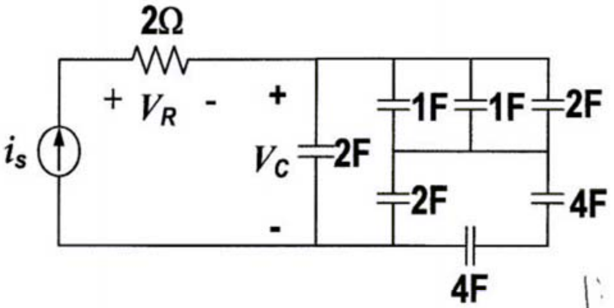
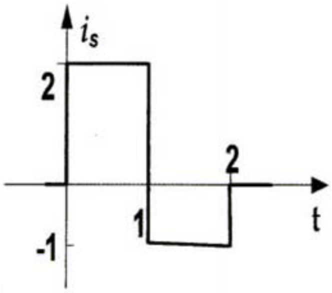
Q#1 For the given 'i_s',

1. Find the equations of 'V_C' for the following ranges **(14 marks)**
   (a) $0<t<1$, (b) $1<t<2$, (c) $2<t<\infty$
2. Plot the result in part 1. **(3 marks)**
3. Plot the results of 'V_R' **(3 marks)**





$$C_{eq} = \left(\left[(4\,||\,4) + 2\right] \,||\, \left[1+1+2\right]\right) + 2$$

$$= \left(\left[2+2\right] \,||\, \left[4\right]\right) + 2$$

$$= (4\,||\,4) + 2 = 2+2 = 4F$$

(3 marks)

$$V_C = \frac{1}{C_{eq}} \int_{-\infty}^{t} i_s(\tau)\,d\tau$$

for $0<t<1$

$$V_C = \frac{1}{4}\int_0^t 2\,d\tau = \frac{1}{2}t \quad \text{(3 marks)}$$

for $1<t<2$

$$V_C = \frac{1}{4}\int_1^t -1\,d\tau + \left(\tfrac{1}{2}t\right)_{t=1} = -\frac{1}{4}(t-1) + \frac{1}{2} = -\frac{1}{4}t + \frac{3}{4} \quad \text{(4 marks)}$$

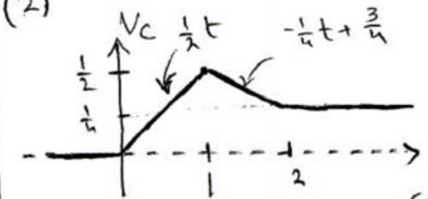
for $t>2$

$$V_C = \frac{1}{4}\int_2^t i_s(\tau)\,d\tau + \left(-\tfrac{1}{4}t + \tfrac{3}{4}\right)_{t=2} = 0 + -\frac{2}{4} + \frac{3}{4} = \frac{1}{4} \quad \text{(4 marks)}$$

$$\therefore V_C = \begin{cases} 0 & t<0 \\ t/2 & 0<t<1 \\ -\frac{1}{4}t + \frac{3}{4} & 1<t<2 \\ \frac{1}{4} & t>2 \end{cases}$$

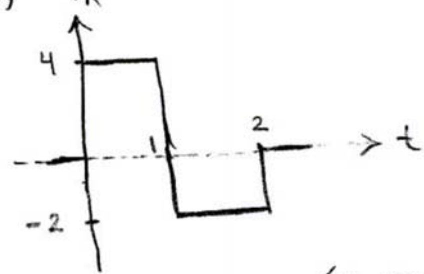
$$V_R = i_s(t)\,R = 2\,i_s(t)$$

(2)



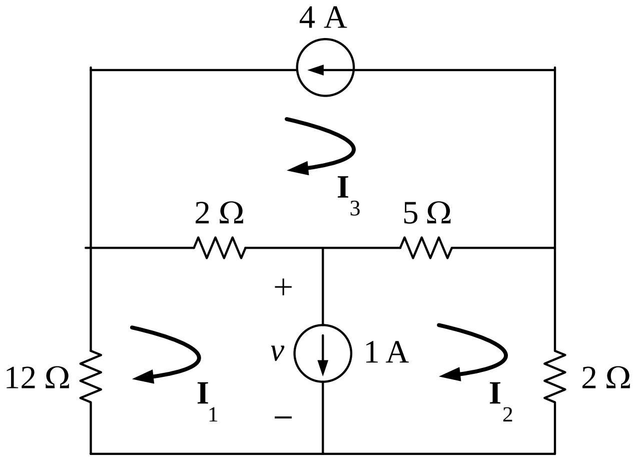
(3 marks)

(3)



(3 marks)

Problem 2



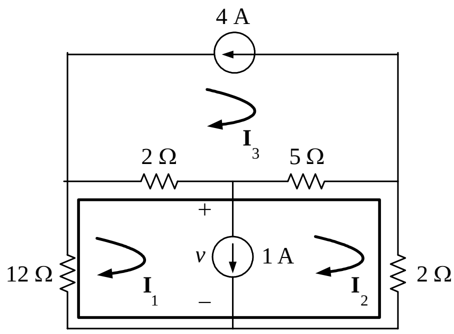
Using mesh analysis find

(a) The mesh currents I1, I2, I3 ?

