ve Computer Algorithms:
and their applications in engineering

Sadiq M. Sait, Ph.D

sadiq@kfupm.edu.sa

Department of Computer Engineering

King Fahd University of Petroleum and Minerals

Dhahran, Saudi Arabia

Talk outline

- The various general iterative non-deterministic algorithms for combinatorial optimization.
  - » Search, examples of hard problems
  - » SA, TS, GA, SimE and StocE
  - » Their background and operation
  - » Parameters
  - » Differences
  - » Applications
  - » Some research problems and related issues: **Convergence, parallelization, hybridization, fuzzification, etc.**

Talk outline

Based on the text book by Sadiq M. Sait and Habib Youssef entitled: Iterative Computer Algorithms: and their applications in engineering to be published by IEEE Computer Society Press, 1999.

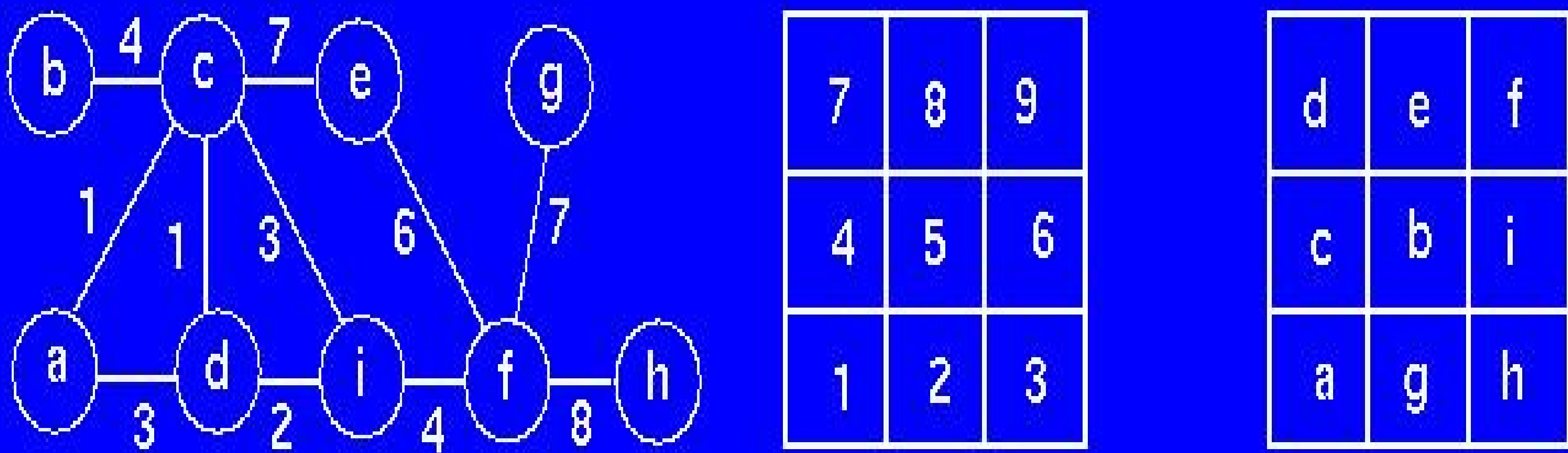
Terminology

- **Combinatorics**: Does a particular arrangement exist?
- **Combinatorial optimization**: Concerned with the determination of an **optimal** arrangement or order
- Hard problems: NP & NP complete.
- **Examples**: QAP, Task scheduling, shortest path, TSP, partitioning (graphs, sets, etc), HCP, VCP, Topology Design, Facility location, **etc**
- **Optimization methods**:
  - » Constructive & Iterative
  - » Aim at improving a certain cost function

