EE 202-01 – Fall 2012(122)

Quiz 5

| SER | ID | NAME **KEY** |
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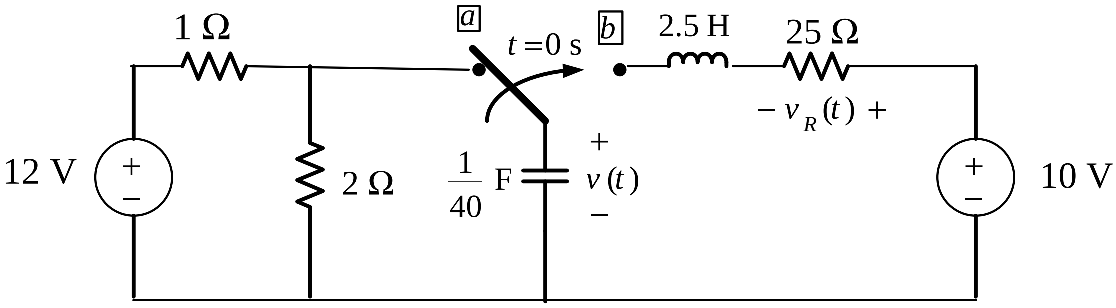
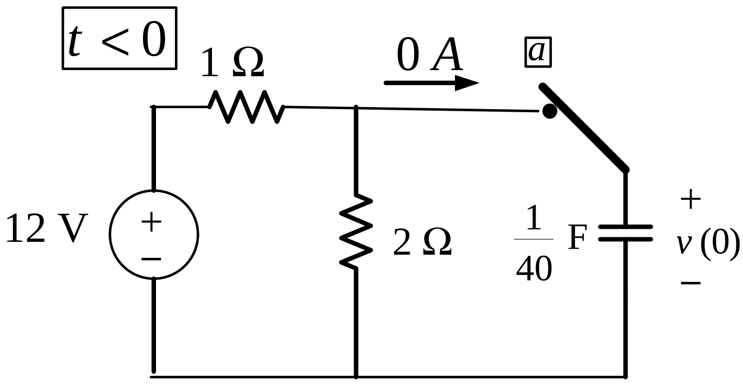



For the circuit shown above , the **switch** was in position **a** for a long time and at t =0 s the **switch** move to position **b** .

Find $v(t)$ and $v_R(t)$ for t > 0 ? ( **Hint** $\alpha = \frac{R}{2L}$ $\omega_0 = \frac{1}{\sqrt{LC}}$ )

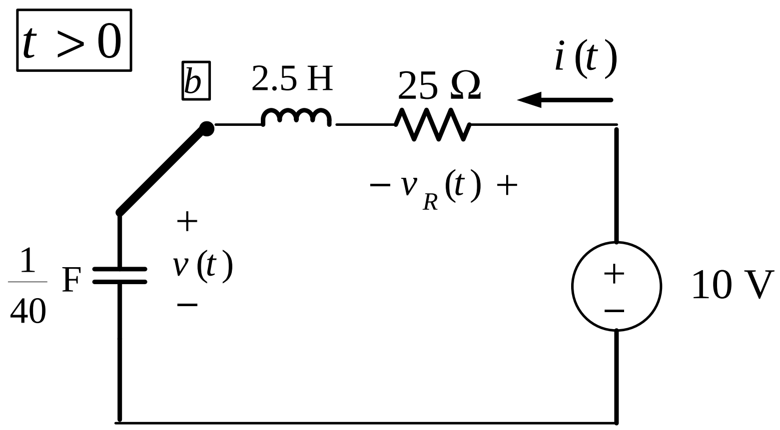
**Solution**




The switch in postion **a**

Capacitor initial voltage $v(0) = \frac{2}{1+2}(12) = 8$ V (voltage divsion)

Inductor initial current $i(0) = 0$ A




The switch in postion **b**, we have series $RLC$ circuit with voltage source

$$\alpha = \frac{R}{2L} = \frac{25}{2(2.5)} = 5 \text{ rad/s} \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.5)\left(\frac{1}{40}\right)}} = 4 \text{ rad/s}$$

$$\alpha > \omega_0 \Rightarrow \text{ overdamped } \Rightarrow v(\text{t}) = v_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

