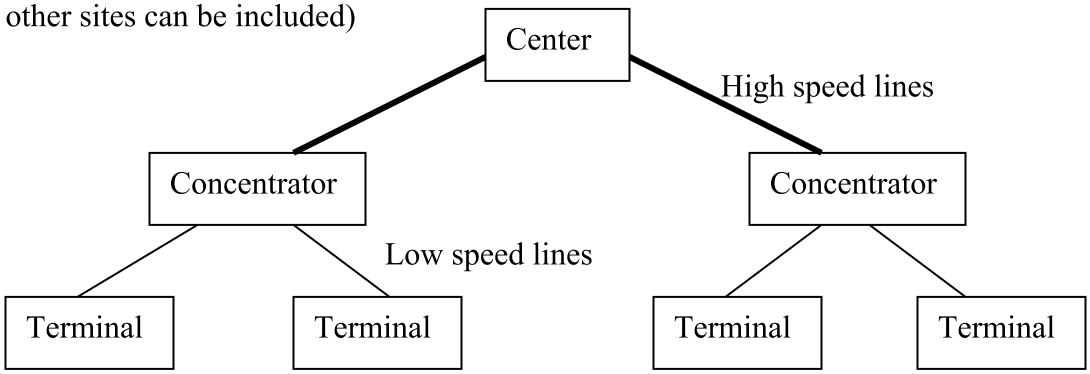
Chapter 5 *Topology design and analysis*

Topics covered:
Topology design. Network design algorithms. Terminal assignment. Concentrator location. Traffic flow analysis and performance evaluation. Network reliability. Network simulation.

5.1 Topology design

5.1.1 Centralized Network design

- **Centralized network:** is where all communication is to and from a single central site.
- The "central site" is capable of making routing decisions.
  → Tree topology provides only one path through the center (For reliability, lines between other sites can be included)



- Three different problems:
  - **Multipoint line topology:** selection of links connecting terminals to concentrators or directly to the center.
  - **Terminal assignment:** association of terminals with specific concentrators.
  - **Concentrator location:** deciding where to place concentrators, and whether or not to use them at all.

5.1.2 Finding Trees in Graphs

- Used to design and analyze networks.
- Connect a number of nodes to a central node:
  - **Node:** Hub, Switch, Router, etc.
  - **Central node:** backbone

- A tree is a graph with no loops, with only one path between any pair of nodes.

- Trees are minimal networks: provide connectivity without any unnecessary additional links:

    - Minimally reliable and robust

    - Networks are more highly connected (but design starts with a tree)

5.1.2.1 Tree Traversals

- Visit all nodes in a tree: edges are traversed twice.

- First, identify a node as the root

- Assume the tree is directed (outward from the root)

- Two algorithms:

    - BFS (Breadth First Search):
        - Nodes closest to root are visited first
        - Implemented using a queue (FIFO)

    - DFS (Depth First Search):
        - Visits an unvisited neighbor of the node just visited.
        - Implemented using a stack (LIFO)

- Both traversals (BFS and DFS) can be preorder traversals (i.e., visit nodes then successors) or post-order traversals (i.e., successors visited first).

- Traversal is generalized to undirected graphs by keeping track of which nodes were visited, and not visiting them again.

- In a BFS or DFS traversal, edges visited form a tree (if the graph is connected) or a forest (if the graph is not connected).

5.1.2.2 Minimum Spanning Trees (MSTs)

- Use DFS to find a spanning tree in a graph, if one exists
  → Arbitrary tree

- Useful to find the "best" tree
  → Minimum Spanning Tree (e.g., minimum total length. Where length is: distance, cost, function(delay), function(reliability), etc.)

- If the graph is not connected → minimum spanning forest
  - For ***n*** nodes, ***c*** components, and ***e*** edges, we have: $n = c + e$
  - For a tree, $c = 1$.

- DFS will not, in general, find the spanning tree with minimum total cost.

5.1.2.2.1 The Greedy Algorithm

- At each stage, select the shortest edge possible.

- May not find a feasible solution when one exists.

- Efficient and simple to implement → widely used.

- Basis of other more complex and effective algorithms.

- In the case of MST, the greedy algorithm guarantees both optimality and reasonable computational complexity.

