EE 202 (Semester 132)

Homework # 1 Solution

Problems from the text book (***Electric Circuits,*** James Nilsson and Susan Riedel, 9th edition, Prentice Hall, 2011)

From Chapter 1: 1.11, 1.13, 1.14, 1.26, and

From Chapter 2: 2.19, 2.21, 2.24, 2.26, 2.27, 2.30

P 1.11 [a] In Car A, the current i is in the direction of the voltage drop across the 12 V battery(the current i flows into the + terminal of the battery of Car A). Therefore using the passive sign convention,

$p = vi = (30)(12) = 360$ W.

Since the power is positive, the battery in Car A is absorbing power, so Car A must have the "dead" battery.

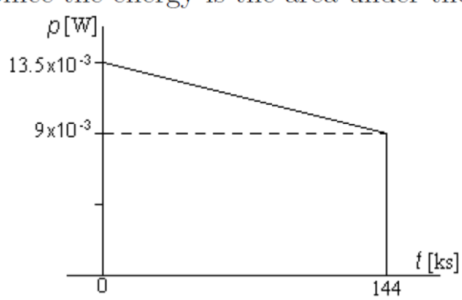
[b] $w(t) = \int_0^t p\,dx; \qquad 1 \text{ min} = 60 \text{ s}$

$$w(60) = \int_0^{60} 360\,dx$$

$$w = 360(60 - 0) = 360(60) = 21{,}600 \text{ J} = 21.6 \text{ kJ}$$

P 1.13 $p = vi; \qquad w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



Note that in constructing the plot above, we used the fact that 40 hr = 144,000 s = 144 ks

$$p(0) = (1.5)(9 \times 10^{-3}) = 13.5 \times 10^{-3} \text{ W}$$

$$p(144 \text{ ks}) = (1)(9 \times 10^{-3}) = 9 \times 10^{-3} \text{ W}$$

$$w = (9 \times 10^{-3})(144 \times 10^3) + \frac{1}{2}(13.5 \times 10^{-3} - 9 \times 10^{-3})(144 \times 10^3) = 1620 \text{ J}$$

P 1.14 Assume we are standing at box A looking toward box B. Then, using the passive sign convention $p = -vi$, since the current i is flowing into the $-$ terminal of the voltage v. Now we just substitute the values for v and i into the equation for power. Remember that if the power is positive, B is absorbing power, so the power must be flowing from A to B. If the power is negative, B is generating power so the power must be flowing from B to A.

[a] $p = -(125)(10) = -1250$ W   1250 W from B to A

[b] $p = -(-240)(5) = 1200$ W   1200 W from A to B

[c] $p = -(480)(-12) = 5760$ W   5760 W from A to B

[d] $p = -(-660)(-25) = -16{,}500$ W   16,500 W from B to A

P 1.26 We use the passive sign convention to determine whether the power equation is $p = vi$ or $p = -vi$ and substitute into the power equation the values for v and i, as shown below:

$$p_a = v_a i_a = (150 \times 10^3)(0.6 \times 10^{-3}) = 90 \text{ W}$$

$$p_b = v_b i_b = (150 \times 10^3)(-1.4 \times 10^{-3}) = -210 \text{ W}$$

$$p_c = -v_c i_c = -(100 \times 10^3)(-0.8 \times 10^{-3}) = 80 \text{ W}$$

$$p_d = v_d i_d = (250 \times 10^3)(-0.8 \times 10^{-3}) = -200 \text{ W}$$

$$p_e = -v_e i_e = -(300 \times 10^3)(-2 \times 10^{-3}) = 600 \text{ W}$$

$$p_f = v_f i_f = (-300 \times 10^3)(1.2 \times 10^{-3}) = -360 \text{ W}$$

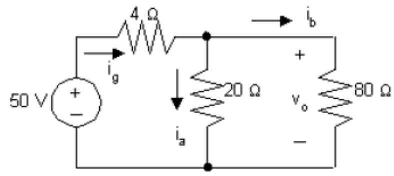
Remember that if the power is positive, the circuit element is absorbing power, whereas is the power is negative, the circuit element is developing power. We can add the positive powers together and the negative powers together — if the power balances, these power sums should be equal:

$\sum P_{\text{dev}} = 210 + 200 + 360 = 770$ W;

$\sum P_{\text{abs}} = 90 + 80 + 600 = 770$ W

Thus, the power balances and the total power developed in the circuit is 770 W.

