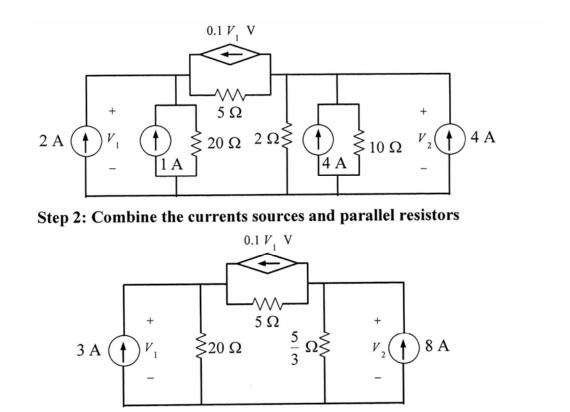
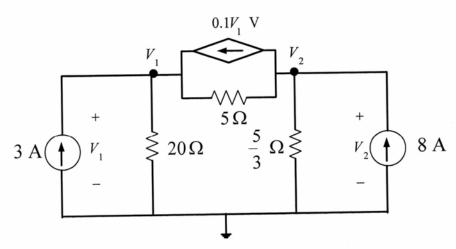
Step 1: Source Transform the voltage sources



Note : Do not transform the currents sources because that will destroy the required voltages namely $V_1 \, V_2$

Step 3 : Solve the remaining circuit using nodal analysis



KCl on the two nodes $\ensuremath{V_1},\ensuremath{\ensuremath{V_2}}$, we have

$$\frac{V_1}{20} + \frac{V_1 - V_2}{5} - 0.1V_1 - 3 = 0$$

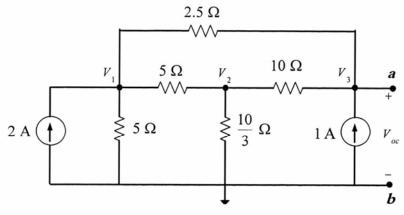
$$\Rightarrow 0.15V_1 - 0.2V_2 = 3 -----(1)$$

$$\frac{V_2}{5/3} + \frac{V_2 - V_1}{5} + 0.1V_1 - 8 = 0$$

$$\Rightarrow -0.1V_1 + 0.8V_2 = 8 -----(2)$$

$$\Rightarrow \begin{bmatrix} 0.15 & -0.2 \\ -0.1 & 0.8 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$$
Solving $\Rightarrow V_1 = 40 \text{ V} \quad V_2 = 15 \text{ V}$

(a) Using Nodal Analysis were $V_{oc} = V_3$



KCl on the three nodes V_1 , V_2 , V_3

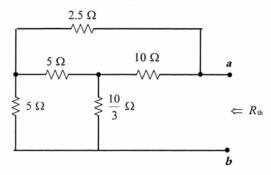
$$\frac{V_1}{5} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{2.5} - 2 = 0 \implies 0.8V_1 - 0.2V_2 - 0.4V_3 = 2 ----(1)$$

$$\frac{V_2 - V_1}{5} + \frac{V_2 - V_3}{10} + \frac{V_2}{10/3} = 0 \implies -0.2V_1 + 0.6V_2 - 0.1V_3 = 0 ----(2)$$

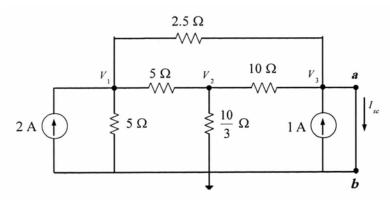
$$\frac{V_3 - V_2}{10} + \frac{V_3 - V_1}{2.5} - 1 = 0 \qquad \Rightarrow \qquad -0.4V_1 - 0.1V_2 + 0.5V_3 = 1 \quad ----(3)$$

$$\begin{bmatrix} 0.8 & -0.2 & -0.4 \\ -0.2 & 0.6 & -0.1 \\ -0.4 & -0.1 & 0.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$
 Solving $\Rightarrow V_{oc} = V_3 = 9.6 \text{ V}$

To find $\,R_{\,_{
m th}}$, we deactivate all independent sources



then find R_{th} , $R_{th} = V_{oc}/I_{sc}$.



Here $V_3 = 0$, KCl on the two nodes V_1 , V_2

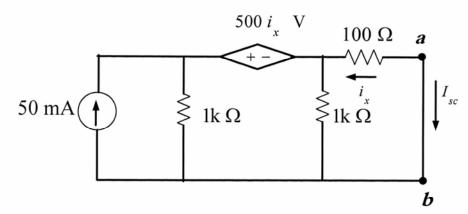
$$\frac{V_1}{5} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{2.5} - 2 = 0 \implies 0.8V_1 - 0.2V_2 = 2 ----(1)$$

$$\frac{V_2 - V_1}{5} + \frac{V_2 - V_3}{10} + \frac{V_2}{10/3} = 0 \implies -0.2V_1 + 0.6V_2 = 0 \quad ----(2)$$