  - Start with empty solution ***s***
  - While elements exist
    - Find ***e***, the best element not yet considered
    - If adding ***e*** to ***s*** is feasible, add it; if not, discard it.

5.1.2.2.2 Kruskal's Algorithm

- A greedy algorithm for finding MSTs.

- Sort the edges, shortest first and then include all edges which do not form cycles with the edges previously selected.

- n: number of nodes

- **Algorithm:**

  1. Sort all edges in ascending order (least cost first)

  2. Select among edges not yet selected, the one with the least cost.

  3. Add it if it does not create a cycle.

  4. If the number of edges selected < ***n-1***, go to step (2), otherwise exit (tree completed)

- **Complexity:**

  O(m log m), m = number of edges

5.1.2.2.3 Prim's Algorithm

- A greedy algorithm for finding MSTs.
- Advantageous if the network is dense.
- Well suited to parallel implementation.
- **Algorithm:**
  1. Start with one node (root node) in the tree
  2. Find node ***i***, not in the tree, which is the nearest to the tree.
  3. Add node ***i*** to the tree and edge ***e*** connecting ***i*** to the tree.
- **Complexity:**

  $O(n^2)$

5.1.2.2.4 Comparison of the Complexity of Kruskal's and Prim's Algorithms

- If the network is dense → $m \sim O(n^2)$ → Prim's algorithm is faster
- If the network is not dense → $m \sim O(n)$ → Kruskal' algorithm is faster

5.1.3 Constrained/Capacitated MST (CMST)

- The algorithms presented in the previous subsections are called "unconstrained MST algorithms"
  - No constraint on flow of information
  - No constraint on the number of ports at each node.
- For the unconstrained spanning tree problem, all these algorithms produce a minimum cost spanning tree.
- **CMST Problem:** Given a central node N_0 and a set of other nodes (N_1, N_2, …, N_n), as et of weights (W_1, W_2, …, W_n) for each node, the capacity of a link, W_{max}, and a cost matrix C_{ij} = Cost(i,j), find a set of trees T_1, T_2, …, T_k such that each N_i belongs to exactly one T_j and each T_j contains N_0.

- **Objective:** Find a tree of minimum cost and which satisfies a number of constraints such as:
  - Flow over a link
  - Number of ports

- **Example:**
  - Assume we are allowed to use one type of links only that can accommodate a maximum of 5 units of flow per unit time.

  - Assume that the flow generated from each node to the central node (N_1) is as follows: f_1=0, f_2=2, f_3=3, f_4=2, f_5=1 (in units/time_unit).

  - Effect of constraint violation:
    - As a result, a queue will build up since node 3 can service only 5 units/time_unit. If node 3 does not have a large queue to accommodate all coming units, some units will be lost. So, these units are retransmitted, which may cause the network to collapse.

- The CSMT problem is NP-hard (i.e., cannot be solved in polynomial time)
  → Resort to heuristics (approximate algorithms)

- These heuristics will attempt to find a good feasible solution, not necessarily the best, that:
  - Minimizes the cost
  - Satisfies all the constraints

- Well-known heuristics:
  - Kruskal
  - Prim
  - Esau-Williams

5.1.3.1 Kruskal's Algorithm for CMST

Algorithm:

1. Sort all edges in ascending order, $\boldsymbol{e} \leftarrow \mathbf{0}$.
2. Select edge with minimum cost (from edges not yet selected)
3. If it satisfies constraints (i.e., no cycles and no violation of flows on links)
   - Then: add it to the tree, $\boldsymbol{e} \leftarrow \boldsymbol{e} + \mathbf{1}$
   - Else: go to step (**2**)
4. If (**e = n – 1**) then exit, else go to step (**2**)

Example:

Given a network with five nodes, labelled **1** to **5**, and characterized by the following cost matrix:

	1	**2**	**3**	**4**	**5**
1	-	3	3	5	10
2	3	-	6	4	8
3	3	6	-	3	5
4	5	4	3	-	7
5	10	8	5	7	-

Node 1 is the central backbone node.

f_{max}=5, f_1=0, f_2=2, f_3=3, f_4=2, f_5=1.