Examples

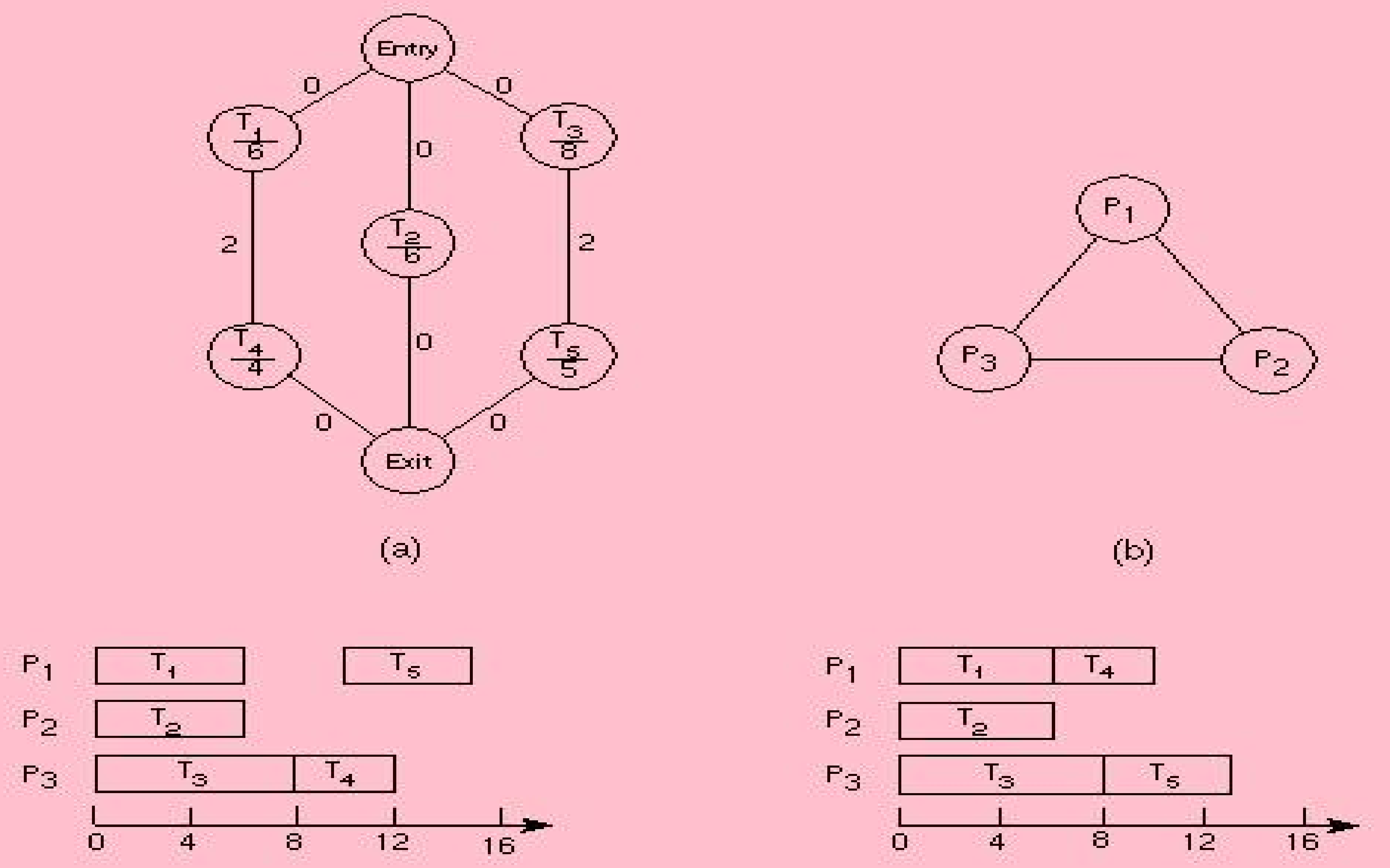
- QAP: Required to assign M modules to L locations (L>=M), in order to minimize a certain objective
  - » wire-length, timing, dissipation, area
  - » Number of solutions is given by **L!**
- Task Scheduling: Given a set of tasks (n) represented by an acyclic DAG, and a set of inter-connected processors (m), it is required to assign the tasks to processors in order to minimize the ñtime to completionò of the tasks.
  - » **Number of solutions given by** M_N

QAP



(a) (b) (c)

Scheduling


Entry
0
0
0
T_1 / 6
T_2 / 6
T_3 / 8
2
2
0
T_4 / 4
T_5 / 5
0
0
Exit
(a)
P_1
P_2
P_3
(b)
P_1 T_1 T_5
P_2 T_2
P_3 T_3 T_4
0 4 8 12 16
P_1 T_1 T_4
P_2 T_2
P_3 T_3 T_5
0 4 8 12 16

Purpose

- To motivate application of iterative search heuristics to hard practical engineering problems.
- To understand some of the underlying principles, parameters, and operators, of these modern heuristics.

