# Capacity Regions for Wireless Ad Hoc Networks*

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***Abstract*—We determine the capacity region of an ad hoc wireless network with an arbitrary number of nodes. This region defines the set of achievable rate vectors between all source-destination pairs in the network under variable rate transmission and both single hop and multihop routing. We also determine the effect of sophisticated transceiver capabilities such as power control, spatial reuse, and successive interference cancellation on the capacity region. The capacity region boundary is obtained using complex linear programming methods. Numerical results indicate that under variable rate transmission, multihop routing and spatial frequency reuse greatly increase the capacity region. Considerable capacity gains are also achieved by successive interference cancellation. On the other hand, gains by power control in addition to variable rate transmission are marginal. Similar trends are observed for the special case of multihop cellular networks.**

## I. Introduction

Wireless ad hoc networks consist of a number of nodes communicating with each other over wireless channels. Two nodes wishing to communicate can do so directly or can use intermediate nodes to forward packets between them. Ad hoc networks lack a backbone intrastructure, so all control functions (e.g. routing, access, adaptivity) must be coordinated between network nodes. The lack of a backbone infrastructure differentiates ad hoc networks from cellular networks, where all nodes communicate directly with a base station and the base station controls all transmission and routing functions.

Ad hoc networks pose many design challenges due to their lack of backbone infrastructure, decentralized control, dynamic topology, and wireless channel characteristics [1]. A recent landmark paper determined the uniformly achievable rate for nodes in an asymptotically large ad hoc network [2]. In this work we investigate the capacity region for ad hoc networks with any number of nodes. This multidimensional region dictates the set of rates that all nodes can achieve to all other nodes in the network. We determine the capacity region under time division routing and variable rate transmission. The Shannon capacity region of ad hoc networks remains an open problem, so our capacity regions only define the maximum achievable rates under our transmission assumptions, which may be suboptimal. Our problem formulation allows us to investigate the impact of different techniques on capacity, including power control, multihop routing, spatial reuse, and successive interference cancellation. We will see that with the exception of power control, all of these techniques significantly increase the ad hoc network capacity region.

The remainder of the paper is organized as follows. In Section II we describe the system model. Section III describes the achievable rate matrices for a given network, and defines the capacity region in terms of these matrices. Capacity region slices for a five node ad hoc network are given in Section IV under various assumptions on the transceiver capabilities, including single hop and multihop routing, spatial reuse, power control, and successive interference cancellation. The capacity of a multihop cellular network is also studied in this section. Our conclusions are given in Section V. Throughout the paper, terms being defined are set in **boldface**.

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## II. System Model

Consider an ad hoc network with $n$ nodes $A_1, A_2, \dots, A_n$. Each node has a transmitter, receiver, and an infinite buffer, and wishes to communicate with some or all of the other nodes, possibly by multihop routing. We assume that nodes cannot transmit and receive at the same time. We also assume that nodes do not broadcast information, so every transmitted packet is intended for a single node.

Node $A_i$ transmits at some maximum power $P_i$ and all transmissions occupy the full bandwidth $W$ of the system. We define the **power vector** to be the vector $P = [P_1\ P_2\ \dots\ P_n]$. When node $A_i$ transmits, node $A_j$ receives the signal with power $G_{ij}P_i$, where $G_{ij}$ denotes the channel gain between nodes $i$ and $j$. The channel gain assumes path loss and shadowing, and is modeled as $G_{ij} = K d_{ij}^{\alpha} S$, where $K$ is a propagation constant, $d_{ij}$ is the distance between nodes $i$ and $j$, $\alpha$ is the path loss exponent, and $S$ is random shadowing with standard deviation $\sigma$. We define the **channel gain matrix** to be the $n \times n$ matrix $G = \{G_{ij}\}$. The receiver of each node is subject to thermal noise and interference from other users, where the interference between nodes $i$ and $j$ is also determined by the link gain $G_{ij}$. We assume the noise to be additive, white and Gaussian, with noise power spectral density $\eta_i$ for node $A_i$. We define the **noise vector** $H = [\eta_1\ \eta_2\ \dots\ \eta_n]$.