**were** $v_f = v(\infty) = 10 \text{ V} \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -5 \pm \sqrt{5^2 - 4^2} = -5 \pm 3$

$$\Rightarrow s_1 = -8 \text{ rad/s} \qquad s_1 = -2 \text{ rad/s}$$

$$\Rightarrow v(\text{t}) = 10 + A_1 e^{-8t} + A_2 e^{-2t}$$

We get $A_1, A_2$ by imposing the initial conditions as follows:

$$v(0) = 8 = 10 + A_1 + A_2 \Rightarrow A_1 + A_2 = -2 \text{ ------(1)}$$

$$\frac{dv(\text{t})}{\text{dt}} = -8A_1 e^{-8t} - 2A_2 e^{-2t}$$

$$\Rightarrow \frac{dv(0)}{\text{dt}} = -8A_1 - 2A_2 \qquad \text{finding } \frac{dv(0)}{\text{dt}} \text{ ?}$$

Since $i(0) = C\frac{dv(0)}{\text{dt}} \Rightarrow \frac{dv(0)}{\text{dt}} = \frac{i(0)}{C} = \frac{0}{1/40} = 0$

$$\Rightarrow \frac{dv(0)}{\text{dt}} = 0 = -8A_1 - 2A_2 \Rightarrow 4A_1 + A_2 = 0 \text{ ------(2)}$$

solving (1) and (2) , we get $A_1 = \frac{2}{3}$ $A_2 = -\frac{8}{3}$

$$\Rightarrow \; v(\text{t}) = 10 + \frac{2}{3}e^{-8t} - \frac{8}{3}e^{-2t} \quad \text{V} \quad t > 0$$

$$i(t) = C\,\frac{dv(\text{t})}{dt} = \left(\frac{1}{40}\right)\left\{\frac{2}{3}e^{-8t}(-8) - \frac{8}{3}e^{-2t}(-2)\right\} = \left(\frac{1}{40}\right)\left\{-\frac{16}{3}e^{-8t} + \frac{16}{3}e^{-2t}\right\}$$

$$= \frac{2}{15}e^{-2t} - \frac{2}{15}e^{-8t} \quad \text{A} \quad t > 0$$

$$\Rightarrow \; v_R(\text{t}) = 25i(t) = 25\left\{\frac{2}{15}e^{-2t} - \frac{2}{15}e^{-8t}\right\} = \frac{10}{3}e^{-2t} - \frac{10}{3}e^{-8t} \quad \text{V} \quad t > 0$$

**Another Solution**

We can start by finding i(t) directly as follows :

$$i(\text{t}) = i_f + A_1e^{-8t} + A_2e^{-2t} \quad \text{A} \quad t > 0$$

here $i_f = 0$ ( at ∞ the capacitor is open)

$$i(\text{t}) = A_1e^{-8t} + A_2e^{-2t} \quad \text{A} \quad t > 0$$

$$i(0) = 0 = A_1 + A_2 \text{------}(1)$$

$$\frac{\text{d}i(\text{t})}{\text{dt}} = -8A_1e^{-8t} - 2A_2e^{-2t}$$

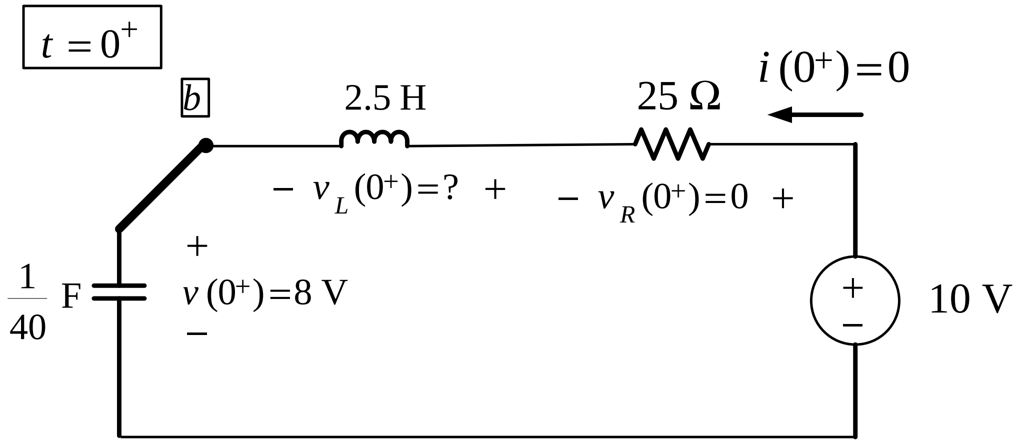
$$\frac{\text{d}i(0)}{\text{dt}} = -8A_1 - 2A_2 \quad \text{finding } \frac{\text{d}i(0)}{\text{dt}} \;?$$

