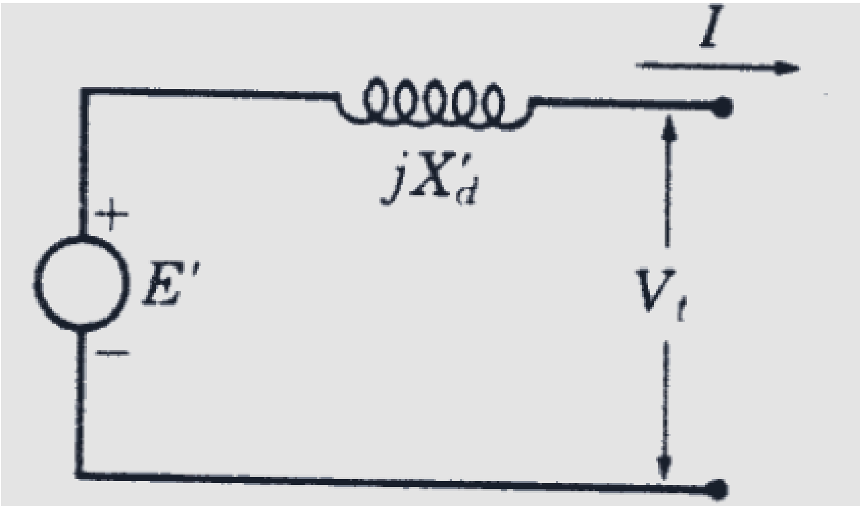
The Equal Area Criterion

One of the methods of solving the swing equation is by graphical technique, and is applicable to one or two machines under the following assumptions:

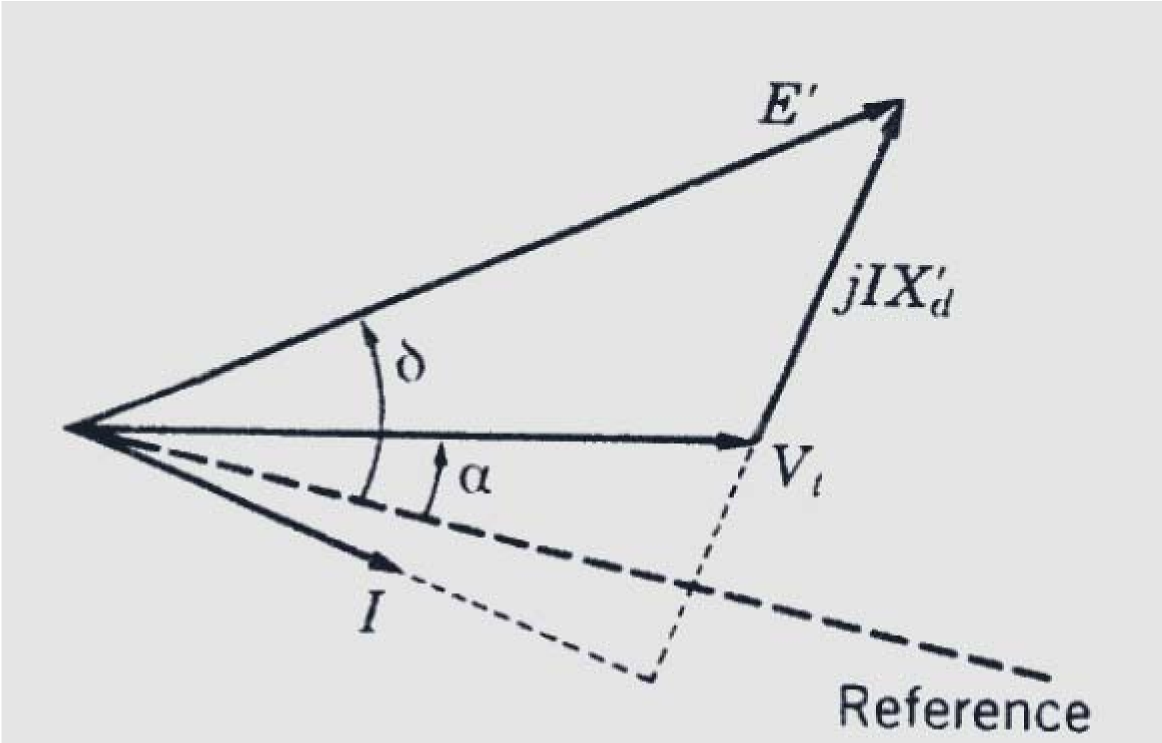
1. P_m, the mechanical power input, does not change during the swing
2. P_e, the electrical power output, is obtained from the steady state solution of the system and is given by