Let $\{A_t : t \in \mathcal{T}\}$ be the set of transmitting nodes at a given time, each node $A_t$ transmitting with power $P_t$. Let us assume that node $A_j \notin \mathcal{T}$ is receiving information from node $A_i, i \in \mathcal{T}$. Then the **signal to interference and noise ratio (SINR)** at node $A_j$ will be

$$\gamma_{ij} = \frac{G_{ij}P_i}{\eta_j + \sum_{k \in \mathcal{T}, k \neq i} G_{kj} P_j}. \tag{1}$$

We assume that the transmit-receive node pairs vary their transmission rate based on $\gamma_{ij}$ to meet a given performance metric. Specifically, nodes $A_i$ and $A_j$ agree on a transmission rate $R_{ij}$ that satisfies $R_{ij} \leq f(\gamma_{ij})$ where $f(.)$ is a function that depends on the transmission and decoding strategy and the performance metric. For example, based on Shannon capacity we can set

$$f(\gamma_{ij}) = W \log_2(1 + \gamma_{ij}). \tag{2}$$

Under the Shannon assumption bits transmitted over a link are received with asymptotically small error as long as the rate $R_{ij} \leq f(\gamma_{ij})$ for $f$ defined in Eq. (2). Alternatively, $f(\gamma_{ij})$ can correspond to the maximum data rate to meet a given BER requirement under a specific modulation scheme such as MQAM [3]. We assume that all transmit-receive node pairs use their maximum achievable rate rate $R_{ij} = f(\gamma_{ij})$ for the given function $f$ and link SINR $\gamma_{ij}$. Note that $f(\gamma_{ij})$ for $\gamma_{ij}$ defined in (1) assumes all interference signals are treated as noise: this assumption will be relaxed when we consider successive interference cancellation.

We assume omniscient nodes with perfect knowledge of the channel gain matrix ($G$) and the noise ($H$) and power ($P$) vectors. The transmission strategy for all nodes for a given channel gain matrix $G$ is agreed to in advance. Thus, no overhead is needed for nodes to determine $G$, $H$, $P$, or the transmission strategy.

## III. Achievable Rate Matrices and Capacity

In this section we determine the capacity region of a network based on its achievable rate matrices, which we now define.

### A. Transmission Schemes and Rate Matrices

Rate matrices provide a mathematical framework for describing the network transmission scheme at any given time. A transmission scheme describes the information flow between different nodes in the network at a given time, and must capture the characteristics of transmit-receive node pairs as well as packet forwarding. Therefore, the transmission scheme at a given time consists of all transmit-receive node pairs in operation at that time and, for each of these pairs, the transmission rate and the original source node of the transmitted information. We assume that nodes cannot transmit and receive simultaneously, and that the rates defined by a given transmission scheme are achievable, i.e. the rate between nodes $i$ and $j$ does not exceed $f(\gamma_{ij})$, where $f$ defines the link rate constraint and $\gamma_{ij}$ is the link SINR between nodes $i$ and $j$ for the given transmission scheme.

To illustrate transmission schemes, consider a four node network where the node pairs $(A_1, A_2)$, $(A_2, A_3)$, $(A_3, A_4)$, $(A_4, A_1)$ can all communicate directly but the node pairs $(A_1, A_3)$ and $(A_2, A_4)$ cannot, perhaps due to excessive shadowing of their links. Therefore, if $A_1$ wants to communicate with $A_3$, it must do so by forwarding packets via intermediate nodes, and similarly for traffic between nodes $A_2$ and $A_4$. Let us consider a transmission scheme that allows multi-hop routing and spatial reuse. Under these assumptions a possible strategy to transmit information between nodes $A_1$ and $A_3$ using nodes $A_2$ and $A_4$ as intermediate nodes would be time sharing between transmission schemes $\mathcal{S}_1$ and $\mathcal{S}_2$ shown in Fig. 1. The figure shows the transmit-receive node pairs in operation for each scheme using an arrow connecting them. The originating node of the information being transmitted and the link transmission rate is shown next to the link arrows. Specifically, in scheme $\mathcal{S}_1$, node $A_1$ sends information to node $A_2$ and node $A_3$ sends information to node $A_4$. The nodes originating the traffic in this scheme are nodes $A_1$ and $A_3$ respectively. The transmission rate for each of these transmissions, given as 10 in the figure, is dictated by the SINR on each link under $\mathcal{S}_1$. The second scheme $\mathcal{S}_2$ forwards the information from the intermediate links to their final destination. Specifically, node $A_2$ sends the information that originated at node $A_1$ to its final destination $A_3$ at rate 10, and node $A_4$ sends the information that originated at node $A_3$ to its final destination $A_1$ at rate 10. If we time share equally between $\mathcal{S}_1$ and $\mathcal{S}_2$ we see that node $A_1$ can send information at rate 5 ($10 \times .5$) to node $A_3$ and node $A_3$ can send information at rate 5 to node $A_1$.

