KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
ELECTRICAL ENGINEERING DEPARTMENT
FIRST SEMESTER 2009/2010
EE 207 MAJOR EXAM I

DATE: Wednesday 11 November 2009
TIME: 6:00-7:30 PM

Student's Name:......Key Solution......

Student's I.D. Number:......................

Section Number:........................

Problem	Maximum Score	Score
1	30	
2	35	
3	35	
Total	*100*	

Problem 1 [30 points]

5 A. Evaluate the following integrals:

a. $\int_0^{10} e^{-t^2}\cos(\omega t)\delta(t+6)dt$ $= 0$ since the delta function exists outside the integral limits.

2

b. $\int_0^{10} e^{-t^2}\cos(\omega t)\delta(t-6)dt$, using the sifting property:

$= e^{-t^2}\cos(wt)\Big|_{t=6}$ 2

$= e^{-(6)^2}\cos(6w)$

$= e^{-36}\cos(6w)$. 1

10 B. Is the following system $y(t) = tx(t^2)$

a. Causal
b. Time varying

There will be no credit unless you show your work.

a. The given system is not causal since, e.g.

2

$y(2) = 2x(4)$. 3

b. The given system is time-varying since the amplitude

2

of the output $y(t)$ varies with time.

3

15 C. Given the following signal $x(t) = e^{-10t}u(t)$,

a. Use definitions to calculate its power and energy?
b. Is this a power signal or an energy signal? Why?

a. To compute the energy of $x(t)$, we use:

$$E \triangleq \underbrace{\lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt}_{} = \lim_{T\to\infty} \int_{-T}^{T} \left[e^{-10t}u(t)\right]^2 dt, \text{ since } x(t) \text{ is a real signal.}$$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{-20t} dt$$

$$= \lim_{T\to\infty} -\frac{1}{20} e^{-20t} \Big|_{t=0}^{T}$$

$$= \lim_{T\to\infty} \frac{1-e^{-20T}}{20} = \frac{1}{20} \text{ Joules.}$$

To compute the power of $x(t)$, we use:

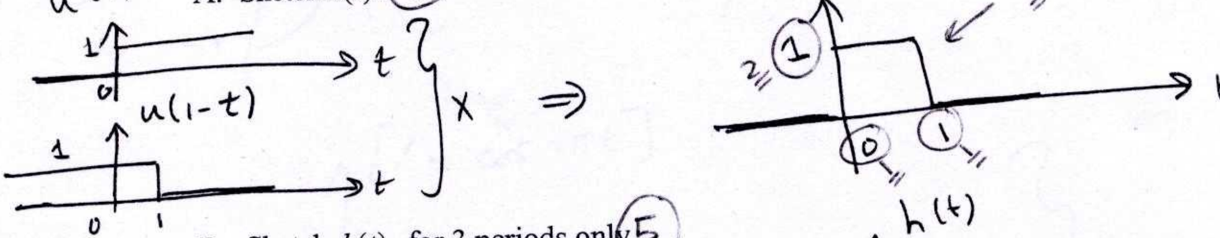
$$P \triangleq \underbrace{\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt}_{} = \underbrace{\lim_{T\to\infty} \frac{1-e^{-20T}}{40T}}_{} = 0 \text{ Watts.}$$

b. $\overbrace{\text{Since } 0 < E = \frac{1}{20} < \infty, \text{ i.e., is finite} \Rightarrow x(t) \text{ is}}$ $\underbrace{\text{an energy signal \& that is why } P = 0.}$