5.1.3.2 Prim's Algorithm for CMST

Algorithm:

1. Start with one node (root node) in the tree.

2. Find node ***i***, not in the tree, which is the nearest to the tree

3. Add node ***i*** to the tree and edge ***e*** connecting ***i*** to the tree if it satisfies constraints (i.e., no violation of flows on links)

Example:

5.1.3.3 Esau-Williams Algorithm for CMST

Node **1** is the central node.

t_{ij}: is the tradeoff of connecting i to j or i directly to the root.

- If ($t_{ij} < 0$) → better to connect i to j
- If ($t_{ij} \geq 0$) → better to connect i directly to the root

Algorithm:

1. Compute $t_{ij} = c_{ij} - c_{i1}$ for all i, j ≠ 1.

2. Select the link (m,n) such that: $t_{mn} = \min(t_{ij})$

3. If $t_{mn} < 0$, then go to step (**4**)
   Else (i.e., $t_{mn} \geq 0$ for all m,n), connect to node 1 all nodes not connected yet, and **exit**.

4. Verify constraints (e.g., does not exceed the maximum weight)
   - If satisfied go to step (**5**)
   - Else: $t_{mn} = \infty$ and $t_{nm} = \infty$, go to step (**2**)

5. Add link (m, n), remove link (m, 1) and update t_{ij} to indicate that **m** is now connected to **n**.
   → $t_{mn} = \infty$ and $t_{nm} = \infty$
   → if $t_{mj} \neq \infty$, $t_{mj} = c_{mj} - \min(c_{k1})$ [$k \in C_m$, where C_i = component containing node **i**]

6. Go to step (**2**)

Example:

Given a network with five nodes, labelled **1** to **5**, and characterized by the following cost matrix:

	1	**2**	**3**	**4**	**5**
1	-	3	3	5	10
2	3	-	6	4	8
3	3	6	-	3	5
4	5	4	3	-	7
5	10	8	5	7	-

W_{max}=5, W_1=0, W_2=2, W_3=3, W_4=2, W_5=1.

5.1.4 Terminal Assignment

5.1.4.1 Problem Statement

- **Terminal Assignment:** Association of terminals with specific concentrators.

Given:

T terminals (stations) $i = 1, 2, \dots, T$

C Concentrators (hubs/switches) $j = 1, 2, \dots, C$

$\mathbf{C_{ij}}$: cost of connecting terminal i to concentrator j

$\mathbf{W_j}$: capacity of concentrator j

Assume that terminal i requires W_i units of a concentrator capacity.

Assume that the cost of all concentrators is the same.

- $x_{ij} = 1$; if terminal i is assigned to concentrator j.
- $x_{ij} = 0$; otherwise.

Objective:

5.1.4.2 Augmenting path algorithm

Based on the following observations:

1. Ideally, every terminal is assigned to the nearest concentrator.

2. Terminals on concentrators that are full are moved only to make room for another terminal that would cause a higher overall cost if assigned to another concentrator.

3. An optimal partial solution with ***k+1*** terminals can be found by finding the least expensive way of adding the **(k+1)th** terminal to the ***k*** terminal solution.

Assignment problem:

Given a cost matrix:
- One column per concentrator
- One row per terminal

Assume that:
- Weight of each terminal is 1 (i.e., each terminal consumes exactly one unit of concentrator capacity)
- A concentrator has a capacity of W terminals (e.g., number of ports)

A feasible solution exists iff $T \leq W * C$

Algorithm:

1. Initially, try to associate each terminal to its nearest concentrator

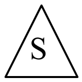
2. If successful in assigning all terminals without violating capacity constraints, then stop (i.e., an optimal solution is found)

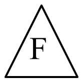
3. Else,
   - **Repeat**
     1. Build a compressed auxiliary graph
     2. Find an optimal augmentation
   - **Until** all terminals are assigned

Building a compressed auxiliary graph:

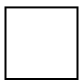
U: set of unassociated terminals

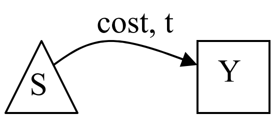
T(Y): set of terminals associated with Y



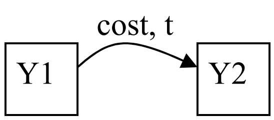
and


are the start and finish of all augmenting paths

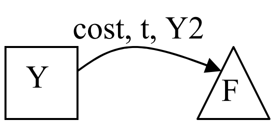

represents a fully loaded concentrator



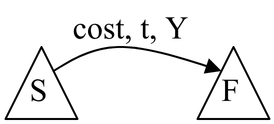
Assign t to a fully loaded concentrator Y. (t Є U)
cost = c(tY) = min c(xY) for x Є U



