Q1
Step 1: Source Transform the voltage sources


Step 2: Combine the currents sources and parallel resistors


Note : Do not transform the currents sources because that will destroy the required voltages namely $\mathrm{V}_{1} \mathrm{~V}_{2}$

Step 3 :
Solve the remaining circuit using nodal analysis


KCl on the two nodes $V_{1}, V_{2}$, we have
$\frac{V_{1}}{20}+\frac{V_{1}-V_{2}}{5}-0.1 V_{1}-3=0$
$\Rightarrow 0.15 V_{1}-0.2 V_{2}=3$
$\frac{V_{2}}{5 / 3}+\frac{V_{2}-V_{1}}{5}+0.1 V_{1}-8=0$
$\Rightarrow-0.1 V_{1}+0.8 V_{2}=8$
$\Rightarrow\left[\begin{array}{cc}0.15 & -0.2 \\ -0.1 & 0.8\end{array}\right]\left[\begin{array}{l}V_{1} \\ V_{2}\end{array}\right]=\left[\begin{array}{l}3 \\ 8\end{array}\right]$
Solving $\Rightarrow V_{1}=40 \mathrm{~V} \quad V_{2}=15 \mathrm{~V}$

Q2
(a) Using Nodal Analysis were $V_{o c}=V_{3}$


KCl on the three nodes $V_{1}, V_{2}, V_{3}$
$\frac{V_{1}}{5}+\frac{V_{1}-V_{2}}{5}+\frac{V_{1}-V_{3}}{2.5}-2=0 \Rightarrow 0.8 V_{1}-0.2 V_{2}-0.4 V_{3}=2$
$\frac{V_{2}-V_{1}}{5}+\frac{V_{2}-V_{3}}{10}+\frac{V_{2}}{10 / 3}=0 \Rightarrow-0.2 V_{1}+0.6 V_{2}-0.1 V_{3}=0$
$\frac{V_{3}-V_{2}}{10}+\frac{V_{3}-V_{1}}{2.5}-1=0 \quad \Rightarrow \quad-0.4 V_{1}-0.1 V_{2}+0.5 V_{3}=1$
$\left[\begin{array}{ccc}0.8 & -0.2 & -0.4 \\ -0.2 & 0.6 & -0.1 \\ -0.4 & -0.1 & 0.5\end{array}\right]\left[\begin{array}{l}V_{1} \\ V_{2} \\ V_{3}\end{array}\right]=\left[\begin{array}{l}2 \\ 0 \\ 1\end{array}\right] \quad$ Solving $\Rightarrow V_{o c}=V_{3}=9.6 \mathrm{~V}$

To find $R_{\text {th }}$, we deactivate all independent sources

then find $\mathrm{R}_{\mathrm{th}}, \mathrm{R}_{\mathrm{th}}=\mathrm{V}_{\mathrm{oc}} / \mathrm{I}_{\mathrm{sc}}$.


Here $V_{3}=0, \mathrm{KCl}$ on the two nodes $V_{1}, V_{2}$
$\frac{V_{1}}{5}+\frac{V_{1}-V_{2}}{5}+\frac{V_{1}-V_{3}}{2.5}-2=0 \Rightarrow 0.8 V_{1}-0.2 V_{2}=2 \ldots-$ (1)
$\frac{V_{2}-V_{1}}{5}+\frac{V_{2}-V_{3}}{10}+\frac{V_{2}}{10 / 3}=0 \Rightarrow-0.2 V_{1}+0.6 V_{2}=0-\ldots-$ (2)
$\left[\begin{array}{cc}0.8 & -0.2 \\ -0.2 & 0.6\end{array}\right]\left[\begin{array}{l}V_{1} \\ V_{2}\end{array}\right]=\left[\begin{array}{l}2 \\ 0\end{array}\right]$ Solving $\Rightarrow V_{1}=2.73 \mathrm{~V} \quad V_{2}=0.91 \mathrm{~V}$
$\frac{V_{1}-V_{3}}{2.5}+\frac{V_{2}-V_{3}}{10}+1=I_{s c} \Rightarrow \frac{2.73}{2.5}+\frac{0.91}{10}+1=I_{s c} \Rightarrow I_{s c}=2.183 \Rightarrow R_{\mathrm{th}}=\frac{V_{o c}}{I_{s c}}=\frac{9.6}{2.183}=4.4 \Omega$
(b) $R_{L}=R_{T H}=4.4 \Omega$
(c) $P_{\max }=\frac{V_{o c}{ }^{2}}{4 R_{T H}}=\frac{9.6^{2}}{4(4.4)}=5.24 \mathrm{~W}$