Terminology

- **Search space**
- **Move** (perturb function)
- **Neighborhood**
- **Non-deterministic algorithms**
- **Optimal/Minimal solution**

Simulated Annealing

- Most popular and well developed technique
- Inspired by the cooling of metals
- Based on the Metropolis experiment
- Accepts bad moves with a probability that is a decreasing function of temperature

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$$\boxed{\mathrm{pr(accept)} = \exp(-\Delta E)/KT}$$

- E represents energy (cost)

The Basic Algorithm

- Start with
  - » a random solution
  - » a reasonably high value of T (problem dependent)
- Call the Metropolis function
- Update parameters
  - » Decrease temperature ($T*\alpha$)
  - » Increase number of iterations in loop, i.e., M, ($M*\beta$)
- Keep doing so until freezing, or, out of time

Metropolis Loop

- Begin Loop: Generate a neighbor solution
- Compute difference in cost between old and neighboring solution
- If cost<0 then accept, else accept only if

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$$Random < e^{-\Delta Cost/T}$$

- Decrement M, repeat loop until M=0

Parameters

- Also known as the cooling schedule:
  - » Comprises
    - ï choice of proper values of initial temperature **To**
    - ï decrement factor α**<1**
    - ï parameter β**>1**
    - ï M (how many times the Metropolis loop is executed)
    - ï stopping criterion

Characteristics

- Given enough time it will converge to an optimal state
- Very time consuming
- During initial iterations, behaves like a random walk algorithm, during later iterations it behaves like a greedy algorithm, a weakness
- Very easy to implement
- Parallel implementations available

Requirements

- **Requirements:**
  - » A representation of the state
  - » A cost function
  - » A neighbor function
  - » A cooling schedule
- **Time consuming steps**:
  - » Computation of cost due to move must be done efficiently (estimates of costs are used)
  - » Neighbor function may also be time consuming

Applications

- Has been successfully applied to a large number of combinatorial optimization problems in
  - » science
  - » engineering
  - » medicine
  - » business
  - » etc

Genetic Algorithms

- Introduced by **John Holland** and his colleagues
- Inspired by Darwinian theory of evolution
- Emulates the natural process of evolution
- Based on theory of natural selection
  - » that assumes that individuals with certain characteristics are better able to survive
- Operate on a set of solutions (termed population)
- Each individual of the population is an encoded string (termed chromosome)

Genetic Algorithms

- Strings (**chromosomes**) represent points in the search space
- Each iteration is referred to as **generation**
- New sets of strings called **offsprings** are created in each generation by **mating**
- Cost function is translated to a fitness function
- From the pool of parents and offsprings, candidates for the next generation are selected based on their **fitness**

Requirements

- To represent solutions as strings of symbols or chromosomes
- **Operators:** To operate on parent chromosomes to generate offsprings (crossover, mutation, inversion)
- Mechanism for **choice of parents** for mating
- A **selection** mechanism
- A mechanism to efficiently compute the fitness

Operators

- Crossover: The main genetic operator
  - » Types: Simple, Permutation based (such as Order, PMX, Cyclic), etc.
- Mutation: To introduce random changes
- Inversion: Not so much used in applications

Crossover

- Example:

Chromosome for the scheduling problem of eight tasks, to be assigned to three processors

[1 2 3 1 3 1 1 2], [1 2 3 3 1 3 2 2] (index of the array refers to the task, and the value the processor it is assigned to)

Simple Crossover

- Cut and catenate
- Let the crossover point be after task 5, as shown. Then the offspring created by the simple crossover will be as follows:
- Chromosome for the scheduling problem of 8 tasks to be assigned to three processors

Parent #1: [1 2 3 1 3 | 1 1 2]

Parent #2: [1 2 3 3 1 | 3 2 2]

Offspring generated = [1 2 3 1 3 3 2 2]

Permutation Crossovers

- Consider the linear placement problem of 8 modules (a, b, ...,g, h,) to 8 slots.