Move t from a fully loaded concentrator Y1 to another fully loaded concentrator Y2. (t Є T(Y1))
cost = c(tY2) – c(tY1)= min (c(xY2) – c(xY1)) for x Є t(Y1)



Move t from Y to a concentrator Y2 with spare capacity. (t Є T(Y))
cost = c(tY2) – c(tY)= min (c(xY2) – c(xY)) for x Є t(Y)



Assign t to a concentrator Y with spare capacity. (t Є U)
cost = c(tY) = min c(xY) for x Є U

Example:

5.2 *Traffic Flow Analysis and Performance Evaluation*

5.2.1 Traffic Flow Analysis Objective

- Estimate:
  - Delay
  - Utilization of resources (links)

- Traffic flow across a network depends on:
  - Topology
  - Routing
  - Traffic workload (from all traffic sources)

- Desirable topology and routing are associated with:
  - Low delays
  - Reasonable link utilization (no bottlenecks)

- Assumptions:
  - Topology is fixed and stable
  - Links and routers are 100% reliable
  - Processing time at the routers is negligible
  - Capacity of all links is given $C = [C_i]$ (in bps [bits per second])
  - Traffic workload is given $\Gamma = [\gamma_{jk}]$ (in pps [packets per second])
  - Routing is given $R = [r_{jk}]$
  - Average packet size is $1/\mu$ bits.

5.2.2 Queuing Analysis

Projections of performance are made on the basis of either:

- The existing load information, or
- The estimated load for the new environment.

Approaches that could be used:

- Do an after-the-fact analysis based on actual values

- Make a simple projection from existing to expected environment

- Develop an analytic model based on queuing theory

- Program and run a simulation tool

5.2.2.1 Queuing Models

- The notation **X/Y/N** is used for queuing models.
  - X = distribution of the interarrival times
  - Y = distribution of service times
  - N = number of servers

- The most common distributions are:
  - G = general independent arrivals or service times
  - M = negative exponential distribution
  - D = deterministic arrivals or fixed length service

- Example: **M/M/1**

- **Single-server queues**

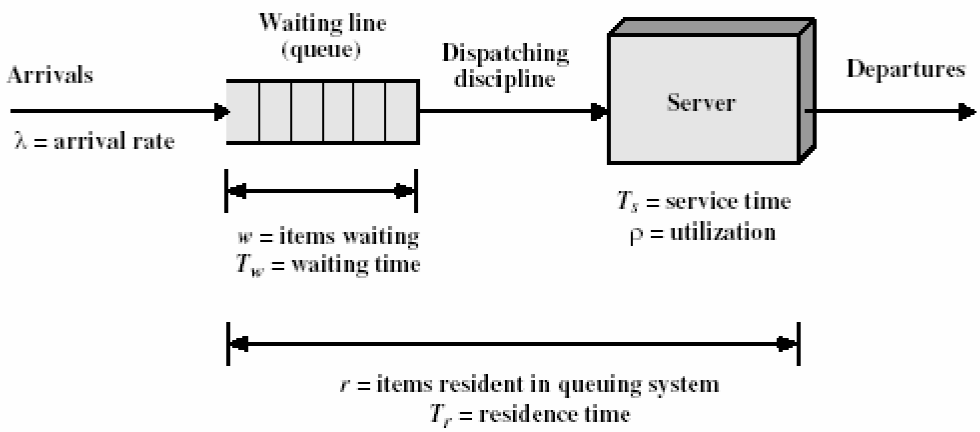


Figure 5.1: Queuing System Structure and Parameters for Single-Server Queue (Taken from "Queuing Analysis" by William Stallings)

<u>**Queue parameters:**</u>

λ = arrival rate; mean number of arrivals per second

$\mathbf{T_s}$ = mean service time for each arrival; amount of time being served, not counting time waiting in the queue

ρ = utilization; fraction of time facility (server or servers) is busy

r = mean number of items in system, waiting and being served (residence time)

$\mathbf{T_r}$ = mean time an item spends in system (residence time)

w = mean number of items waiting to be served

$\mathbf{T_w}$ = mean waiting time (including items that have to wait and items with waiting time = 0)

Basic Queuing relationship:

- $\rho = \lambda * T_s$
- $r = w + \rho$
- $\lambda_{max} = 1/T_s$
- **$r = \lambda * T_r$ (Little's formula)**
- $w = \lambda * T_w$ (Little's formula)
- $T_r = T_w + T_s$
- **$r = \rho/(1-\rho)$**

- **Multiserver queue**

  N = number of servers

  ρ = utilization of each server

  $N\rho$ = utilization of all servers (= $\lambda * T_s$)

5.2.2.2 M/M/1 Queues – Application to Networks

- Each link is seen as a service station servicing packets.