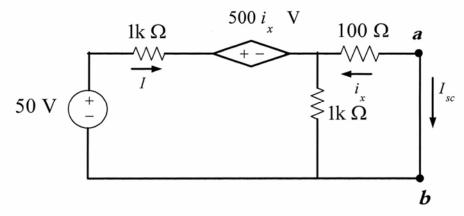
$$\begin{bmatrix} 0.8 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$
 Solving $\Rightarrow V_1 = 2.73$ V $V_2 = 0.91$ V

$$\frac{V_1 - V_3}{2.5} + \frac{V_2 - V_3}{10} + 1 = I_{sc} \implies \frac{2.73}{2.5} + \frac{0.91}{10} + 1 = I_{sc} \implies I_{sc} = 2.183 \implies R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{9.6}{2.183} = 4.4 \Omega$$

(b)
$$R_L = R_{TH} = 4.4 \Omega$$
 (c) $P_{\text{max}} = \frac{V_{oc}^2}{4R_{TH}} = \frac{9.6^2}{4(4.4)} = 5.24 \text{ W}$



Source Transformation



KVL on the outer loop

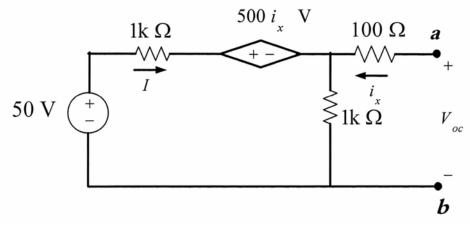
$$-50+1000I+500i_{x}-100i_{x}=0 \implies 1000I+400i_{x}=50 ---(1)$$

KVL on the inner loop

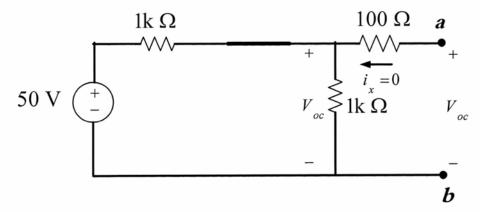
$$-1000(I + i_x) - 100i_x = 0 \implies 1000I + 1100i_x = 0 \qquad ---(2)$$

Solving (1) and (2) for
$$i_x = -\frac{1}{14} A \implies I_{sc} = -i_x = \frac{1}{14} A$$

Finding $R_{\rm th}$: Since the circuit has a dependent source we cannot find $R_{\rm th}$ by combining resistors . We find $R_{\rm th}$ as $R_{\rm th} = \frac{V_{oc}}{I_{sc}}$. Therefore we seek V_{oc} as follows:

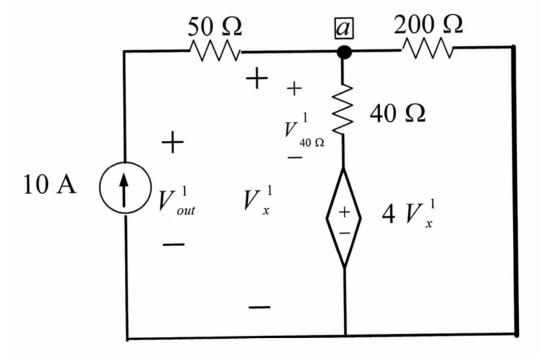


Here $i_x = 0$ which implies also that the dependent source $500i_x = 0$



$$V_{oc} = \frac{1k}{1k + 1k} (50) = 25 \text{ V}$$

 $\Rightarrow R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{25}{1/14} = 350 \Omega$



Apply KCL at node a

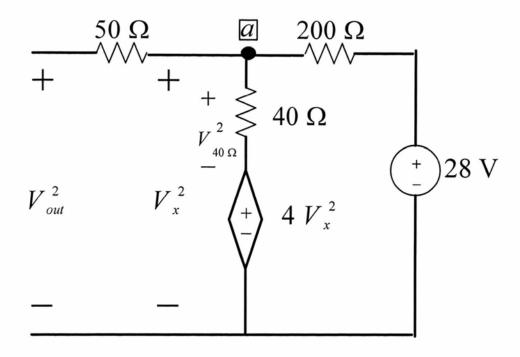
$$\frac{V_{x}^{1}}{200} + \frac{V_{40 \Omega}^{1}}{40} - 10 = 0 \quad ----(1)$$

Since
$$V_{40 \Omega}^{1} = V_{x}^{1} - 4V_{x}^{1} = -3V_{x}^{1}$$

(1)
$$\Rightarrow \frac{V_x^1}{200} + \frac{-3V_x^1}{40} - 10 = 0 \Rightarrow V_x^1 = -\frac{1000}{7} = -142.857 \text{ V}$$

KVL
$$\Rightarrow -V_{out}^{1} +50(10) + V_{x}^{1} = 0 \Rightarrow V_{out}^{1} = \frac{2500}{7} = 357.143 \text{ V}$$

Step 2: Deactivate the independent current source



Apply KCL at node a

$$\frac{V_{x}^{2}-28}{200}+\frac{V_{40 \Omega}^{2}}{40}=0 \quad ---(2)$$

Since
$$V_{40 \Omega}^2 = V_x^2 - 4V_x^2 = -3V_x^2$$

(2)
$$\Rightarrow \frac{V_x^2 - 28}{200} + \frac{-3V_x^2}{40} = 0 \Rightarrow V_x^2 = -2 \text{ V}$$

 $V_{out}^2 = V_x^2 = -2 \text{ V}$

Step 3: Combine the results in step 1 and step 2

$$V_{out} = V_{out}^{1} + V_{out}^{2} = 357.143 + (-2) = 355.143 \text{ V}$$