(b) the voltage v(t) ?

Solution

$$\mathbf{I}_3 = -4\ \text{A}\ \ \text{(by inspection)}$$



KVL on the super mesh

$$2\left(\mathbf{I}_1 - \mathbf{I}_3\right) + 5\left(\mathbf{I}_2 - \mathbf{I}_3\right) + 2\mathbf{I}_2 + 12\mathbf{I}_1 = 0$$

$$\Rightarrow 14\mathbf{I}_1 + 7\mathbf{I}_2 = 7\mathbf{I}_3 = 7(-4) = -28 \qquad (1)$$

$$\mathbf{I}_1 - \mathbf{I}_2 = 1 \qquad (2)$$

$$\Rightarrow \begin{bmatrix} 14 & 7 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} -28 \\ 1 \end{bmatrix}$$

solving

$$\mathbf{I}_1 = \frac{\begin{vmatrix} -28 & 7 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 14 & 7 \\ 1 & -1 \end{vmatrix}} = \frac{28-7}{-14-7} = \frac{21}{-21} = -1 \text{ A}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 14 & -28 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 14 & 7 \\ 1 & -1 \end{vmatrix}} = \frac{14+28}{-14-7} = \frac{42}{-21} = -2 \text{ A}$$

(c) KVL on the first mesh

$$2\left(\mathbf{I}_1 - \mathbf{I}_3\right) + v + 12\mathbf{I}_1 = 0$$

$$\Rightarrow v = -14\mathbf{I}_1 + 2\mathbf{I}_3 = -14(-1) + 2(-4) = 6 \text{ V}$$

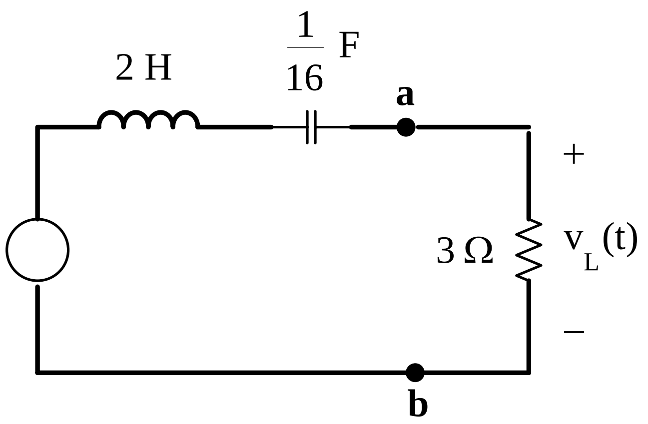
OR KVL on the second mesh

$$5\left(\mathbf{I}_2 - \mathbf{I}_3\right) + 2\mathbf{I}_2 - v = 0$$

$$\Rightarrow v = 7\mathbf{I}_2 - 5\mathbf{I}_3 = 7(-2) - 5(-4) = 6 \text{ V}$$

Problem 3

$v_s(t)=10\cos(2t + 30^o)$ V

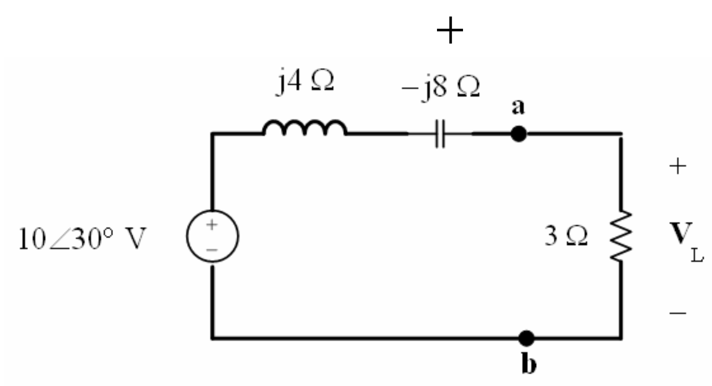


Using the Thevenin Reduction Method only, find $v_L(t)$ the voltage across the load resistor $3\ \Omega$?