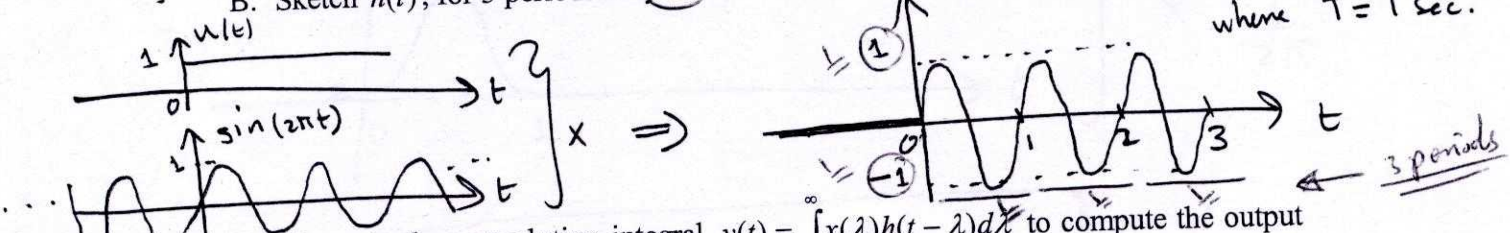
Problem 2 [35 points]

Assume that a linear time-invariant (LTI) system is characterized by the impulse response $h(t) = \sin(2\pi t)u(t)$. Also, consider $x(t) = u(t)u(1-t)$ to be the input signal to this LTI system. Answer the following:

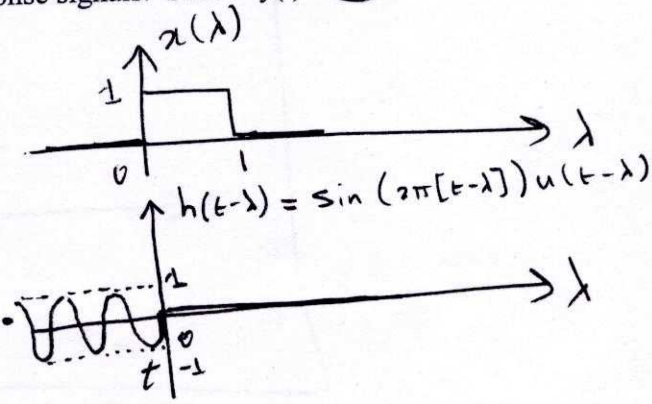
A. Sketch $x(t)$. 5



B. Sketch $h(t)$, for 3 periods only. 5



C. Use only the convolution integral $y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$ to compute the output based on the above input and impulse response signals. Sketch y(t) 25



For $t < 0$ $\quad y(t) = 0$. 5

For $0 \le t \le 1$

$$y(t) = \int_0^t \sin(2\pi[t-\lambda])\, d\lambda \quad 5$$

now using change of variables, i.e.,

$$\sigma = 2\pi(t-\lambda) \Rightarrow d\sigma = -2\pi\, d\lambda$$

$$\text{or } -\frac{1}{2\pi}d\sigma = d\lambda$$

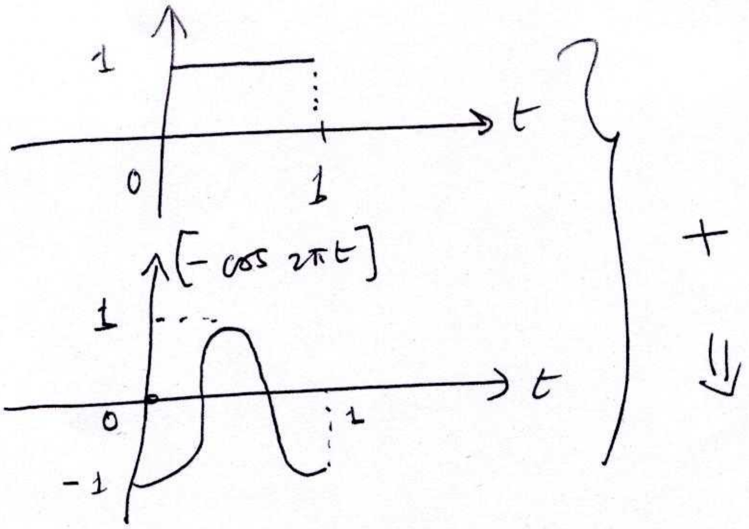
now if $\lambda = 0 \Rightarrow \sigma = 2\pi t$ and if $\lambda = t \Rightarrow \sigma = 2\pi(t-t) = 0$

