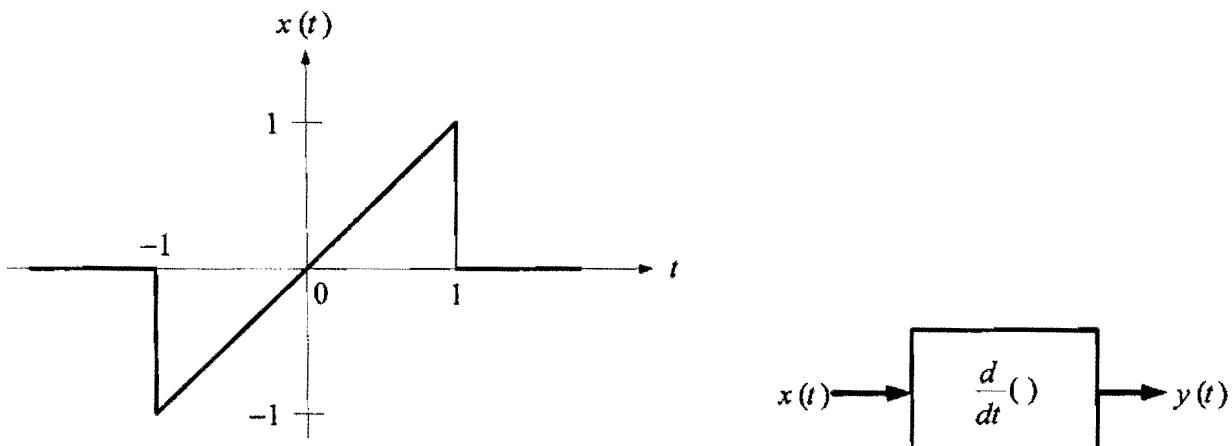


EE 207-03 - Winter 2012(112)  
Quiz 4

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| SER | ID | NAME |
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The input signal  $x(t)$  and output signal  $y(t)$  shown above,

- Using the Fourier Transform Properties and Table find the Fourier Transform input signal  $X(\omega)$  ?
- Find the transfer function  $H(\omega)$  ?
- Using the Fourier Transform Properties and Table find the output signal  $y(t)$  ?

Solution

$$\begin{aligned}
 \text{(a)} \quad \frac{d x(t)}{dt} &= \text{rect}(t/2) - [\delta(t+1) + \delta(t-1)] \\
 &\Rightarrow (j\omega) X(f) = 2 \text{sinc}(\omega) - [e^{j\omega} + e^{-j\omega}] \\
 &= 2 \text{sinc}(\omega) - 2 \cos(\omega)
 \end{aligned}$$

$$\Rightarrow X(f) = \frac{2 \text{sinc}(\omega) - 2 \cos(\omega)}{j\omega}$$

$$\text{(b)} \quad y(t) = \frac{d x(t)}{dt} \Rightarrow Y(\omega) = (j\omega) X(\omega) \Rightarrow \frac{Y(\omega)}{X(\omega)} = j\omega$$

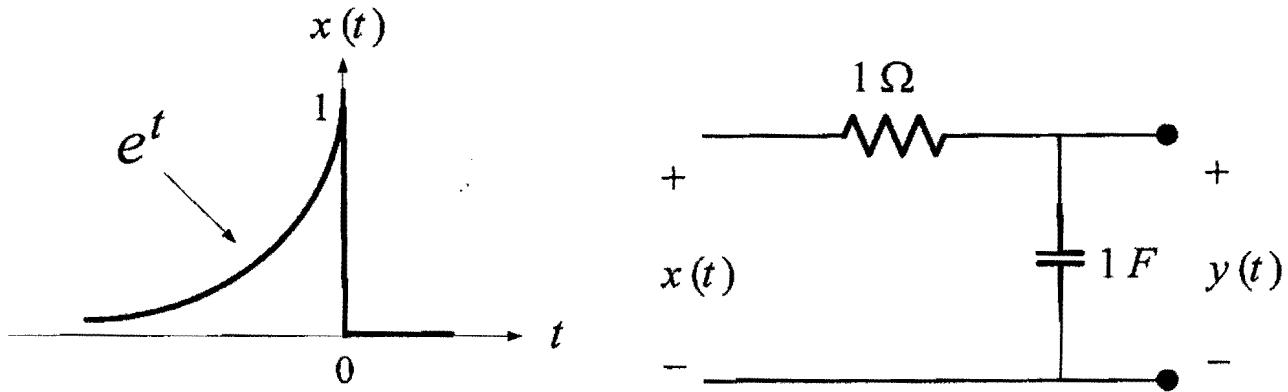
$$(c) \quad Y(\omega) = H(\omega) X(\omega)$$

$$= (j\omega) \frac{2 \operatorname{sinc}(\omega) - 2 \operatorname{Cs}(\omega)}{j\omega}$$

$$= 2 \operatorname{sinc}(\omega) - 2 P_\omega(\omega)$$

$$\Rightarrow y(t) = \operatorname{rect}(t/2) - [\delta(t+1) + \delta(t-1)]$$

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The input signal  $x(t)$  and output signal  $y(t)$  shown above,

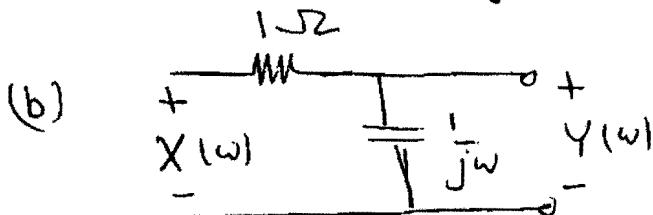
- Find the Fourier Transform input signal  $X(\omega)$  ?
- The transfer function  $H(\omega)$  ?
- The output signal  $y(t)$  ?

Solution

$$(a) X(\omega) = \int_{-\infty}^{\infty} e^t \cdot e^{-j\omega t} dt = \int_{-\infty}^{\infty} e^{(1-j\omega)t} dt$$

$$= \frac{1}{1-j\omega} e^{(1-j\omega)t} \Big|_{-\infty}^{\infty} = \frac{1}{1-j\omega} [e^{\infty} - e^{-\infty}]$$

$$= \frac{1}{1-j\omega} \quad \Rightarrow H(\omega) = \frac{1}{1+j\omega}$$



$$Y(\omega) = \frac{1/j\omega}{1 + \frac{1}{j\omega}} X(\omega)$$
 ~~$H(\omega) = \frac{1}{1 + \frac{1}{j\omega}}$~~

$$Y(\omega) = H(\omega)X(\omega)$$

$$= \left( \frac{1}{1+j\omega} \right) \left( \frac{1}{1-j\omega} \right) = \frac{1}{1+\omega^2}$$

From Table 5.2

$$e^{-\alpha|t|} \quad \alpha > 0 \quad \leftrightarrow \quad \frac{2\alpha}{\alpha^2 + \omega^2}$$

$$\rightarrow y(t) = \frac{1}{2} e^{-|\alpha t|}$$