$$P_e = \frac{E^{'}V}{x_e}\sin\delta; \qquad x_e = x_d^{'} + x_{ext}$$

$$= P_{\max}\sin\delta$$

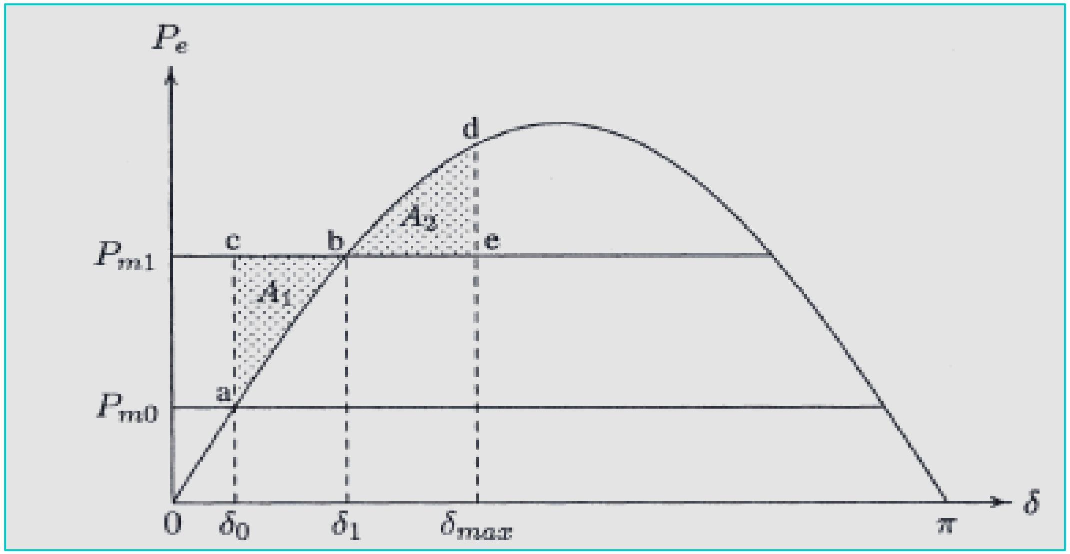


3. The machines are represented by transient voltage E′ behind the transient reactance xd′



4. The rotor angle δ is in phase with excitation voltage E′
5. Damping powers are negligible

Let us start with a one machine system connected to an infinite bus through reactance X_{ext}. Observe the power angle characteristic given below.

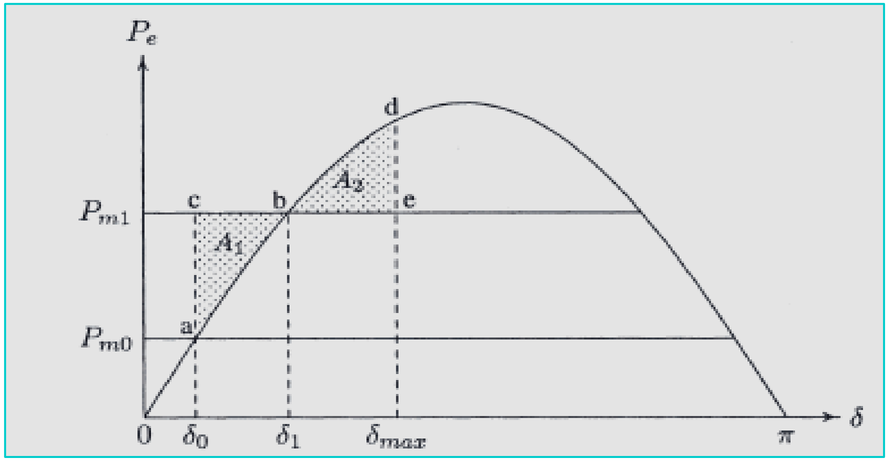


Let the machine operate initially at steady state with $\delta = \delta_o$ corresponding to mechanical power $P_{mo} = P_{eo}$ as shown in the figure. We know the electrical power output is given as,

$$P_e = P_{max} \sin \delta$$

Consider a sudden step increase in input power to P_{m1}. Since $P_{m1} > P_{eo}$, the machine accelerates and δ increases. The excess energy stored is the area A_1.

With increase in δ electrical power increases and when $\delta = \delta_1$ the electrical power matches the new input power P_{m1} and the acceleration $(d^2 \delta/dt^2)$ is zero.



Though the acceleration is zero, the machine runs above synchronous speed, hence δ and electrical power continues to increase. Now $P_m < P_e$ causing rotor to decelerate towards synchronous speed until $\delta = \delta_{max}$. At this point, the kinetic energy stored by the rotor while accelerating has been completely returned to the system. The energy given up is represented by the area A_2.

At d the rotor speed again synchronous although the rotor angle has advance to δ_{max}. The accelerating power is still negative (retarding) and so the rotor cannot remain at synchronous speed but must continue to slow down. The relative velocity is negative and rotor angle moves back from δ_{max} along the power angle curve to point b at which point the rotor speed is less than synchronous (deceleration is zero). From b to a the machines accelerates until synchronous speed is achieved at a (the zero damping case). And the cycle continues to repeat.