P 2.19 [a]



$$20i_{\rm a} = 80i_{\rm b} \qquad i_g = i_{\rm a} + i_{\rm b} = 5i_{\rm b}$$

$$i_{\rm a} = 4i_{\rm b}$$

$$50 = 4i_g + 80i_{\rm b} = 20i_{\rm b} + 80i_{\rm b} = 100i_{\rm b}$$

$$i_{\rm b} = 0.5 \text{ A, therefore, } i_{\rm a} = 2 \text{ A} \quad \text{and} \quad i_g = 2.5 \text{ A}$$

[b] $i_{\rm b} = 0.5$ A

[c] $v_o = 80i_{\rm b} = 40$ V

[d] $$p_{4\Omega} = i_g^2(4) = 6.25(4) = 25 \text{ W}$$

$$p_{20\Omega} = i_{\rm a}^2(20) = (4)(20) = 80 \text{ W}$$

$$p_{80\Omega} = i_{\rm b}^2(80) = 0.25(80) = 20 \text{ W}$$

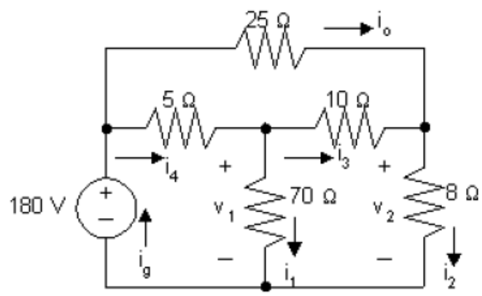
[e] $p_{50\rm V}$ (delivered) $= 50i_g = 125$ W

Check:

$$\sum P_{\rm dis} = 25 + 80 + 20 = 125 \text{ W}$$

$$\sum P_{\rm del} = 125 \text{ W}$$

P 2.21 [a]



$$v_2 = 180 - 100 = 80\text{V}$$

$$i_2 = \frac{v_2}{8} = 10\text{A}$$

$$i_3 + 4 = i_2, \qquad i_3 = 10 - 4 = 6\text{A}$$

$$v_1 = 10i_3 + 8i_2 = 10(6) + 8(10) = 140\text{V}$$

$$i_1 = \frac{v_1}{70} = \frac{140}{70} = 2\text{A}$$

Note also that

$$i_4 = i_1 + i_3 = 2 + 6 = 8\,\text{A}$$

$$i_g = i_4 + i_o = 8 + 4 = 12\,\text{A}$$

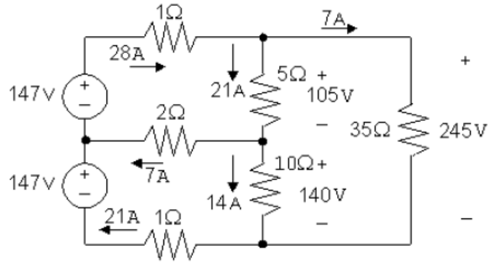
[b]

$$\begin{aligned} p_{5\Omega} &= 8^2(5) = 320\text{ W} \\ p_{25\Omega} &= (4)^2(25) = 400\text{ W} \\ p_{70\Omega} &= 2^2(70) = 280\text{ W} \\ p_{10\Omega} &= 6^2(10) = 360\text{ W} \\ p_{8\Omega} &= 10^2(8) = 800\text{ W} \end{aligned}$$

[c] $\sum P_{\text{dis}} = 320 + 400 + 280 + 360 + 800 = 2160\text{W}$

$$P_{\text{dev}} = 180i_g = 180(12) = 2160\text{W}$$

P 2.24 [a] Start by calculating the voltage drops due to the currents i_1 and i_2. Then use KVL to calculate the voltage drop across and 35 Ω resistor, and Ohm's law to find the current in the 35 Ω resistor. Finally, KCL at each of the middle three nodes yields the currents in the two sources and the current in the middle 2 Ω resistor. These calculations are summarized in the figure below:



$$p_{147(\text{top})} = -(147)(28) = -4116 \text{ W}$$

$$p_{147(\text{bottom})} = -(147)(21) = -3087 \text{ W}$$

[b]