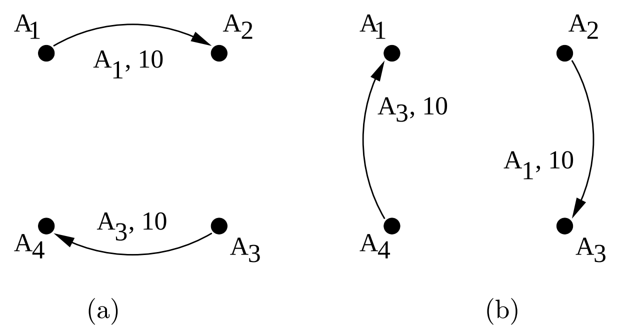


Fig. 1. Transmission schemes $\mathcal{S}_1$ (a) and $\mathcal{S}_2$ (b).

Although transmission schemes are useful for describing the state of the network at a given time, they are not convenient for mathematical manipulation. We will therefore formally define transmission schemes using rate matrices. For a network of size $n$ we define the **rate matrix** $R(\mathcal{S})$ of a transmission scheme $\mathcal{S}$ as an $n \times n$ square matrix with elements $R_{ij}$ such that:

$$R_{ij} = \begin{cases} R & \text{If node } A_j \text{ receives information at rate } R \text{ with node } A_i \text{ as the original information source.} \\ -R & \text{If node } A_j \text{ transmits information at rate } R \text{ that originated at node } A_i. \\ 0 & \text{otherwise.} \end{cases}$$

Positive entries in the rate matrix correspond to information being received, while negative entries correspond to information being sent or forwarded. For example, the rate matrices of schemes $\mathcal{S}_1$ and $\mathcal{S}_2$ are, respectively,

$$R_1 = \begin{bmatrix} -10 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -10 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \; R_2 = \begin{bmatrix} 0 & -10 & 10 & 0 \\ 0 & 0 & 0 & 0 \\ 10 & 0 & 0 & -10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A rate matrix mathematically captures all the information of a transmission scheme: the transmit-receive node pairs in operation and, for each pair, the original source node of the transmitted information and the transmission rate. We note that since information must be preserved, i.e. the total amount of information that is transmitted from a node must be received by other nodes, the elements along any row of a rate matrix must sum to zero.

### B. Time Division Routing

Transmission schemes and their corresponding rate matrices completely describe the behavior of a network at any time. Under time division routing the network may divide its time between multiple transmission schemes or equivalently, between multiple rate matrices. Due to the one-to-one correspondence between transmission schemes and their corresponding rate matrices, a weighted combination of transmission schemes has a rate matrix equal to the weighted sum of the corresponding rate matrices. Therefore, if $R_1, \ldots, R_N$ are a set of achievable rate matrices, the matrix $R = \sum_{i=1}^{N} a_i R_i$ is also an achievable rate matrix for any coefficients $a_i \geq 0$ such that $\sum_{i=1}^{N} a_i \leq 1$. Fig. 2 shows the transmission strategy corresponding to using transmission scheme $\mathcal{S}_1$ 75% of the time and transmission scheme $\mathcal{S}_2$ 25% of the time. The corresponding rate matrix is

$$0.75R_1 + 0.25R_2 = \begin{bmatrix} -7.5 & 5 & 2.5 & 0 \\ 0 & 0 & 0 & 0 \\ 2.5 & 0 & -7.5 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (3)$$

which is equivalent to the transmission scheme shown in the right side of Fig. 2.

