current and power can be computed from algebraic equations.

3. Generated voltage is considered to be unaffected by machine saturation and speed variations.

The Rotor Dynamics and Swing Equation

Consider a generating unit consisting of a three phase synchronous generator and its prime mover.





The rotor motion is determined by Newton's second law, given by,

$$\begin{aligned} J\alpha_m &= T_m - T_e \\ &= T_a \end{aligned} \qquad (1)$$

where,

J	Total moment of inertia of the rotating mass, kgm^2
α_m	rotor angular acceleration, rad/sec^2
T_m	Mechanical input torque, Nm (+ve for generator)
T_e	Electrical output torque including losses, Nm (+ve for generator)
T_a	Net accelerating/decelerating torque, Nm

The rotor angular velocity, ω_m, and acceleration are related to the rotor angular position, Θ_m, as [Note: subscript m refers to mechanical quantities]:

$$\alpha_m = \frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} \qquad (2)$$

$$\omega_m = \frac{d\theta_m}{dt}$$

δ_m is measured with respect to a synchronously rotating frame. Therefore, we define,

$$\theta_m = \omega_{ms} t + \delta_m \qquad (3)$$

where,

ω_{ms}	Synchronous angular speed of the rotor, rad/sec
δ_m	Rotor angular position from the synchronously rotating reference frame (mech.)

Substituting (2) in (1),

$$J\frac{d^2\theta_m}{dt^2} = J\frac{d^2\delta_m}{dt^2} = T_m - T_e \qquad (4)$$

It is convenient to work with power rather than torque, and to work in p.u. rather than actual units.

Multiply both sides of (4) by ω_m and divide by rated volt-ampere, S_{rated}, to get

$$\begin{aligned}\frac{J\omega_m}{S_{rated}}\frac{d^2\delta_m}{dt^2} &= \frac{\omega_m T_m - \omega_m T_e}{S_{rated}} \\ &= \frac{P_m}{S_{rated}} - \frac{P_e}{S_{rated}} \\ &= P_m(p.u.) - P_e(p.u.)\end{aligned} \qquad (5)$$

Define the normalized inertia constant as,

$$H = \frac{stored\ kinetic\ energy\ at\ sysnchronous\ speed}{rated\ VA}$$
$$= \frac{\frac{1}{2} J\omega_{ms}^2}{S_{rated}} \quad Joules/VA,\ or\ p.u.\ seconds \tag{6}$$

Substitute back in (5) to get,

$$2H\frac{\omega_m}{\omega_{ms}^2}\frac{d^2\delta_m}{dt^2} = P_m - P_e \quad (in\ p.u.) \tag{7}$$

The electrical and mechanical quantities are related by,

$$\alpha = \frac{P}{2}\alpha_m \qquad \delta = \frac{P}{2}\delta_m \qquad \omega = \frac{P}{2}\omega_m$$

This gives,

$$2H\frac{\omega}{\omega_s^2}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{8}$$

Recognizing the fact that frequency of the machine is not appreciably different from the synchronous frequency, the above equation is approximated as,

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \qquad \text{(in per unit)} \qquad (9)$$

In the above, ω_s is the synchronous frequency (also, called the base frequency ω_o) in electrical radian/sec ($\omega_s = 2\pi f$), and δ is in electrical radian.

Equation (9) is a second order differential equation, called the electromechanical swing equation or just the **swing equation**. This can also be broken up into 2 first order equations as,

$$\frac{d\delta}{dt} = \omega - \omega_s$$
$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = P_m - P_e \qquad (10)$$

The quantity 2H/ωs is sometimes expressed as M.

Swing Equation for Multiple Machines

In a multi-machine system, a number of machines can be replaces by an equivalent machine, if the rotors of these machines swing together (coherent). Consider 2 such machines

$$\frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$
$$\frac{2H_2}{\omega_s}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \qquad (11)$$

If the machines are coherent, then

$$\delta_1 - \delta_2 = k$$
$$Giving,$$
$$\frac{d^2\delta_1}{dt^2} = \frac{d^2\delta_1}{dt^2} = \frac{d^2\delta_1}{dt^2}(say) \qquad (12)$$

Add the 2 swing equations in (11), and write

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \qquad (13)$$

$$
\begin{aligned}
&\text{where,}\\
&H = H_1 + H_2\\
&P_m = P_{m1} + P_{m2}\\
&P_e = P_{e1} + P_{e2}
\end{aligned}
$$

If the H's are specified on different ratings, they should be converted to the same base before the equivalencing is carried out. The relationship is

$$H_{new} = H_{old}\frac{S_{old}}{S_{new}} \qquad (14)$$

For any pair of non-coherent machines in a system, swing equations similar to (13) can be written. Reorganize the 2 swing equations in (11) and write,

$$\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \frac{\omega_s}{2}\left[\frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2}\right] \qquad (15)$$

Multiply each side by H1H2/(H1+H2), and rearrange

$$\frac{2}{\omega_s}\left(\frac{H_1 H_2}{H_1 + H_2}\right)\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \frac{P_{m1}H_2 - P_{m2}H_1}{H_1 + H_2} - \frac{P_{e1}H_2 - P_{e2}H_1}{H_1 + H_2} \qquad (16)$$

And it follows that,

$$\frac{2H_{12}}{\omega_s}\frac{d^2\delta_{12}}{dt^2} = P_{m12} - P_{e12}$$

where,

$$H_{12} = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_{m12} = \frac{P_{m1}H_2 - P_{m2}H_1}{H_1 + H_2}$$

$$P_{e12} = \frac{P_{e1}H_2 - P_{e2}H_1}{H_1 + H_2} \qquad (17)$$