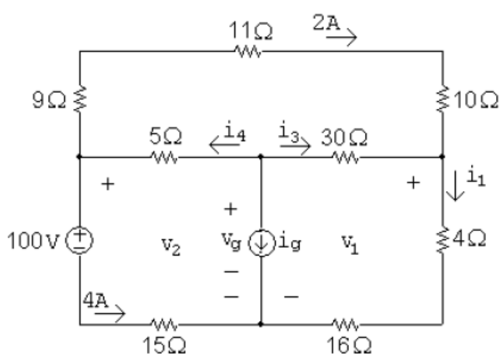
$$\sum P_{\text{dis}} = (28)^2(1) + (7)^2(2) + (21)^2(1) + (21)^2(5) + (14)^2(10) + (7)^2(35)$$

$$= 784 + 98 + 441 + 2205 + 1960 + 1715 = 7203 \text{ W}$$

$$\sum P_{\text{sup}} = 4116 + 3087 = 7203 \text{ W}$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{sup}} = 7203$ W

P 2.26 [a]



$$v_2 = 100 + 4(15) = 160\,\text{V}; \qquad v_1 = 160 - (9 + 11 + 10)(2) = 100\,\text{V}$$

$$i_1 = \frac{v_1}{4+16} = \frac{100}{20} = 5\,\text{A}; \qquad i_3 = i_1 - 2 = 5 - 2 = 3\,\text{A}$$

$$v_g = v_1 + 30i_3 = 100 + 30(3) = 190\,\text{V}$$

$$i_4 = 2 + 4 = 6\,\text{A}$$

$$i_g = -i_4 - i_3 = -6 - 3 = -9\,\text{A}$$

[b] Calculate power using the formula $p = Ri^2$:

$p_{9\,\Omega} = (9)(2)^2 = 36\,\text{W}$; $p_{11\,\Omega} = (11)(2)^2 = 44\,\text{W}$

$p_{10\,\Omega} = (10)(2)^2 = 40\,\text{W}$; $p_{5\,\Omega} = (5)(6)^2 = 180\,\text{W}$

$p_{30\,\Omega} = (30)(3)^2 = 270\,\text{W}$; $p_{4\,\Omega} = (4)(5)^2 = 100\,\text{W}$

$p_{16\,\Omega} = (16)(5)^2 = 400\,\text{W}$; $p_{15\,\Omega} = (15)(4)^2 = 240\,\text{W}$

[c] $v_g = 190\,\text{V}$

[d] Sum the power dissipated by the resistors:

$$\sum p_{\text{diss}} = 36 + 44 + 40 + 180 + 270 + 100 + 400 + 240 = 1310\,\text{W}$$

The power associated with the sources is

$$p_{\text{volt-source}} = (100)(4) = 400\,\text{W}$$

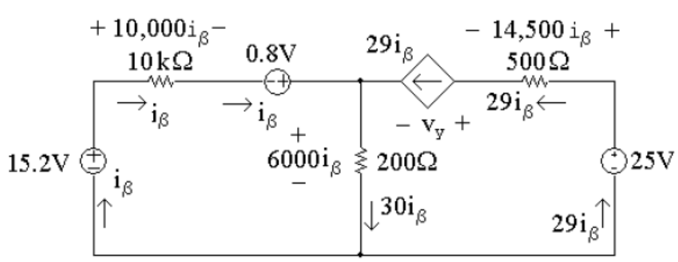
$$p_{\text{curr-source}} = v_g i_g = (190)(-9) = -1710\,\text{W}$$

Thus the total power dissipated is $1310 + 400 = 1710$ W and the total power developed is 1710 W, so the power balances.