Source Transformation


KVL on the outer loop

$$
-50+1000 I+500 i_{\mathrm{x}}-100 i_{\mathrm{x}}=0 \Rightarrow 1000 I+400 i_{\mathrm{x}}=50---(1)
$$

KVL on the inner loop

$$
\begin{equation*}
-1000\left(I+i_{\mathrm{x}}\right)-100 i_{\mathrm{x}}=0 \Rightarrow 1000 I+1100 i_{\mathrm{x}}=0 \tag{2}
\end{equation*}
$$

Solving (1) and (2) for $i_{\mathrm{x}}=-\frac{1}{14} \mathrm{~A} \Rightarrow I_{s c}=-i_{\mathrm{x}}=\frac{1}{14} \mathrm{~A}$

Finding $R_{\mathrm{th}}$ : Since the circuit has a dependent source we cannot find $R_{\mathrm{th}}$ by combining resistors. We find $R_{\mathrm{th}}$ as $R_{\mathrm{th}}=\frac{V_{o c}}{I_{s c}}$. Therefore we seek $V_{o c}$ as follows :


Here $i_{x}=0$ which implies also that the dependent source $500 i_{x}=0$


$$
\begin{aligned}
& V_{o c}=\frac{1 k}{1 k+1 k}(50)=25 \mathrm{~V} \\
& \Rightarrow R_{t h}=\frac{V_{o c}}{I_{s c}}=\frac{25}{1 / 14}=350 \Omega
\end{aligned}
$$

Q4
Step 1 : Deactivate the independent voltage source


Apply KCL at node a
$\frac{V_{x}^{1}}{200}+\frac{V_{40 \Omega}^{1}}{40}-10=0$
Since $V_{40 \Omega}^{1}=V_{x}^{1}-4 V_{x}^{1}=-3 V_{x}^{1}$

$$
\begin{aligned}
& (1) \Rightarrow \frac{V_{x}^{1}}{200}+\frac{-3 V_{x}^{1}}{40}-10=0 \Rightarrow V_{x}^{1}=-\frac{1000}{7}=-142.857 \mathrm{~V} \\
& \mathrm{KVL} \Rightarrow-V_{\text {out }}^{1}+50(10)+V_{x}^{1}=0 \Rightarrow V_{\text {out }}^{1}=\frac{2500}{7}=357.143 \mathrm{~V}
\end{aligned}
$$

Step 2 : Deactivate the independent current source


Apply KCL at node a
$\frac{V_{x}^{2}-28}{200}+\frac{V_{40 \Omega}^{2}}{40}=0$

Since $V_{40 \Omega}^{2}=V_{x}^{2}-4 V_{x}^{2}=-3 V_{x}^{2}$
(2) $\Rightarrow \quad \frac{V_{x}^{2}-28}{200}+\frac{-3 V_{x}^{2}}{40}=0 \Rightarrow V_{x}^{2}=-2 \mathrm{~V}$
$V_{\text {out }}^{2}=V_{x}^{2}=-2 \mathrm{~V}$
Step 3: Combine the results in step 1 and step 2

$$
V_{o u t}=V_{o u t}^{1}+V_{o u t}^{2}=357.143+(-2)=355.143 \mathrm{~V}
$$