  λ_i = arrival rate (in pps); mean number of packets that arrive to link i in one second.

  μC_i = average service rate (in pps); mean number of packets that will get out of the link i in one second. (= $1/T_s$)

- Utilization of link i is:

$\rho_i =$

- Stability condition of a network is:

- The external workload offered to the network is:

γ =

Where:

γ = total workload in packets per second

γ_{jk} = workload between source j and destination k

N = total number of sources and destinations

- The internal workload on link i is:

λ_i =

Where:

γ_{jk} = workload between source j and destination k

Π_{jk} = path followed by packets to go from source j and destination k

- The total internal workload is:

λ =

Where:

λ = total load on all of the links in the network

λ_i = load on link i

L = total number of links

- The average length for all paths is given by:

E[number of links in a path] = λ/γ

- The average number of items waiting and being served for link i is:

r_i =

- The number of packets waiting and being served in the network can be expressed as:

$$\gamma^*T =$$

Where:

T = average delay experienced by a packet through the network.

$$T =$$

- T_{ri} is the residence time at each queue. If we assume that each queue can be treated as an independent M/M/1 model (Jackson's Theorem), then:

$$T_{ri} =$$

Where: T_{si} is the service time for link i

$$T_{si} =$$

Where:

- C_i = data rate on the link (in bps)
- $M = 1/\mu$ = average packet length in bits

Example:

5.3 *Network Reliability*

5.3.1 Introduction

- A network model is a set of facilities. A facility could be a device or a link.

- A network must contain some slack to allow it to function even if some of its facilities have failed.

- Any network facility is either:

    - Working (**p**)

    - Failing (q = **1-p**)

- **MTBF:** Mean Time Between Failures (**f**).

- **MTTR:** Mean Time To Repair (**r**)

- For any facility i, we'll know from measurements of f_i and r_i:

    P_i = Prob [facility i is working] =

    Therefore:

- We assume that all facilities are independent:

    P(ij) = Prob[facility i and facility j are working] =

    P(i|j) = Prob[facility i or facility j is working] =

- Simplest measure of network reliability:

    $P_c(G)$ = Prob[Network is connected]

    Where: **c** stands for the connectivity of the network, and
    G stands for the graph representing the network

$P_c(G)$ = Prob[All nodes are working and there is a spanning tree of working links]

$P_c(G)$ =

- Since enumerating all trees in G requires an exponential amount of effort, **$P_c(G)$** is very difficult (if not impossible) to compute.

  → We seek simpler measures of network reliability.

5.3.2 Reliability of Tree Networks

- A typical enterprise/campus network includes trees:

- Given a tree T:

  $P_c(T)$ = Prob[A tree network, T, being connected]
  = Prob[All components (nodes and links) are working]

  $P_c(T)$ =

- **$P_c(T)$** can also be computed recursively:

  $P_c(T)$ =

  Where: **T-i** is the tree T without node i, and
  j is the link between node i and the rest of the tree

- Given a particular tree with root r:

  $P_c(i)$ = Prob[node i can communicate with root r]

  $P_c(i)$ =

  Where: **j** is the link between nodes i and k, and
  k is the predecessor of node i

  $P_c(r)$ =

- The expected number of nodes communicating with the root r is:

  E(r) =

- This expression can be computed efficiently for any node as follows:

  E(i) = the expected number of nodes communicating with the node i

  E(i) =

- If node i is a leaf, then:

  E(i) =

Example:

- The expected number of node pairs communicating through the root r is:

EPR(r) =

Example:

5.4 References

1. "Telecommunications Network Design Algorithms" by Aaron Kershenbaum, 1993

2. "Queuing Analysis" by William Stalling, 2000