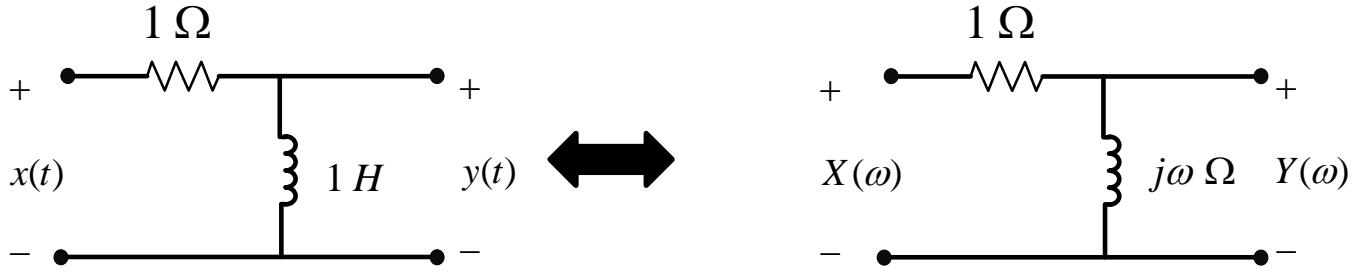| Sec	Ser	ID	Name **Solution**

An input output system is as shown below

(a) Find h(t) the impulse response ?

(b) If the input is the step function , $x(t) = u(t)$, find the out put y(t) using the Fourier Transform Table and Properties **only** ?

Note $\dfrac{1}{x(1+x)} = \dfrac{1}{x} - \dfrac{1}{(1+x)}$

Solution (a) $Y(\omega) = \dfrac{j\omega}{1+j\omega}X(\omega) \Rightarrow H(\omega) = \dfrac{Y(\omega)}{X(\omega)} = \dfrac{j\omega}{1+j\omega}$

$\Rightarrow h(t) = \dfrac{d}{dt}(e^{-t})u(t) = -e^{-t}u(t) + (e^{-t})_{t=0}\delta(t) = \delta(t) - e^{-t}u(t)$

(b) $\quad X(\omega) = \dfrac{1}{j\omega} + \pi\delta(\omega)$

$$Y(\omega) = X(\omega)H(\omega) = \left[\frac{1}{j\omega} + \pi\delta(\omega)\right]\left[1 - \frac{1}{1+j\omega}\right] = \left[\frac{1}{j\omega} + \pi\delta(\omega)\right] - \left[\underbrace{\frac{1}{j\omega(1+j\omega)}}_{\frac{1}{j\omega} - \frac{1}{(1+j\omega)}} + \underbrace{\frac{1}{1+j\omega}(\pi\delta(\omega))}_{\frac{1}{1+j0}(\pi\delta(\omega))}\right]$$

$$= \left[\frac{1}{j\omega} + \pi\delta(\omega)\right] - \left[\frac{1}{j\omega} - \frac{1}{(1+j\omega)} + \pi\delta(\omega)\right] = \frac{1}{(1+j\omega)}$$

$\Rightarrow \quad y(t) = e^{-t}u(t)$