Solution

Transferring the circuit to the phasor domain

$v_s(t)=10\cos(2t + 30^o)$ V $\Rightarrow$ $10\angle 30^o$ V



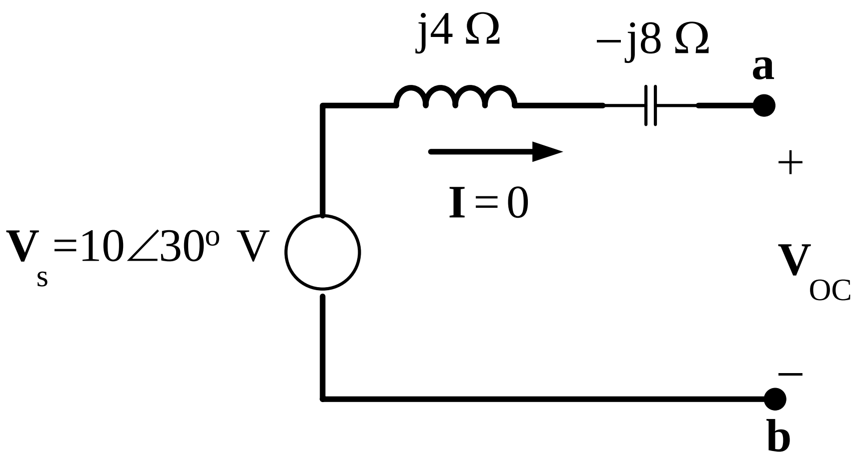
$Z_L = j\omega L=j(2)(2)=j4\ \Omega$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j(2)\left(\frac{1}{16}\right)} = -j8\ \Omega$$

$Z_R = 3\ \Omega$

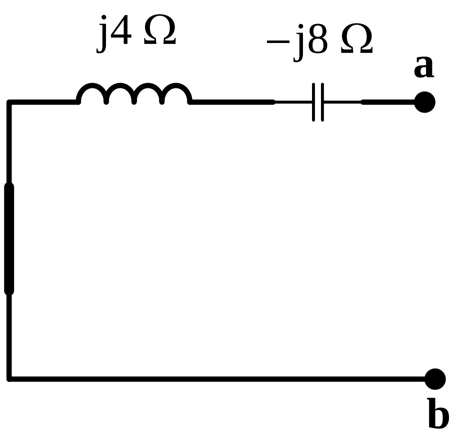
Finding the Thevenin equivalent

Removing the load resistant



$$\Rightarrow \mathbf{V}_{OC} = \mathbf{V}_s = 10\angle 30^o \text{ V}$$

Deactivating the source (shorting it)



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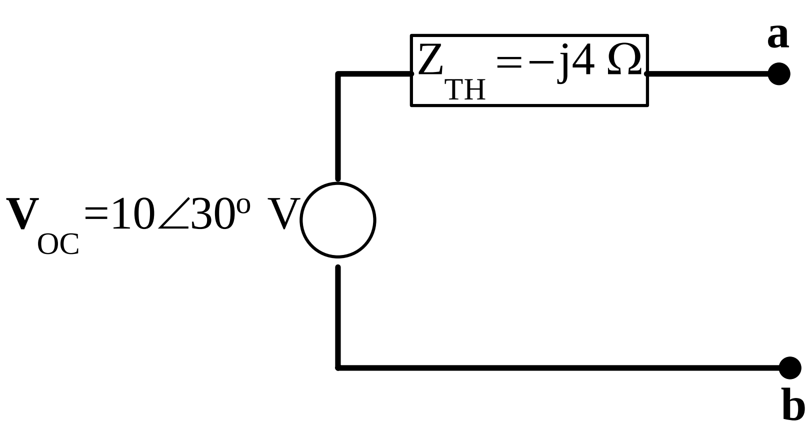
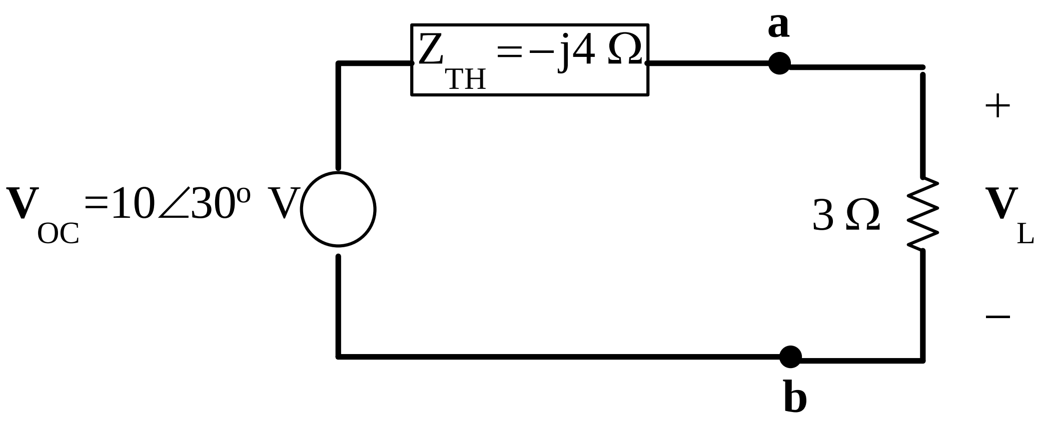
$$\Leftarrow Z_{TH} = \left(j4\ \Omega\right) + \left(-j8\ \Omega\right) = -j4\ \Omega$$





Using Voltage division

$$\mathbf{V}_L = \frac{3}{(-j4\ \Omega)+3} 10\angle 30^o = \frac{3}{5\angle -53.13^o} 10\angle 30^o = 6\angle 83.13^o \text{ V}$$

$$\Rightarrow \quad v_L(t) = 6\cos(2t + 83.13^o) \quad \text{V}$$