### C. Ad Hoc Network Capacity Region

Since an achievable rate matrix describes the set of achievable rate vectors in a given network at any time, it would be reasonable to define the capacity of the network under time division as the set of all weighted sums of achievable rate matrices. However, some weighted sums of rate matrices will have off-diagonal components that are negative. Such rate matrices correspond to scenarios where some nodes forward more information from a source than they receive from that source (possibly indirectly, through routing). Clearly, this is not a stable condition, and we therefore exclude these sums from the capacity region. All other weighted sums of achievable rate matrices are considered part of the network capacity region. Some of these sums correspond to noncausal routing, as a node forwards traffic from another node before that traffic actually arrives. This situation does not pose a problem under the assumption of infinite backlog, since forwarding nodes will always have an infinite number of packets to forward. We neglect causality in our routing model since it significantly complicates the problem and obscures our main results.

Based on this motivation we define the **ad hoc network capacity region** under time division routing as the convex hull of the achievable rate matrices with the restriction that the weighted sums must have nonnegative off diagonal elements. Specifically, if $\{R_1, \ldots, R_N\}$ denotes the set of achievable rate matrices for a given network, the network capacity is

$$\begin{aligned} C &= C(\{R_i\}) \\ &= \{\sum_{i=1}^{N} a_i R_i \ : \ a_i \geq 0, \sum_{i=1}^{N} a_i = 1\} \cap \mathcal{P}_n \\ &= Co(\{R_i\}) \cap \mathcal{P}_n \end{aligned}$$

where $\mathcal{P}_n$ is the subset of all $n \times n$ matrices with all their off-diagonal components non-negative and $Co\{R_i\}$ denotes the convex hull of the set $\{R_i\}$ of matrices.

The meaning of the capacity region is the following: Let $R$ be a matrix in the capacity region. Then there is a time division of achievable rate matrices such that when the network operates under this time division and $i \neq j$, $R_{ij}$ is the rate with which node $A_i$ sends its own information to node $A_j$, possibly through multiple hops and time division, and $-R_{ii}$ is the total rate with which node $A_i$ is passing information to all other nodes.

Since the elements in each row of all matrices in the capacity region must sum to 0, the capacity region is a subset of the $n \times (n-1)$ Euclidean space. This dimensionality is expected, since there are $n$ nodes, each with $(n-1)$ other nodes with which it may want to communicate. To capture the capacity of an ad hoc network with a single parameter, we define the **uniform capacity** $C_u$ of a network as the maximum aggregate communication rate, if all nodes wish communicate with all other nodes, using a common rate. The uniform capacity is equal to $R \times n(n-1)$, where $R$ is the largest $R$ for which the matrix with all its off-diagonal elements equal to $R$ belongs to the capacity region, and $n(n-1)$ is the total number of source-destination pairs for a network of $n$ nodes.

### D. Computational Issues

Our goal is to determine the capacity region for an ad-hoc network, defined as the intersection of the convex hull of the network's achievable rate matrices with the set $\mathcal{P}_n$. Therefore, checking if a point is in the network's capacity region is equivalent to checking if the point belongs to $\mathcal{P}_n$, which is trivial, and checking if it belongs to the convex hull of the network's achievable rate matrices. Since the set of achievable rate matrices is isomorphic to a set of vectors of length $n(n-1)$, this problem represents a standard problem in computational geometry, and can be solved by a variety of different techniques . However, the complexity of this problem is quite large, since even for a small five node network the capacity region is twenty dimensional. For graphing purposes we will only be interested in two dimensional slices of the capacity region. We can determine the boundary of such slices in a simplified manner by following a line starting at the origin that is perpendicular to any tangent of the boundary and finding where this line crosses the region boundary, then repeating this process for all such tangents. The crossing point is found by checking when points $R$ along the line cease to be in the capacity region. Checking if a point $R$ is in the capacity region can be cast as the following linear program, which is much faster to solve than finding the entire region:

$$\begin{aligned} \text{minimize:} \quad & g(x) = \sum_{i=1}^{N} x_i \\ \text{subject to:} \quad & 1 \geq x_i \geq 0 \\ & R = \textstyle\sum_{i=1}^{N} x_i R_i, \end{aligned}$$

where $\{R_1, \ldots, R_N\}$ is the set of all achievable rate matrices for the network. If, after solving this problem, $g(x)_{min} \leq 1$, then $R$ can be obtained via a time-division strategy of achievable rate matrices, so $R \in C(\{R_i\})$. If $g(x)_{min} > 1$ then $R \notin C(\{R_i\})$.