Since $v_L(0) = L\dfrac{di(0)}{dt} \Rightarrow \dfrac{di(0)}{dt} = \dfrac{v_L(0)}{L}$

$$\frac{di(0)}{dt} = -8A_1 - 2A_2 = \frac{v_L(0)}{L} \qquad \text{finding } v_L(0)\ ?$$

To find $v_L(0)$ we must look at the circuit at the instant $0^+$

(just after the switch move to position **b**)




**KVL** $\quad -8 - v_L(0^+) - v_R(0^+) + 10 = 0$

$$\Rightarrow v_L(0^+) = -8 - v_R(0^+) + 10 = -8 - 0 + 10 = 2$$

$$\Rightarrow \frac{di(0)}{dt} = \frac{v_L(0)}{L} = \frac{2}{2.5} = 0.8$$

$$\Rightarrow \frac{di(0)}{dt} = -8A_1 - 2A_2 = 0.8$$

$$\Rightarrow -20A_1 - 5A_2 = 2 \quad \text{------(2)}$$

solving (1) and (2) we get $A_1 = -\dfrac{2}{15} \qquad A_2 = \dfrac{2}{15}$

$$\Rightarrow i(t) = \frac{2}{15}e^{-2t} - \frac{2}{15}e^{-8t} \quad \text{A} \quad t > 0$$

Now, we can find $v(t)$ using **KVL** as follows:

$$-v(t) - v_L(t) - v_R(t) + 10 = 0$$

$$\Rightarrow \ v(t) = 10 - v_L(t) - v_R(t)$$

$$= 10 - (2.5)\frac{di(t)}{dt} - (25)i(t)$$

$$= 10 + \frac{2}{3}e^{-8t} - \frac{8}{3}e^{-2t} \quad \text{V} \quad t > 0$$

$$v_R(t) = (25)i(t) = 25\left(\frac{2}{15}e^{-2t} - \frac{2}{15}e^{-8t}\right)$$

$$= \frac{10}{3}e^{-2t} - \frac{10}{3}e^{-8t} \quad \text{V} \quad t > 0$$

EE 202-02 – Fall 2012(122)

Quiz 5

| SER | ID | NAME **KEY** |
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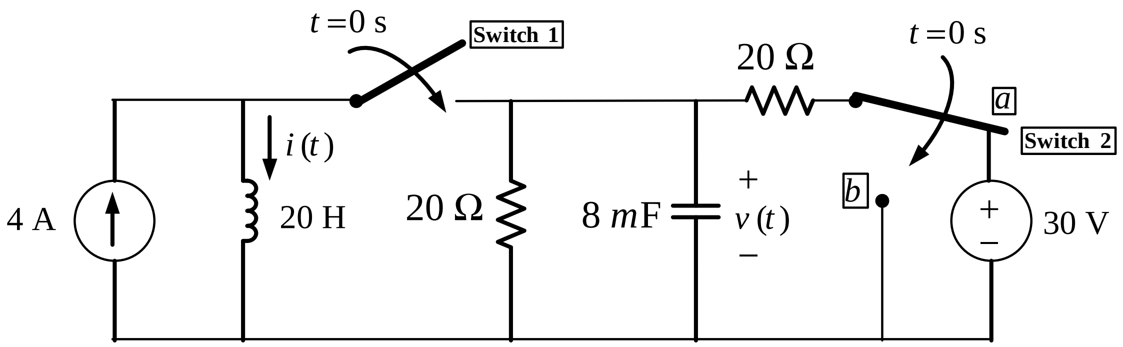
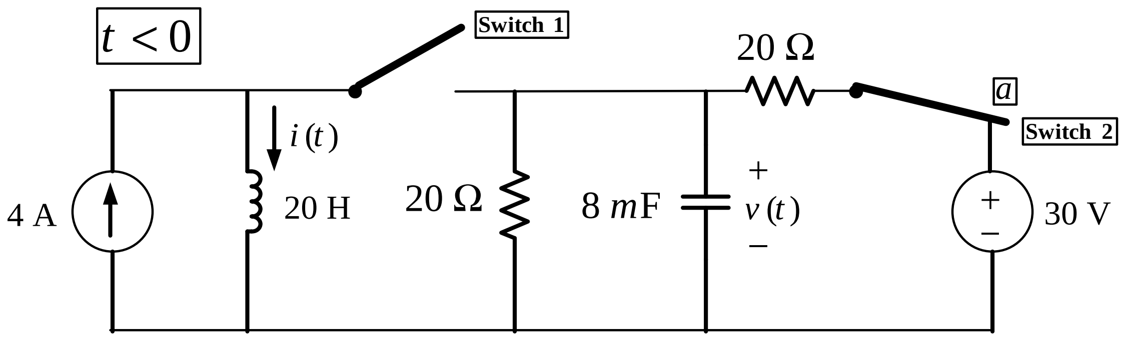