P 2.27 First note that we know the current through all elements in the circuit except the $200\,\Omega$ resistor (the current in the three elements to the left of the $200\,\Omega$ resistor is i_β; the current in the three elements to the right of the $200\,\Omega$ resistor is $29i_\beta$). To find the current in the $200\,\Omega$ resistor, write a KCL equation at the top node:

$$i_\beta + 29i_\beta = i_{200\,\Omega} = 30i_\beta$$

We can then use Ohm's law to find the voltages across each resistor in terms of i_β. The results are shown in the figure below:



[a] To find i_β, write a KVL equation around the left-hand loop, summing voltages in a clockwise direction starting below the 15.2V source:

$-15.2\,\text{V} + 10{,}000 i_1 - 0.8\,\text{V} + 6000 i_\beta = 0$

Solving for i_β

$10{,}000 i_\beta + 6000 i_\beta = 16\,\text{V} \qquad \text{so} \qquad 16{,}000 i_\beta = 16\,\text{V}$

Thus,

$$i_\beta = \frac{16}{16{,}000} = 1\,\text{mA}$$

Now that we have the value of i_β, we can calculate the voltage for each component except the dependent source. Then we can write a KVL equation for the right-hand loop to find the voltage v_y of the dependent source. Sum the voltages in the clockwise direction, starting to the left of the dependent source:

$-v_y - 14{,}500 i_\beta + 25\,\text{V} - 6000 i_\beta = 0$

Thus,

$$v_y = 25\,\text{V} - 20{,}500 i_\beta = 25\,\text{V} - 20{,}500(10^{-3}) = 25\,\text{V} - 20.5\,\text{V} = 4.5\,\text{V}$$

[b] We now know the values of voltage and current for every circuit element. Let's construct a power table:

Element	Current (mA)	Voltage (V)	Power Equation	Power (mW)
15.2 V	1	15.2	$p = -vi$	−15.2
10 kΩ	1	10	$p = Ri^2$	10
0.8 V	1	0.8	$p = -vi$	−0.8
200 Ω	30	6	$p = Ri^2$	180
Dep. source	29	4.5	$p = vi$	130.5
500 Ω	29	14.5	$p = Ri^2$	420.5
25 V	29	25	$p = -vi$	−725

The total power generated in the circuit is the sum of the negative power values in the power table:

$-15.2\,\text{mW} + -0.8\,\text{mW} + -725\,\text{mW} = -741\,\text{mW}$

Thus, the total power generated in the circuit is 741 mW. The total power absorbed in the circuit is the sum of the positive power values in the power table:

$10\,\text{mW} + 180\,\text{mW} + 130.5\,\text{mW} + 420.5\,\text{mW} = 741\,\text{mW}$

Thus, the total power absorbed in the circuit is 741 mW and the power in the circuit balances.

P 2.30 [a] $100 - 20i_\sigma + 18i_\Delta = 0$

$-18i_\Delta + 5i_\sigma + 40i_\sigma = 0 \quad \text{so} \quad 18i_\Delta = 45i_\sigma$

Therefore, $\quad -100 - 20i_\sigma + 45i_\sigma = 0, \quad \text{so} \quad i_\sigma = 4\,\text{A}$

$18i_\Delta = 45i_\sigma = 180; \text{ so } i_\Delta = 10\,\text{A}$

$v_o = 40i_\sigma = 160\,\text{V}$

[b] i_g = current out of the positive terminal of the 100 V source
v_{d} = voltage drop across the $8i_\Delta$ source

$i_g = i_\Delta + i_\sigma + 8i_\Delta = 9i_\Delta + i_\sigma = 94\,\text{A}$

$v_d = 160 - 20 = 140\,\text{V}$

$$\begin{aligned}
\sum P_{\text{gen}} &= 100i_g + 20i_\sigma i_g = 100(94) + 20(4)(94) = 16,920\ \text{W} \\
\sum P_{\text{diss}} &= 18i_\Delta^2 + 5i_\sigma(i_g - i_\Delta) + 40i_\sigma^2 + 8i_\Delta v_d + 8i_\Delta(20) \\
&= (18)(100) + 20(94 - 10) + 16(40) + 80(140) + 80(20) \\
&= 16{,}920\ \text{W; Therefore,} \\
\sum P_{\text{gen}} &= \sum P_{\text{diss}} = 16,920\ \text{W}
\end{aligned}$$