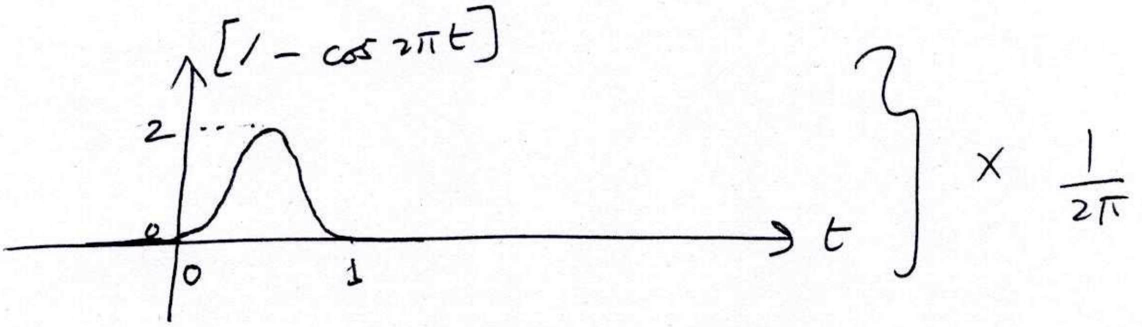
$$\Rightarrow y(t) = \int_{2\pi t}^{0} \sin\sigma \frac{d\sigma}{-2\pi} = \frac{-1}{2\pi}\left[-\cos\sigma\right]\Big|_{\sigma=2\pi t}^{0}$$

$$= \frac{1}{2\pi}\left[1 - \cos 2\pi t\right] \quad 5$$

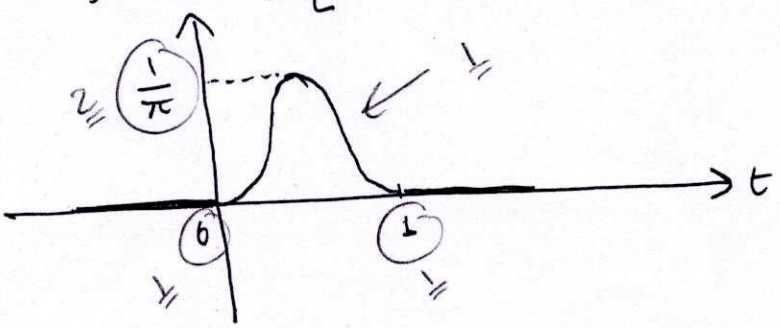
For $t > 1$ 5

$y(t) = 0$, since we are going integrate a full period of the sine wave within the $(0-1)$ window.



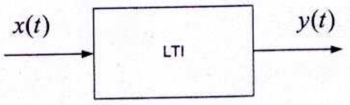


$$y(t) = \frac{1}{2\pi}\left[1 - \cos 2\pi t\right], \text{ for } 0 \le t \le 1$$



Problem 3 [35 points]

Given



where the above linear time-invariant (LTI) system is characterized by its impulse response that is given by:

$$h(t) = \frac{1}{100} e^{-t/100} u(t).$$

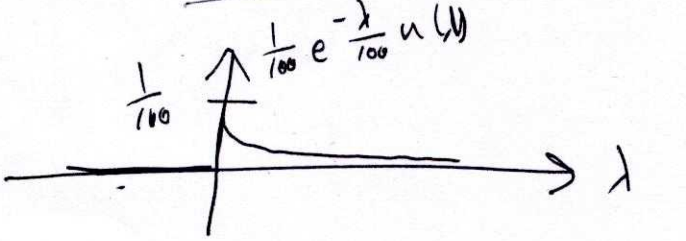
A. Find the output $y(t)$ if the input $x(t) = u(t)$. 10