For the circuit shown above , **switch 1** closes at t = 0 and **switch 2** was in position **a** for a long time and at t =0 s **switch 2** move to position **b** . Find $i(t)$ and $v(t)$ for t > 0 ? ( **Hint** $\alpha = \frac{1}{2RC}$ $\omega_0 = \frac{1}{\sqrt{LC}}$ )

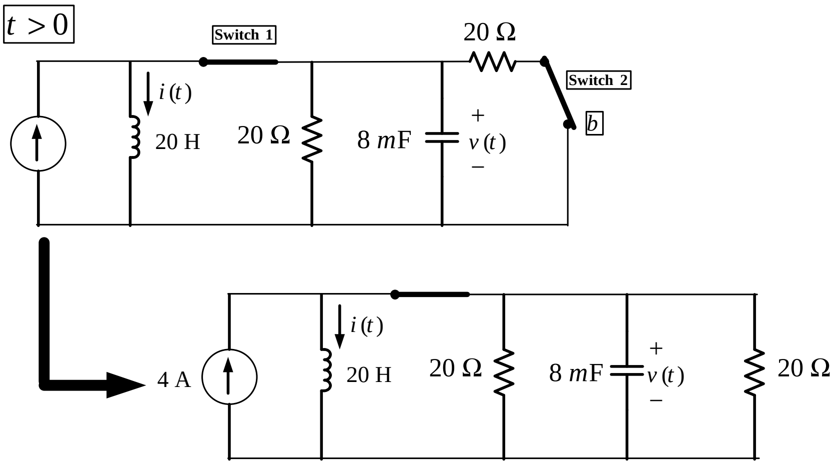
**Solution**




switch 1 open and switch 2 in postion **a**

Inductor initial current $i(0) = 4$ A

Capacitor initial voltage $v(0) = \frac{20}{20+20}(30) = 15$ V (voltage divsion)




switch 1 close and switch 2 in postion **b**, we have a parallel $RLC$ circuit with a current source

$$\alpha = \frac{1}{2RC} = \frac{1}{2\left(20\|20\right)\left(8\text{X}10^{-3}\right)} = 6.25 \text{ rad/s} \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\left(8\text{X}10^{-3}\right)}} = 2.5 \text{ rad/s}$$

$\alpha > \omega_0 \;\Rightarrow$ overdamped $\Rightarrow\; i\,(\text{t}) = i_f + A_1 e^{s_1 t} + A_2 e^{s_2 t}$

**were** $i_f = i\,(\infty) = 4$ A $\qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -6.25 \pm \sqrt{(6.25)^2 - (2.5)^2} = -6.25 \pm 5.7282$

$$\Rightarrow\; s_1 = -11.978 \text{ rad/s} \quad s_2 = -0.5218 \text{ rad/s}$$

$$\Rightarrow\; i\,(\text{t}) = 4 + A_1 e^{-11.978t} + A_2 e^{-0.5218t}$$

We get $A_1, A_2$ by imposing the initial conditions as follows:

$$i\,(0) = 4 = 4 + A_1 + A_2 \;\Rightarrow\; A_1 + A_2 = 0 \text{ ------(1)}$$

$$\frac{di\,(\text{t})}{\text{dt}} = -11.978 A_1 e^{-11.978t} - 0.5218 A_2 e^{-0.5218t}$$

$$\Rightarrow\; \frac{di\,(0)}{\text{dt}} = -11.978 A_1 - 0.5218 A_2 \qquad \text{finding } \frac{di\,(0)}{\text{dt}} \;?$$

$$\text{Since } v\,(0) = 15 = L\,\frac{di\,(0)}{\text{dt}} \;\Rightarrow\; \frac{di\,(0)}{\text{dt}} = \frac{15}{20} = 0.75$$

$$\Rightarrow\; \frac{di\,(0)}{\text{dt}} = 0.75 = -11.978 A_1 - 0.5218 A_2 \text{ ------(2)}$$

