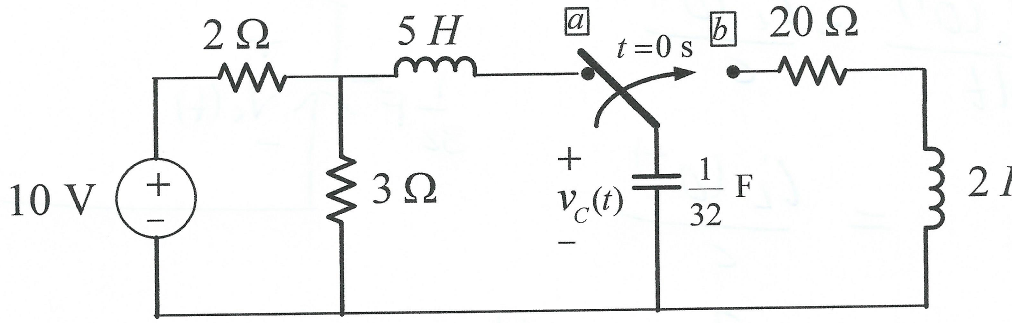
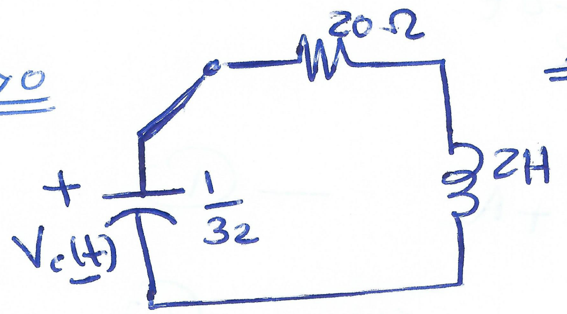
EE 202-02-Winter 2014(141)
QZ4

Sec	Ser	ID	Name KEY



For the circuit shown above , the switch was in position ***a*** for a long time and at t = 0 the switch move to position ***b*** . Find $v_c(t)$ $t \geq 0$?

t > 0



⇒ Natural Response seires RLC

$\Rightarrow \alpha = \frac{R_{eq}}{2L} = \frac{20}{2(2)} = 5$ rad/s

$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2(\frac{1}{32})}} = 4$ rad/s

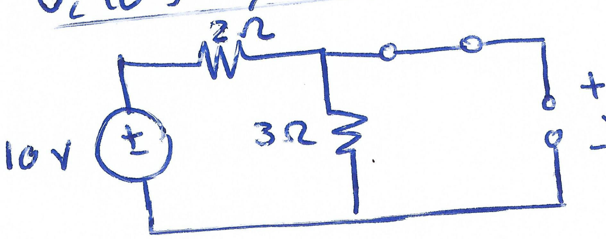
⇒ overdamp response

v_c $S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w_0^2} = -5 \pm \sqrt{5^2 - 4^2}$
$= -2, -8$

$v_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} = A_1 e^{-2t} + A_2 e^{-8t}$ $t \geq 0$

To find A_1, A_2 we need $v_c(0^+)$, $\frac{dv_c(0^+)}{dt}$

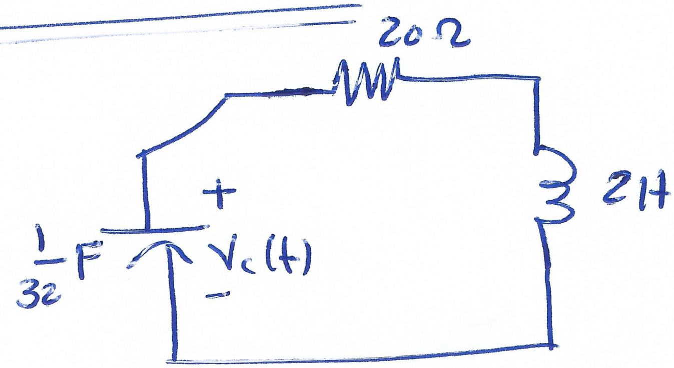
$v_c(0^+)$ ⇒ we look at the circuit t < 0



$v_c(0^-) = \frac{3}{3+2} 10 = 6$ V

⇒

To find $\frac{dv_c(0^+)}{dt}$, we look at $t = 0^+$



$$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{c}$$

$$= \frac{i_L(0^+)}{c}$$

$$= \frac{0}{c} = 0$$

$$U_c(t) = A_1 e^{-2t} + A_2 e^{-8t}$$

$$\Rightarrow V_c(0) = 6 = A_1 + A_2 \quad \text{— ①}$$

$$\frac{dU_c(0^+)}{dt} = 0 = -2A_1 - 8A_2 \quad \text{— ②}$$

solving ① and ②, $\Rightarrow$ $A_1 = 8$, $A_2 = -2$

$$\Rightarrow U_c(t) = 8e^{-2t} - 2e^{-8t} \quad t > 0$$