Parent #1: [**h d a e b** | **c g f**]

Parent #2: [**d b c g a** | **f h e**]

Offspring generated = [**h d a e b f h e**]

The above **offspring is not a valid solution** since modules **e** and **h** are assigned to more than one location, and modules **c**, and **g** are lost

Order Crossovers

Parent #1: [h d a e b | c g f]

Parent #2: [d b f c a | g h e]

Offspring generated = [h d a e b | f c g]

The above offspring represents a valid solution

Mutation

- Similar to the perturb function used in simulated annealing.
- The idea is to produce incremental random changes in the offsprings
- Important, because crossover is only an inheritance mechanism, and offsprings cannot inherit characteristics which are not in any member of the population.
- Size of the population is generally small.

Mutation

- Example: Consider the population below

s1= 0 1 1 0 0 1

s2= 1 0 1 1 0 0

s3= 1 1 0 1 0 1

s4= 1 1 1 0 0 0

Observe that the second last gene in all chromosomes is always ñ0ò, and the offsprings generated by simple crossover will never get a 1.

Decisions to be made

- What is an efficient chromosomal representation?
- Probability of crossover (P_c)? Generally close to 1
- Probability of mutation (P_m) kept very very small, 1% - 5% (**Schema theorem**)
- Type of crossover? and, what mutation scheme?
- Size of the population? How to construct the initial population?
- What selection mechanism to use, and the generation gap (i.e., what percentage of population to be replaced during each generation?)