Note that in order to determine the capacity region, the set of all achievable rate matrices must first be determined. As the number of nodes increases, the number of achievable rate matrices increases factorially. Moreover, the more capabilities we assign to the transceivers such as multihop

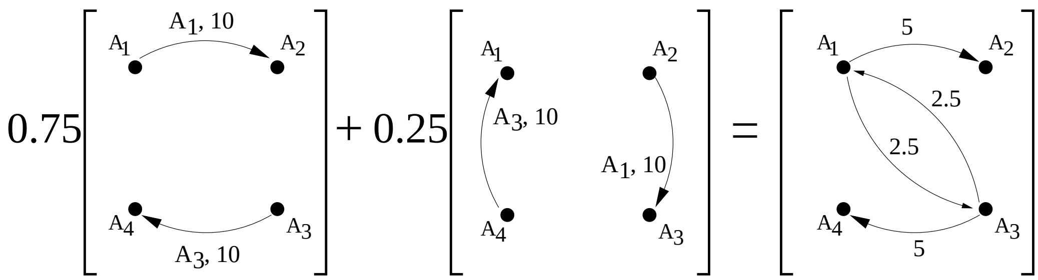


Fig. 2. The time division transmission scheme $\mathcal{T} = 0.75\mathcal{S}_1 + 0.25\mathcal{S}_2$.

routing, simultaneous transmissions, power control, and successive interference cancellation, the larger the set of achievable rate matrices that must be considered. Therefore the complexity of the capacity calculation becomes intractable for large networks. Multihop cellular networks, where all nodes send to a centralized base station possibly via multiple hops, have a restricted structure which reduces the computational complexity of their capacity regions, as described in more detail in [4], [5].

## IV. Numerical Results

In this section we determine the capacity region for a given ad hoc network under increasingly more sophisticated transmission strategies. This will illustrate the capacity gains that can be obtained from these strategies. The network under consideration is a five node network with the topology shown in Fig. 3. This network topology was obtained by uniformly and independently distributing five nodes in the box $[-1,1]\times[-1,1]$. The power gains between nodes $A_i$ and $A_j$ are given by $G_{ij} = Kd_{ij}^{-\alpha}S$ where $d_{ij}$ is the distance between the nodes, the propagation constant $K = 10^{-6}$, the path loss exponent $\alpha = 4$, and the random shadowing $S$ is generated from a log normal distributionwith unit mean and variance $\sigma = 1$. Note that all node pairs in this network have $G_{ij} > 0$, so the network is fully connected and all nodes can talk directly to all other nodes. However, since the link gains of different node pairs may be very different, multihop routing over channels with better gains may improve performance over single hop routing. The transmitter powers are $P_i = 10$ W, the white noise power spectral density is $n = 10^{-12}$ W/Hz, and the bandwidth is $W = 10^6$ Hz. The achievable link data rates are set to the Shannon limit defined by Eq. (2). Although we present numerical results for a single realization of the random network topology and shadowing parameters, we have studied many such realizations and found that the same general trends hold for all realizations [5].

### A. Single Hop Routing, No Spatial Reuse

We first determine the network capacity region when only single hop routing is allowed (no forwarding) and only one node is active at any time. By only allowing one active node at a time, link data rates are higher since there is no interference, but the network does not take advantage of spatial reuse. Since there are $n$ nodes in the system, and each of them has $n-1$ possible receivers, the network has $N_a = n(n-1)$ possible transmission schemes. Their corresponding achievable rate matrices are denoted by $R_i^a, i = 1, \ldots, N_\alpha$. Determining these rate matrices is straightforward using Eq. (2), $G$, $P$, and $H$. The capacity

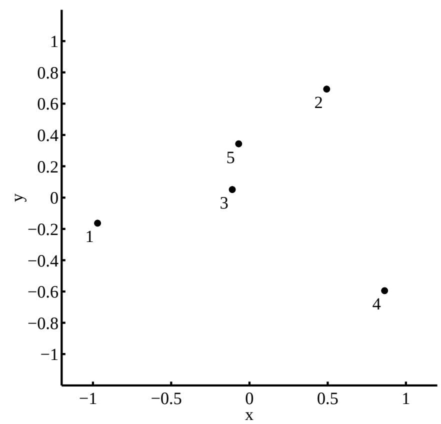


Fig. 3. Network topology for the five node network.

