Home Work #1: Chapter 2

Modeling	P2.11, P2.13
Block Diagram Reduction	E2.8, E2.13
Signal Flow Graph	AP2.2, E2.22

Solution

P2.11 The transfer functions from $V_c(s)$ to $V_d(s)$ and from $V_d(s)$ to $\theta(s)$ are:

$$V_d(s)/V_c(s) = \frac{K_1 K_2}{(L_q s + R_q)(L_c s + R_c)} \text{, and}$$

$$\theta(s)/V_d(s) = \frac{K_m}{(Js^2 + fs)((L_d + L_a)s + R_d + R_a) + K_3 K_m s} .$$

The block diagram for $\theta(s)/V_c(s)$ is shown in Figure P2.11, where

$$\theta(s)/V_c(s) = \frac{\theta(s)}{V_d(s)} \frac{V_d(s)}{V_c(s)} = \frac{K_1 K_2 K_m}{\Delta(s)}$$

where

$$\Delta(s) = s(L_c s + R_c)(L_q s + R_q)((Js + b)((L_d + L_a)s + R_d + R_a) + K_m K_3)$$

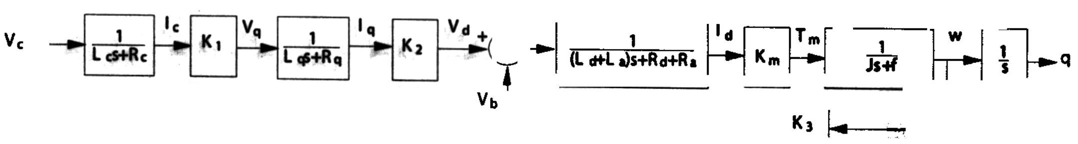


FIGURE P2.11
Block diagram.

AP2.2 The closed-loop transfer function form $R_1(s)$ to $Y_2(s)$ is

$$\frac{Y_2(s)}{R_1(s)} = \frac{G_1 G_4 G_5(s) + G_1 G_2 G_3 G_4 G_6(s)}{\Delta}$$

where

$$\Delta = [1 + G_3 G_4 H_2(s)][1 + G_1 G_2 H_3(s)]$$

If we select

$$G_5(s) = -G_2 G_3 G_6(s)$$

then the numerator is zero, and $Y_2(s)/R_1(s) = 0$. The system is now decoupled.

E2.22 The closed-loop transfer function is

$$\frac{Y(s)}{R(s)} = T(s) = \frac{K_1 K_2}{s^2 + (K_1 + K_2 K_3 + K_1 K_2)s + K_1 K_2 K_3}$$

P2.13 The motor torque is given by

$$T_m(s) \quad (J_m s^2 + b_m s)\theta_m(s) + (J_L s^2 + b_L s)n\theta_L(s)$$
$$n((J_m s^2 + b_m s)/n^2 + J_L s^2 + b_L s)\theta_L(s)$$

where

$$n \quad \theta_L(s)/\theta_m(s) \qquad \text{gear ratio}$$

$$T_m(s) \quad K_m I_g(s)$$

$$I_g(s) = \frac{1}{(L_g + L_f)s + R_g + R_f} V_g(s) \; ,$$

and

$$V_g(s) \quad K_g I_f(s) \quad \frac{K_g}{R_f + L_f s} V_f(s)$$

Combining the above expressions yields

$$\frac{\theta_L(s)}{} = \frac{K_g K_m}{}$$

where

$$\Delta_1(s) \quad J_L s^2 + b_L \; + \frac{J_m s^2 + b_m s}{n^2}$$

and

$$\Delta_2(s) = (L_g s + L_f s + R_g + R_f)(R_f + L_f s)$$

E2.8 The block diagram is shown in Figure E2.8.

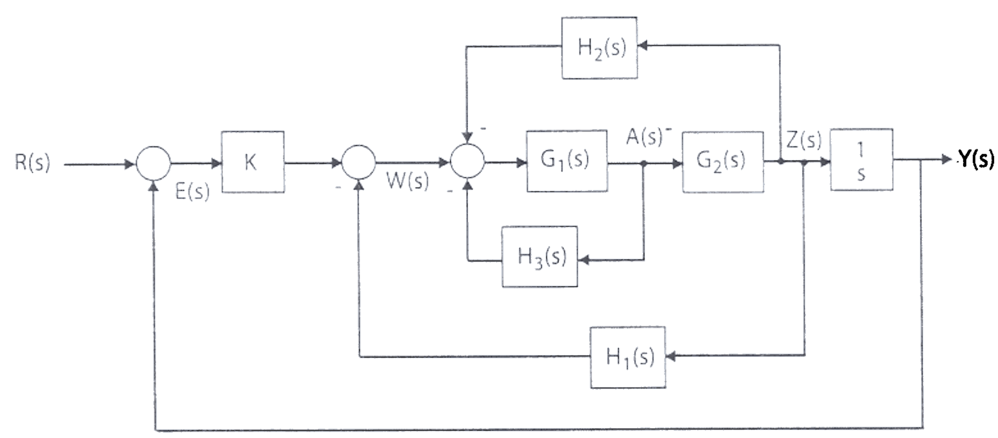


FIGURE E2.8
Block diagram model.