The excess energy stored in the rotor inertia during acceleration of the rotor is shown in the shaded area A1, and that returned during decoration, as the shaded area A_2. For the net energy to be zero, A_1 must be equal to A_2.

This is the equal area criterion.

Let us work this out mathematically. The swing equation is,

$$M\frac{d^2\delta}{dt^2}=P_m-P_e=P_a$$

Multiply both sides by d δ/dt

$$M\frac{d\delta}{dt}\frac{d^2\delta}{dt^2}=P_a\frac{d\delta}{dt}$$

$$Or,\quad \frac{M}{2}\frac{d}{dt}[\frac{d\delta}{dt^2}]^2=P_a\frac{d\delta}{dt}$$

Multiplying by dt and integrating from δ_0 to δ_{max}

$$\frac{M}{2}[\frac{d\delta}{dt^2}]^2\Big|_{\delta_o}^{\delta_{max}}=\int_{\delta_o}^{\delta_{max}}(P_m-P_e)d\delta$$

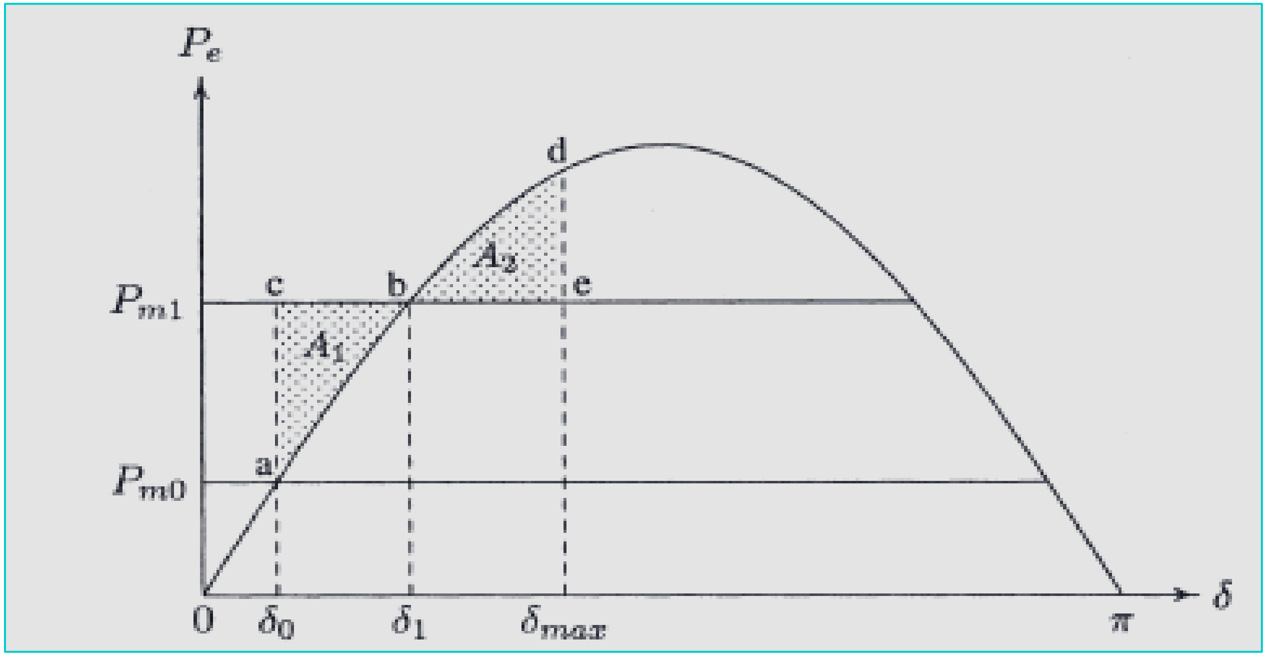
The rotor speed d δ/dt=0 at δ_o as well as at δ_{max} (synch. speed), giving the left hand side to be zero. Or,

$$\int_{\delta_o}^{\delta_{\max}} (P_m - P_e)\,d\delta = 0$$

$$Or,\quad \int_{\delta_o}^{\delta_1} (P_m - P_e)\,d\delta + \int_{\delta_1}^{\delta_{\max}} (P_m - P_e)\,d\delta = 0$$