region will therefore be:

$$C^a = Co\{R_i^\alpha, i = 1, \ldots, N_a\} \cap \mathcal{P}_n \qquad (4)$$

We plot the capacity region $C^a$ corresponding to single hop routing with no spatial reuse for a two dimensional slice along the plane $R_{ij} = 0, \{ij\} \neq \{13\}, \{45\}$ in line (a) of Fig. 4. This slice captures a background rate of zero for node pairs other than $(1,3)$ and $(4,5)$. Therefore only nodes 1 and 4 send data: the other nodes never transmit since their individual rates are zero and under single hop routing they cannot help in forwarding packets. Note that the slice is a straight line, as expected, since without spatial reuse only one data stream can be serviced at any time. The uniform capacity of the network is $C_u^a = 0.072$.

### B. Multihop Routing, No Spatial Reuse

Next we consider the case where multihop routing is allowed, but no spatial reuse, so only one node is transmitting at a given time. Since there are $n$ nodes in the systems, and each has $n-1$ different possible receivers and $n$ possible nodes to forward data for (including itself), there are now $N_b = n^2(n-1)$ possible transmission schemes and their corresponding achievable rate matrices. We define this set of achievable rate matrices as $R_i^b, i = 1, \ldots, N_b$. Determining these achievable rate matrices is straightforward using Eq. (2), $G$, $P$, and $H$. The capacity region under these assumptions will therefore be:

$$C^b = Co\{R_i^b, i = 1, \ldots, N_b\} \cap \mathcal{P}_n. \qquad (5)$$

In Fig. 4 we have drawn a slice (line (b)) of the capacity region $C^b$ along the zero background rate plane $R_{ij} = 0, \{ij\} \neq \{13\}, \{45\}$. We note that this slice is again a straight line, as expected, since without spatial reuse only

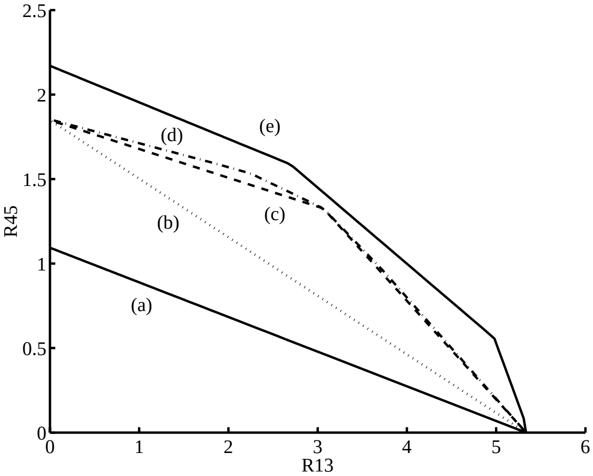


Fig. 4. Capacity region slice along the plane $R_{ij} = 0, \{ij\} \neq \{13\}, \{45\}$. Lines (a)-(e) correspond to capacity regions ($C^a$)-($C^e$). The axis units are bps/Hz.

one data stream can be serviced at any time. a weighted sum of achievable link rates. We also note a significant increase in the size of the capacity region as compared with the previous case ($C^a$ corresponding to line (a)). This is due to the fact that under multihop routing the nodes can avoid transmitting directly to their destination over paths with small gains, and instead use multiple hops over channels with much more favorable gains. This increase is also seen in the uniform capacity of the network, which increases by 94% to $C_u^b = 0.140$.

### *C. Multihop Routing with Spatial Reuse*

We now consider a network with both multihop routing and spatial reuse. In this case a network of size $n$ has

$$N^c = \sum_{i=1}^{\lfloor \frac{N}{2} \rfloor} \frac{n(n-1)\dots(n-2i+1)}{i!}(n-1)^i \qquad (6)$$

achievable rate matrices $R_i^c, i = 1, \dots, N_c$ and its capacity region will be

$$C^c = Co\{R_i^c, i = 1, \dots, N_c\} \cap \mathcal{P}_n. \qquad (7)$$

In Fig. 4 we have drawn a slice of the capacity region ($C^c$ corresponding to line (c)) along the zero background rate plane $R_{ij} = 0, \{ij\} \neq \{13\}, \{45\}$. We note that the slice is no longer a straight line, as the network can now use spatial separation to maintain multiple active transmissions, and at any time instant it is possible that more than one streams are serviced. The introduction of spatial reuse increases uniform capacity by 21% to $C_u^c = 0.169$, even for this small network of five nodes.