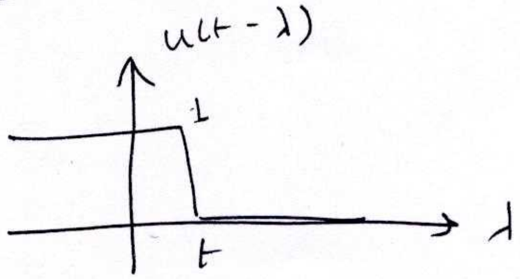
We can solve this using two ways:

Method 1: by the conv. integral:

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda \quad 1$$

$$= \int_{-\infty}^{\infty} \left[\frac{1}{100} e^{-\lambda/100} u(\lambda)\right]\left[u(t-\lambda)\right] d\lambda \quad 2$$





So for $t < 0 \Rightarrow y(t) = 0$. 2

and for $t \geq 0 \Rightarrow y(t) = \int_{0}^{t} \frac{1}{100} e^{-\frac{\lambda}{100}} d\lambda$ 3

$$= - e^{-\lambda/100} \Big|_{\lambda=0}^{t}$$

$$= \left[1 - e^{-t/100}\right]$$

OR $$y(t) = \left[1 - e^{-\frac{t}{100}}\right] u(t).$$

Method 2: since $$\delta(t) \longrightarrow \boxed{LTI} \longrightarrow h(t) = \frac{1}{100} e^{-t/100} u(t)$$

and we know that $$u(t) = \int_{-\infty}^{t} \delta(\lambda)\, d\lambda$$

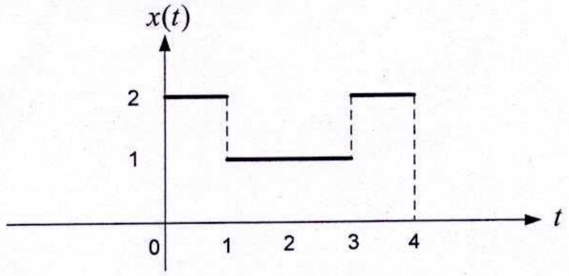
$$\Rightarrow \int_{-\infty}^{t} \delta(\lambda)\, d\lambda \longrightarrow \boxed{LTI} \longrightarrow \int_{-\infty}^{t} h(\lambda)\, d\lambda$$

$$\Rightarrow y(t) = \int_{-\infty}^{t} \frac{1}{100} e^{-\frac{\lambda}{100}} u(\lambda)\, d\lambda$$

$$= \int_{0}^{t} \frac{1}{100} e^{-\lambda/100}\, d\lambda$$

$$= \left[1 - e^{-\frac{t}{100}}\right] u(t).$$

B. Given the input signal to be as follows: 10



Express $x(t)$ in terms of a combination of step functions.

$$x(t) = \underbrace{2}_{1/2}\Big[\underbrace{u(t) - u(t-1)}_{1/2}\Big] + \Big[\underbrace{u(t-1)}_{1/2} - \underbrace{u(t-3)}_{1/2}\Big] + \underbrace{2}_{1/2}\Big[\underbrace{u(t-3) - u(t-4)}_{1/2}\Big]$$

$$= \underbrace{2u(t)}_{1/2} \underbrace{- u(t-1)}_{1/2} + \underbrace{u(t-3)}_{1/2} \underbrace{- 2u(t-4)}_{1/2}.$$

on this final answer

15 C. Find the output $y(t)$ for the input $x(t)$ given above in part (B). Do not use the convolution integral.

Since the above system is LTI & we know its step response from part A., then using linearity & time-inv. properties:

$$y(t) = \underbrace{2\left[\frac{1}{10000}\left(1-e^{-t/100}\right)u(t)\right]}_{3} \underbrace{- \frac{1}{10000}\left(1-e^{-\frac{(t-1)}{100}}\right)u(t-1)}_{3}$$

$$\underbrace{+ \frac{1}{10000}\left(1-e^{-(t-3)/100}\right)u(t-3)}_{3} \underbrace{- 2\left[\frac{1}{10000}\left(1-e^{-\frac{(t-4)}{100}}\right)u(t-4)\right]}_{3}.$$