Starting at the output we obtain

$$Y(s) = \frac{1}{s} Z(s) \quad _s G_2(s)A(s).$$

But $A(s) \quad G_1(s)\left[-H_2(s)Z(s) \quad H_3(s)A(s) + W(s)\right]$ and $Z(s) \quad sY(s)$, so

$$Y(s) \quad G_1(s)G_2(s)H_2(s)Y(s) \quad G_1(s)H_3(s)Y(s) + \frac{1}{s}G_1(s)G_2(s)W(s).$$

Substituting $W(s) \quad KE(s) \quad H_1(s)Z(s)$ into the above equation yields

$$Y(s) = -G_1(s)G_2(s)H_2(s)Y(s) \quad G_1(s)H_3(s)Y(s)$$
$$\frac{1}{s}G_1(s)G_2(s)\left[KE(s) \quad H_1(s)Z(s)\right]$$

and with $E(s) = R(s) - Y(s)$ and $Z(s) = sY(s)$ this reduces to

$$Y(s) = [-G_1(s)G_2(s)(H_2(s) + H_1(s)) - G_1(s)H_3(s) - \frac{1}{s}G_1(s)G_2(s)K]Y(s) + \frac{1}{s}G_1(s)G_2(s)KR(s).$$

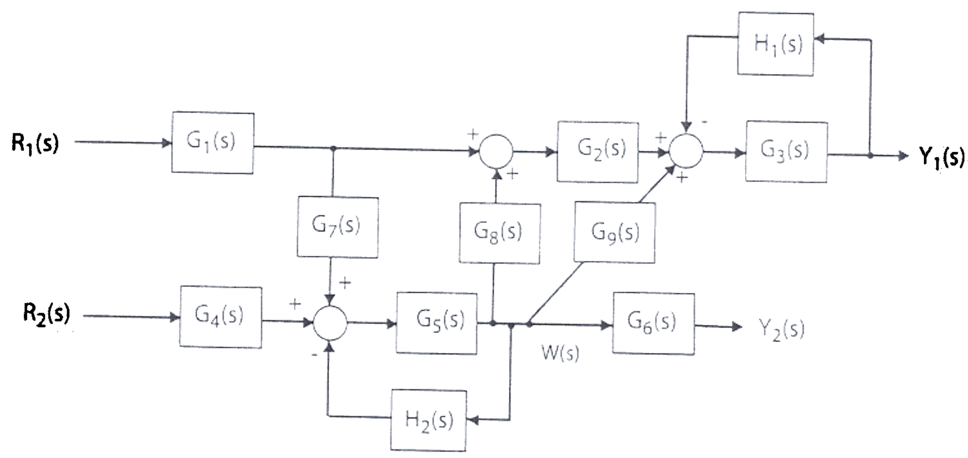
Solving for $Y(s)$ yields the transfer function

$$Y(s) = T(s)R(s),$$

where

$$T(s) = \frac{KG_1(s)G_2(s)/s}{1 + G_1(s)G_2(s)[(H_2(s) + H_1(s)] + G_1(s)H_3(s) + KG_1(s)G_2(s)/s}$$

E2.13 Since we want to compute the transfer function from $R_2(s)$ to $Y_1(s)$, we can assume that $R_1 = 0$ (application of the principle of superposition). Then, starting at the output $Y_1(s)$ we obtain



$$Y_1(s) = G_3(s)[-H_1(s)Y_1(s) + G_2(s)G_8(s)W(s) + G_9(s)W(s)]$$

or

$$1 + G_3(s)H_1(s)]Y_1(s) = [G_3(s)G_2(s)G_8(s)W(s) + G_3(s)G_9(s)]W(s).$$

Considering the signal $W(s)$ (see Figure E2.13), we determine that

$$W(s) = G_5(s)[G_4(s)R_2(s) - H_2(s)W(s)],$$

or

$$[1 + G_5(s)H_2(s)]W(s) = G_5(s)G_4(s)R_2(s).$$

Substituting the expression for $W(s)$ into the above equation for $Y_1(s)$ yields

$$\frac{Y_1(s)}{R_2(s)} = \frac{G_2(s)G_3(s)G_4(s)G_5(s)G_8(s) + G_3(s)G_4(s)G_5(s)G_9(s)}{1 + G_3(s)H_1(s) + G_5(s)H_2(s) + G_3(s)G_5(s)H_1(s)H_2(s)}$$