EE 202-02-Winter 2013(132)

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In the circuit shown above, the switch having been in position $a$ for a long. At $t=0$ time The switch is moved to position $b$.
Find $v(\mathrm{t})$ for $t>0$

The initial capacitor voltage is obtained when the switch is in position a.

$$
\mathrm{v}(0)=[2 /(2+1)] 12=8 \mathrm{~V}
$$

The initial inductor current is $\mathrm{i}(0)=0$.
When the switch is in position b , we have the RLC circuit with the voltage source.

$$
\begin{gathered}
\alpha=\mathrm{R} /(2 \mathrm{~L})=10 /(2 \mathrm{x} 2.5)=2 \\
\omega_{0}=1 / \sqrt{\mathrm{LC}}=1 / \sqrt{(5 / 2) \times(1 / 40)}=4
\end{gathered}
$$

Since $\alpha<\omega_{0}$, we have an underdamped case.

$$
\begin{gathered}
\mathrm{s}_{1,2}=-\alpha \pm \sqrt{\alpha^{2}-\omega_{\mathrm{o}}}=-2 \pm \sqrt{(2)^{2}-16}=-2 \pm \mathrm{j} 3.464 \\
\text { Thus, } \mathrm{v}(\mathrm{t})=\mathrm{v}_{\mathrm{f}}+\left[\left(\mathrm{A}_{1} \cos 3.464 \mathrm{t}+\mathrm{A}_{2} \sin 3.464 \mathrm{t}\right) \mathrm{e}^{-2 \mathrm{t}}\right]
\end{gathered}
$$

where $\mathrm{v}_{\mathrm{f}}=\mathrm{v}(\infty)=10$, the final capacitor voltage. We now impose the initial conditions to get $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$.

$$
\mathrm{v}(0)=8=10+\mathrm{A}_{1} \text { leads to } \mathrm{A}_{1}=-2
$$

The initial capacitor current is the same as the initial inductor current.

$$
\mathrm{i}(0)=\mathrm{C}(\mathrm{dv}(0) / \mathrm{dt})=0 \text { therefore, } \mathrm{dv}(0) / \mathrm{dt}=0
$$

But, dv/dt $=3.464\left[\left(-\mathrm{A}_{1} \sin 3.464 \mathrm{t}+\mathrm{A}_{2} \cos 3.464 \mathrm{t}\right) \mathrm{e}^{-2 \mathrm{t}}\right]$

$$
-2\left[\left(\mathrm{~A}_{1} \cos 3.464 \mathrm{t}+\mathrm{A}_{2} \sin 3.464 \mathrm{t}\right) \mathrm{e}^{-2 \mathrm{t}}\right]
$$

$$
\mathrm{dv}(0) / \mathrm{dt}=0-2 \mathrm{~A}_{1}+3.464 \mathrm{~A}_{2}, \text { which leads to } \mathrm{A}_{2}=-4 / 3.464=-1.1547
$$

$\Rightarrow v(\mathrm{t})=\left\{10+\left[(-2 \cos 3.464 t-1.1547 \sin 3.464 t) \mathrm{e}^{-2 t}\right]\right\} \mathrm{V}$

