CH 1

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$P(A)=\sum_{n=1}^{N}P(A|B_n)P(B_n)$$

$$P(B_n|A)=\frac{P(A|B_n)P(B_n)}{P(A|B_1)P(B_1)+....+P(A|B_n)P(B_n)}$$

$$\binom{N}{k}=\frac{N!}{k!(N-k)!}$$

$$P(A\ \ occurs\ \ k\ \ times)=\binom{N}{k}p^k(1-p)^{N-k}$$

CH 2

$$F_X(x)=P\{X\le x\}$$

$$F_X(x)=\sum_{i=0}^{N}P(x_i)u(x-x_i)$$

$$f_X(x)=\frac{dF_X(x)}{dx}$$

$$F_X(x)=\int_{-\infty}^{x}f_X(\xi)d\xi$$

$$F_X(x)=F\left(\frac{x-a_X}{\sigma_X}\right)$$

$$F_X(x|B)=P\{X\le x|B\}$$

$$=\frac{P\{X\le x\cap B\}}{P(B)}$$

$$f_X(x|B)=\frac{dF_X(x|B)}{dx}$$

CH 3

$$E[X]=\overline{X}=\sum_{i=1}^{N}x_iP(x_i)$$

$$E[X]=\int_{-\infty}^{\infty}xf_X(x)dx$$

$$E[g(x)]=\int_{-\infty}^{\infty}g(x)f_X(x)d\ \ x$$

$$m_n=E[X^n]$$

$$\mu_n=E[(X-\overline{X})^n]$$

$$\Phi_X(\omega)=\int_{-\infty}^{\infty}f_X(x)e^{j\omega x}dx$$

$$m_n=(-j)^n\left.\frac{d^n\Phi_X(\omega)}{d\omega^n}\right|_{\omega=0}$$

$$M_X(v)=\int_{-\infty}^{\infty}f_X(x)e^{vx}dx$$

$$m_n=\left.\frac{d^nM_X(v)}{dv^n}\right|_{v=0}$$

$$f_Y(y)=\sum_n\frac{f_X(x_n)}{\left|\left.\frac{dy}{dx}\right|_{x=x_n}\right|}$$

CH 4

$$F_{X,Y}(x,y)=P\{X\le x,\ Y\le y\}$$

$$P\{x1<X\le x2,\ y1<Y\le y2\}=$$
$$F_{X,Y}(x2,y2)+F_{X,Y}(x1,y2)$$
$$-F_{X,Y}(x1,y2)-F_{X,Y}(x2,y1)$$

$$F_{X,Y}(x,\infty)=F_X(x);$$
$$F_{X,Y}(\infty,y)=F_Y(y);$$

$$f_{X,Y}(x,y)=\frac{\partial^2F_{X,Y}(x,y)}{\partial x\partial y}$$

$$P\{x1<X\le x2,\ y1<Y\le y2\}$$
$$=\int_{y1}^{y2}\int_{x1}^{x2}f_{X,Y}(x,y)dxdy$$

$$f_X(x)=\int_{-\infty}^{\infty}f_{X,Y}(x,y)dy;$$
$$f_Y(y)=\int_{-\infty}^{\infty}f_{X,Y}(x,y)dx$$

$$f_X(x)=\frac{dF_X(x)}{dx}$$
$$f_Y(y)=\frac{dF_Y(y)}{dy}$$

$$f_X(x|y)=\frac{f_{X,Y}(x,y)}{f_Y(y)};$$

CH 5

$$\overline{g} = E[g(x)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y)dxdy$$

$$m_{nk} = E[X^n Y^k] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^n y^k f_{X,Y}(x,y)dxdy$$

$$R_{XY} = m_{11} = E[XY]$$

$$\mu_{nk} = E[(X-\overline{X})^n (Y-\overline{Y})^k]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (X-\overline{X})^n (Y-\overline{Y})^k f_{X,Y}(x,y)dxdy$$

$$C_{XY} = \mu_{11} = E[(X-\overline{X})(Y-\overline{Y})]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (X-\overline{X})(Y-\overline{Y}) f_{X,Y}(x,y)dxdy$$

$$C_{XY} = R_{XY} - \overline{X}\,\overline{Y} = R_{XY} - E[X]E[Y]$$

$$\rho = \frac{C_{XY}}{\sigma_X \sigma_Y}; \ -1 \le \rho \le 1$$

$$f_{u,v}(u,v) = f_{X,Y}(x,y)\left|J\right|;$$

$$where \quad J = \det\begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

CH 6,7 & 8

$$\mu_X(t) = E[X(t)] = \int_{-\infty}^{\infty} x f_{X(t)}(x)dx$$

$$R_X(t_1,t_2) = E[X(t_1)X(t_2)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_{X(t_1),X(t_2)}(x_1 x_2)dx_1 dx_2$$

$$A[X(t)] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X(t)d\ t$$

$$\mu_Y(t) = E[Y(t)] = \mu_X H(0\ )$$

$$h(t) = \int_{-\infty}^{\infty} H(f)\exp(j2\pi\ ft)df$$

$$H(f) = \int_{-\infty}^{\infty} h(t)\exp(-j2\pi\ ft)dt$$

$$S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)\exp(-j2\pi\tau)d\tau$$

⇕ (Fourier Transform pairs)

$$R_X(\tau) = \int_{-\infty}^{\infty} S_X(f)\exp(j2\pi\tau)df$$

$$S_Y(f) = \left|H(f)^2\right| S_X(f)$$

Gaussian

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-(x-a_X)^2/2\sigma_x^2}$$

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \exp\left\{\frac{-1}{2(1-\rho^2)}\left[\frac{(x-\overline{X})^2}{\sigma_X^2} - \frac{2\rho(x-\overline{X})(y-\overline{Y})}{\sigma_X\sigma_Y} + \frac{(y-\overline{Y})}{\sigma_Y^2}\right]\right\}$$

$$f(\boldsymbol{x}) = \frac{1}{\sqrt{(2\pi)^n \det(\Sigma)}} \exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{m})^\top \Sigma^{-1}(\boldsymbol{x}-\boldsymbol{m})\right]$$

$$\boldsymbol{m} = \mathsf{E}[\boldsymbol{X}] = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix} \qquad \Sigma = \mathsf{E}[(\boldsymbol{X}-\boldsymbol{m})(\boldsymbol{X}-\boldsymbol{m})^\top] = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \dots & \Sigma_{1n} \\ \Sigma_{21} & \Sigma_{22} & \dots & \Sigma_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \Sigma_{n1} & \Sigma_{n2} & \dots & \Sigma_{nn} \end{bmatrix}$$