solving (1) and (2) , we get $A_1 = -0.0655 \quad A_2 = 0.0655$

$$\Rightarrow \quad i\,(\mathrm{t}) = 4 - 0.0655e^{-11.978t} + 0.0655e^{-0.5218t} \quad \mathrm{A} \quad t > 0$$

$$v\,(t) = v_L\,(t) = L\,\frac{di\,(\mathrm{t})}{dt} = (20)\left\{-0.0655e^{-11.978t}(-11.978) + 0.0655e^{-0.5218t}(-0.5218)\right\}$$
$$= 15.69e^{-11.978t} - 0.683558e^{-0.5218t} \quad \mathrm{V} \quad t > 0$$

**Another Solution**

We can start by finding $v\,(\mathrm{t})$ directly as follows :

$$v\,(\mathrm{t}) = v_f + A_1 e^{-11.978t} + A_2 e^{-0.5218t} \quad \mathrm{V} \quad t > 0$$

here $v_f = 0$ ( at ∞ the inductor is short ⇒ the capacitro voltage = 0)

$$v\,(\mathrm{t}) = A_1 e^{-11.978t} + A_2 e^{-0.5218t} \quad \mathrm{V} \quad t > 0$$

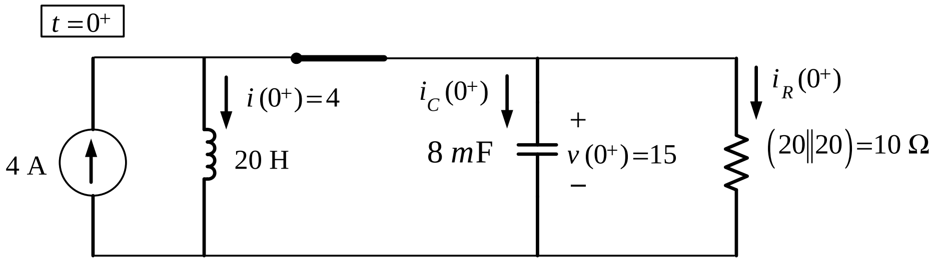
$$v\,(0) = 15 = A_1 + A_2 \text{------}(1)$$

$$\frac{\mathrm{d}v\,(\mathrm{t})}{\mathrm{dt}} = -11.978A_1 e^{-11.978t} - 0.5218A_2 e^{-0.5218t}$$

$$\frac{\mathrm{d}v\,(0)}{\mathrm{dt}} = -11.978A_1 - 0.5218A_2 \quad \text{finding } \frac{\mathrm{d}v\,(0)}{\mathrm{dt}}\ ?$$

Since $i_C\,(0) = C\,\frac{dv\,(0)}{\mathrm{dt}} \Rightarrow \frac{dv\,(0)}{\mathrm{dt}} = \frac{i_C\,(0)}{C}$ finding $i_C\,(0)$ ?

To find $i_C\,(0)$ we must look at the circuit at the instant $0^+$




$$\textbf{KCL}\quad -4+i_L(0^+)+i_C(0^+)+i_R(0^+)=0$$

$$-4+4+i_C(0^+)+\frac{15}{10}=0$$

$$\Rightarrow i_C(0^+)=-1.5\ \text{A}$$

$$\Rightarrow \frac{dv(0)}{\text{dt}}=\frac{i_C(0^+)}{C}=\frac{-1.5}{(8*10^{-3})}=-187.5$$

$$\frac{\text{d}v(0)}{\text{dt}}=-187.5=-11.978A_1-0.5218A_2$$

$$11.978A_1+0.5218A_2=187.5 \text{ ------(2)}$$

solving (1) and (2) we get $A_1=15.6835 \quad A_2=-0.6835$

$$\Rightarrow v(\text{t})=15.6835e^{-11.978t}-0.6835e^{-0.5218t}\ \text{V}\quad t>0$$

Now, we can find $i$ (t) using **KCL** as follows:

$$-4+i(\text{t})+i_C(\text{t})+i_R(\text{t})=0$$

$$\Rightarrow i(\text{t})=4-i_C(\text{t})-i_R(\text{t})=4-\left(8\text{X}10^{-3}\right)\frac{\text{d}v(\text{t})}{\text{dt}}-\frac{v(\text{t})}{10}$$

$$\Rightarrow i(\text{t})=4-0.0655e^{-11.978t}+0.0655e^{-0.5218t}\quad \text{A}\quad t>0$$