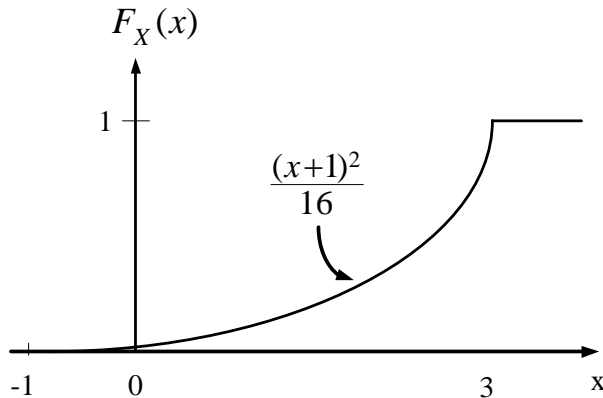


EE 315-Winter 2014(132)  
QZ2

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A random variable  $X$  has the Cumulative Distribution Function  $F_X(x)$



Let the event  $B$  defined as  $B = \{X < 2\}$  find the followings :

- (a)  $P\left(|X| < \frac{1}{2}\right)$  ?
- (b)  $F_X(1|B)$  ?
- (c) The probability density function  $f_X(x)$  **and plot it** ?
- (d)  $f_X(x|B)$  **and plot it** ?

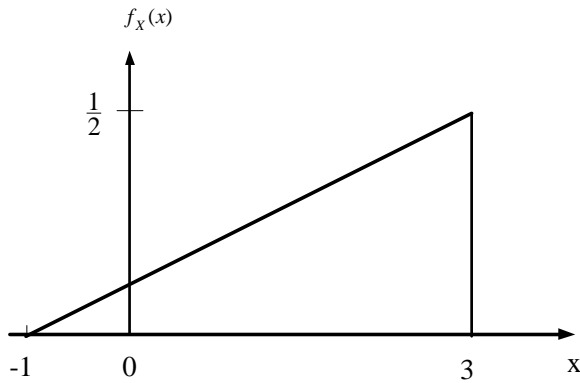
**Solution**

$$(a) P\left(|X| < \frac{1}{2}\right) = P\left(-\frac{1}{2} < X < \frac{1}{2}\right) = F_X\left(\frac{1}{2}\right) - F_X\left(-\frac{1}{2}\right) = \frac{\left(\frac{1}{2}+1\right)^2}{16} - \frac{\left(-\frac{1}{2}+1\right)^2}{16} = \frac{1}{8} = 0.125$$

$$(b) F_X(1|B) = P(X < 1|B) = P(X < 1|X < 2) = \frac{P(X < 1 \cap X < 2)}{P(X < 2)} = \frac{P(X < 1)}{P(X < 2)}$$

$$= \frac{F_X(1)}{F_X(2)} = \frac{\frac{(1+1)^2}{16}}{\frac{(2+1)^2}{16}} = \frac{(1+1)^2}{(2+1)^2} = \frac{4}{9} = 0.4444$$

$$(c) \quad f_X(x) = \frac{d}{dx} F_X(x) = \frac{d}{dx} \frac{(x+1)^2}{16} = \frac{1}{8}(x+1)$$



$$(d) \quad f_X(x|B) = \frac{d}{dx} F_X(x|B) = \frac{d}{dx} \begin{cases} \frac{F_X(x)}{F_X(2)} & -1 < x < 2 \\ 1 & x > 2 \end{cases} = \begin{cases} \frac{\frac{1}{8}(x+1)}{F_X(2)} & -1 < x < 2 \\ 0 & x > 2 \end{cases}$$

$$= \begin{cases} \frac{\frac{1}{8}(x+1)}{\frac{(2+1)^2}{16}} & -1 < x < 2 \\ 0 & x > 2 \end{cases} = \begin{cases} \frac{2}{9}(x+1) & -1 < x < 2 \\ 0 & x > 2 \end{cases}$$