### *D. Power Control*

We have so far assumed that nodes either transmit at their maximum power or remain silent. If we relax this condition and allow each node to transmit at different power levels below the maximum power, then we increase the set of achievable rate matrices and thereby the capacity region. Since there are uncountably many possible power levels, we restrict our attention to power control strategies where node $i$ transmits at one of $p$ possible power levels: $\{\frac{1}{p}P^i_{max}, \frac{2}{p}P^i_{max}, \dots, P_i^{max}\}$. The network will then have a set of

$$N_d = \sum_{i=1}^{\lfloor \frac{N}{2} \rfloor} \frac{n(n-1)\dots(n-2i+1)}{i!}(n-1)^i p^i \qquad (8)$$

achievable rate matrices, resulting in the capacity region

$$C^d = Co\{R_i^d, i = 1, \dots, N_d\} \cap \mathcal{P}_n. \qquad (9)$$

As in the previous cases, the achievable rate matrices are straightforward to compute.

In line (d) of Fig. 4 we have drawn a slice of the capacity region $C^d$ along the zero background rate plane $R_{ij} = 0, \{ij\} \neq \{13\}, \{45\}$ for two-level power control ($p = 2$). We observe that this simple power control does not significantly change the capacity region. Moreover, the uniform capacity with this power control changes less than 1%, to $C_u^c = 0.170$. Although more levels of power control might increase capacity somewhat, it appears that such gains would be negligible. This result is consistent with other results on variable-rate transmission with power control, which indicate that if the variable rate transmission is used to adjust to the link SINR, additional power control does not significantly improve performance [3].

### *E. Successive Interference Cancellation*

The rate restriction $R_{ij} \leq f(\gamma_{ij})$ assumes that under transmission schemes with many simultaneous transmissions, each node decodes only its intended signal, and treat all other signals as noise. However, under successive interference cancellation (SIC) nodes may decode some signals intended for other nodes first, subtract out this interference, and then decode their own signals. This strategy may cause a node to restrict the transmission rate of an interfering nodes, since the given node must be able to decode the interfering signal. However, this restriction is balanced by the fact that the given node's rate will increase due to the removal of interference. For example, consider a four node network where node $A_1$ sends to $A_2$ and node $A_4$ sends to $A_3$. Then node $A_1$'s signal will interfere with node $A_3$'s reception, and node $A_4$'s signal will interfere with node $A_2$'s reception. In this scenario node $A_2$ could decode node $A_1$'s signal and treat node $A_4$'s signal as noise, or node $A_2$ could first decode and remove the signal from node $A_4$ and then decode the desired signal from node $A_1$. This second decoding strategy will impose an additional constraint on the transmission rate of node $A_4$, since this rate must be commensurate with the link SINR and decoding strategy between nodes $A_4$ and $A_3$ as well as the link SINR and decoding strategy between nodes $A_4$ and $A_2$ (since node $A_2$ as well as node $A_3$ must be able to decode $A_4$'s signal). Node $A_3$ can decode in a similar manner, either treating node $A_1$'s transmission as noise or first subtracting it off before decoding the desired signal from node $A_4$. We see therefore that SIC significantly increases the set of achievable rate matrices. It can be shown that if a node's decoding strategy includes SIC and power control with $p$ possible power levels, there are

$$N_e = \sum_{i=1}^{\lfloor \frac{N}{2} \rfloor} \frac{n(n-1)\dots(n-2i+1)}{i!}(n-1)^i p^i G(i) \qquad (10)$$

different achievable rate matrices, where $G(i) = 1 * 2 * \ldots * (i-1) + 2 * \ldots * (i-1) + \ldots + (i-1)$. The capacity region is then

$$C^e = Co\{R_i^e, i = 1, \ldots, N_e\} \cap \mathcal{P}_n. \quad (11)$$

Line (e) of Fig. 4 shows a slice of the capacity region $C^e$ along the zero background rate plane $R_{ij} = 0, \{ij\} \neq \{13\}, \{45\}$. This slice indicates that SIC significantly increases the capacity region even without power control. Moreover, the uniform capacity increased by 23% from the previous case, to $C_u^e = 0.21$.