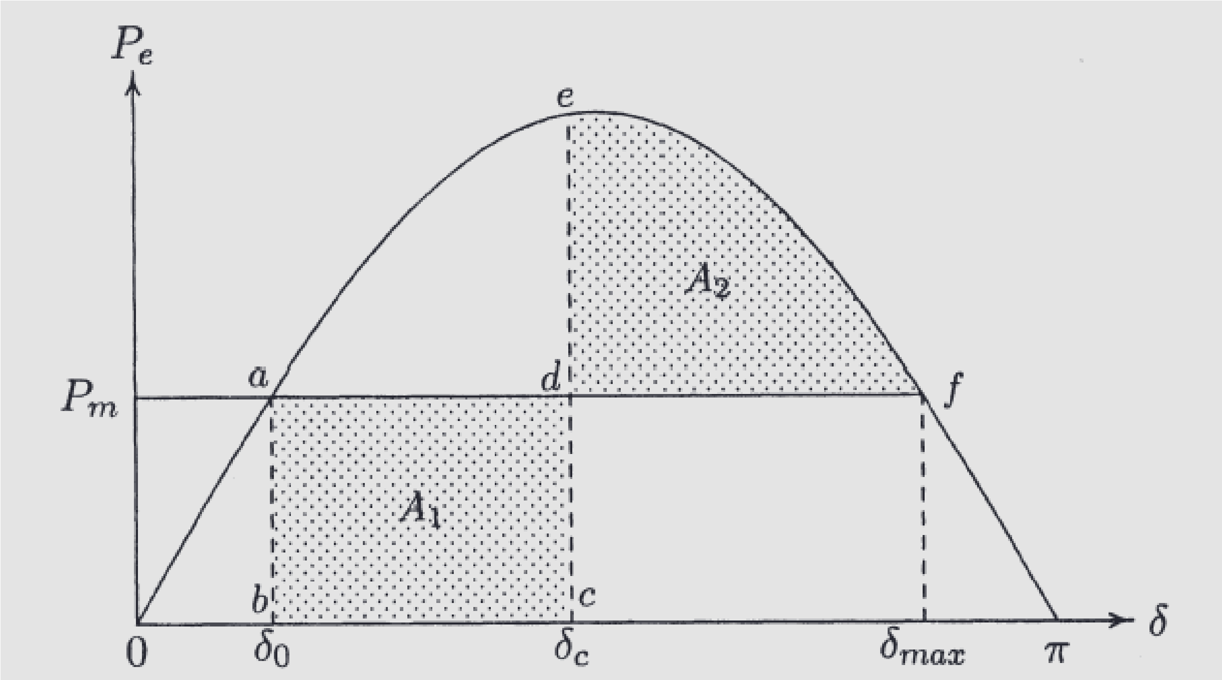
$$Or,\quad \int_{\delta_o}^{\delta_1} (P_m - P_e)\,d\delta = \int_{\delta_1}^{\delta_{\max}} (P_e - P_m)\,d\delta$$

$$Or,\quad A_1 = A_2$$



The Critical Clearing Time

Consider a symmetrical three phase fault at the generator terminal cleared by it-self and the original system conditions are restored. The power angle characteristic is shown below. The steady state power is P_m when the fault is applied.



The shaded area (A_1) representing the kinetic energy is dependent upon the time taken to clear the fault. If δ_c is increased, A_1 is increased. Equal area criterion requires A_2 to be increased. If the delay in clearing is prolonged so that δ swings past δ_m, the rotor speed is more than synchronous and again accelerating power is encountered. This will lead to further increase in δ and hence instability will result. Therefore, there is a critical angle for clearing the fault in order to satisfy the requirements for equal area criterion for stability. This angle, called critical clearing angle (δ_c) and corresponding critical clearing time (t_c) is obtained as follows.

The area A_1 is,

$$A_1 = \int_{\delta_o}^{\delta_c} P_m d\delta = P_m(\delta_c - \delta_o)$$

While area A_2 is,

$$A_2 = \int_{\delta_c}^{\delta_m} (P_{\max} \sin\delta - P_m) d\delta$$
$$= P_{\max}(\cos\delta_c - \cos\delta_m) - P_m(\delta_m - \delta_c)$$

Equating the expression for A_1 and A_2,

$$\cos\delta_c = \frac{P_m}{P_{\max}}(\delta_m - \delta_o) + \cos\delta_m$$

From the power angle curve, $\delta_m = \pi - \delta_o$, and $P_m = P_{max}\sin\delta_o$

Substituting,

$$\delta_c = \cos^{-1}[(\pi - 2\delta_o)\sin\delta_o - \cos\delta_o]$$

δ_c and t_c are related by,

$$\delta_c = \frac{\omega_s P_m}{4H} t_c^2 + \delta_o$$

$$Or, \quad t_c = \sqrt{\frac{4H(\delta_c - \delta_o}{\omega_s P_m}}$$

In a 2 machine system, if the fault occurs on the machine bus there will be no power transfer. However, if fault occurs on some section of a double circuit line, power transfer may be possible. Clearing the fault may not restore original circuit conditions. The equal area criterion can be applied to such cases also. The figure below shows such a scenario.