We summarize the uniform capacity of the various transceiver capabilities we have considered in the following table. We see that by far the largest capacity gain is obtained by allowing intermediate nodes to forward packets using multihop routing. Spatial reuse and successive interference cancellation also contribute significant gains. However, adding power control on top of the underlying variable rate transmission leads to negligible gain.

| | |
|---|---|
| Single hop routing, no spatial reuse | 0.072 |
| Multihop routing, no spatial reuse | 0.140 |
| Multihop routing and spatial reuse | 0.169 |
| Adding power control | 0.170 |
| Adding SIC | 0.21 |

TABLE I

Uniform capacity of the five node ad-hoc network.

### F. Nonzero Background Rates

In Fig. 4 we show capacity region slices under different transceiver capabilities assuming the background rates for all node pairs other than $(1,3)$ and $(4,5)$ are zero. In this section we show the impact on the capacity region slice of nonzero background rates for these other node pairs. Specifically, in Fig. 5 we plot two slices of the capacity regions $C^e$ and $C^d$, one along the zero background rate plane $R_{ij} = 0, \{ij\} \neq \{13\}, \{45\}$ (lines (e) and (d)), and one along the nonzero background rate plane $R_{ij} = 0.1, \{ij\} \neq \{13\}, \{45\}$ (lines (e') and (d')). The (e') and (d') slices correspond to cases where node pairs other than $(1,3)$ and $(4,5)$ have a background rate of 0.1. We note that the shape of the slices changes as the background rate increases above zero. More importantly, we note that adding a background rate of 0.1 for all 18 other node pairs does not decrease the capacity region as much as the sum of the total additional background rate $(1.8 = 18 \times 0.1)$.

### G. Multihop Cellular

We have also determined the uniform capacity for a multihop cellular system with nine nodes transmitting to a centrally located base station. Details can be found in [5]. Table II gives the uniform capacity for this multihop cellular system under various transceiver capabilities. We see that the same trends given above for ad hoc networks also apply to multihop cellular systems. In particular, the uniform capacity increases by more than an order of magnitude under multihop routing.

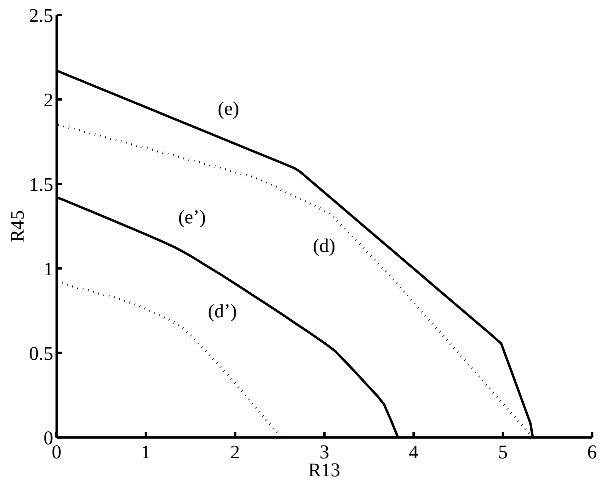


Fig. 5. Comparison of capacity regions $C^d$ and $C^e$ under a background rate of zero (lines e and d) and a background rate of 0.1 (lines e' and d'). Axis units are in bps/Hz.

| | |
|---|---|
| Single hop routing, no spatial reuse | 0.023 |
| Multihop routing, no spatial reuse | 0.68 |
| Multihop routing and spatial reuse | 0.75 |
| Adding power control | 0.77 |

TABLE II

Uniform capacity of a nine node cellular system.

## V. Conclusions and Future Work

We have developed a mathematical framework for finding the capacity region of ad hoc networks and multihop cellular systems under time division routing and variable rate transmission. We then applied this framework to determine the capacity gain that can be obtained using various sophisticated transceiver capabilities. We show that multihop routing greatly increases capacity. Significant gains are also realized with spatial reuse and successive interference cancellation, but gains from power control are marginal. We have verified these finding for a wide range of network topologies and fading scenarios [5]. We are currently investigating the impact of time-varying multipath fading on capacity, as well as the optimal routing strategies that achieve the boundary points on the network capacity region.

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