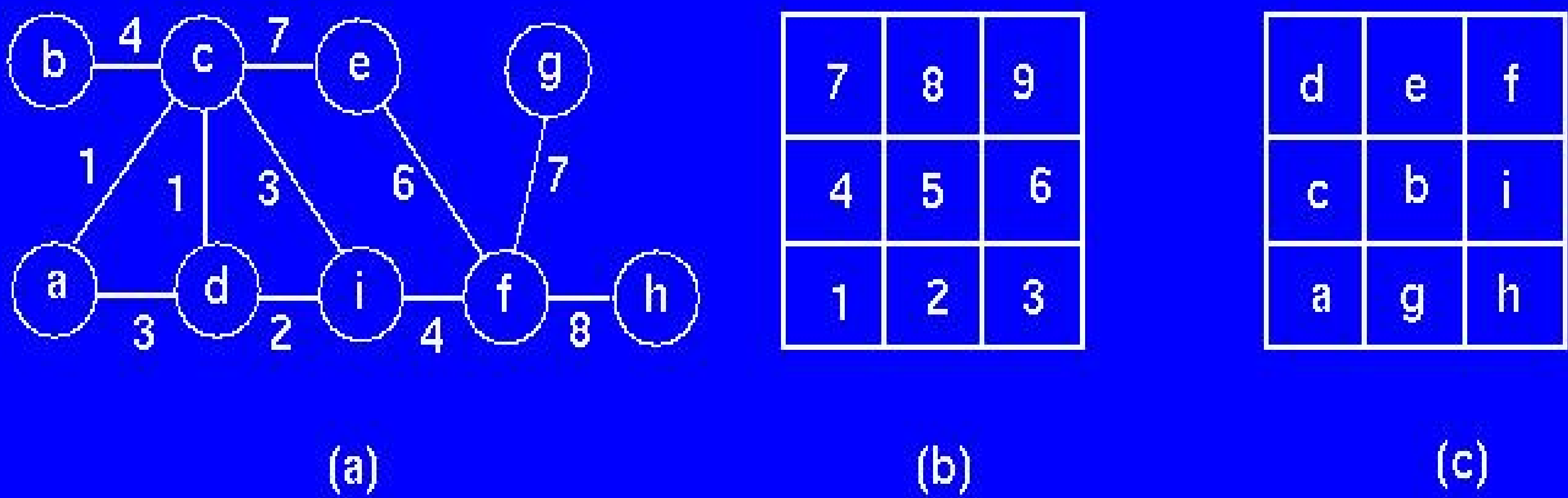
Problems

- Mapping cost function to fitness
- Premature convergence can occur. Scaling methods are proposed to avoid this
- Requires more memory and time
- Several parameters, and can be very hard to tune

Applications

- Classical hard problems (TSP, QAP, Knapsack, clustering, N-Queens problem, the Steiner tree problem, Topology Design, etc.,)
- Problems in high-level synthesis and VLSI physical design,
- Others such as:
  - » Scheduling,
  - » Power systems, telecommunications (maximal distance codes, telecom NW design), etc.
  - » Fuzzy control (GAs used to identify fuzzy rule set)

Example



aghcbidef is a possible chromosome

Some Variations

- 2-D chromosomes
- Gray versus Binary encoding
- Multi-objective optimization with GAs
- Constant versus dynamically decreasing population
- Niches, crowding and speciation
- Scaling
- etc

Tabu Search

- Introduced by **Fred Glover**
- Generalization of Local Search
- At each step, the local neighborhood of the current solution is explored and the best solution is selected as the next solution
- This best neighbor solution is accepted even if it is worse than the current solution (hill climbing)

Central Idea

- Exploitation of memory structures
- Short term memory
  - » Tabu list
  - » Aspiration criterion
- Intermediate memory for intensification
- Long term memory for diversification

Basic Short-Term TS

1. Start with an initial feasible solution
2. Initialize Tabu list and aspiration level
3. Generate a **subset** of neighborhood and find the best solution from the generated ones
4. If move in not in **tabu list** then accept

   else

   If move satisfies **aspiration criterion** then accept
5. Repeat above 2 steps until **terminating condition**

Intensification/Diversification

- **Intensification**: Intermediate term memory is used to target a specific region in the space and search around it thoroughly
- **Diversification**: Long term memory is used to store information such as frequency of a particular move, etc., to take search into unvisited regions.

Implementation related issues

- Size of candidate list?
- Size of tabu list?
- What aspiration criterion to use?
- Fixed or dynamic tabu list?
- What intensification strategy?
- What diversification scheme to use?
- And several others

Tabu list and Move Attributes

- Moves or attributes of moves are stored in tabu lists (storing entire solutions is expensive)
- Tabu list size is generally small (short-term)
- Tabu list size may be fixed or changed dynamically
- Possible data structures are queues and arrays

Related Issues

- Design of evaluator functions
- Candidate list strategies
- Target analysis
- Strategic oscillation
- Path relinking
- Parallel implementation
- Convergence aspects
- Applications (again several)

Simulated Evolution

- Like GAs, also mimics biological evolution
- Each element of the solution is thought of as an individual with some fitness (goodness)
- The basic procedure consists of
  - » evaluation
  - » selection, and,
  - » allocation
- Based on compound moves

Evaluation

- Goodness is defined as the **ratio** of optimal cost to the actual cost

$$\forall i, g_i = \frac{\mathrm{O}_{\mathrm{i}}}{\mathrm{C}_{\mathrm{i}}}$$

- Selection is based on the goodness of the element of a solution
- The optimal cost is determined only once
- The actual cost of some individuals changes with each iteration

Selection

- Selection: The higher the goodness value, higher the chance of the module staying in its current location $$P_i = \min(1 - g_i, 1)$$

  where **gi** is the goodness of element **i**
- That is, low goodness maps to a high probability of the module being altered.
- The selection operator has a non-deterministic nature and this gives SimE the hill climbing capability
- Selection is generally followed by sorting

Allocation

- This is a complex form of genetic mutation (compound move)
- This operator takes two sets (selection **S** and remaining set **R**) and generates a new population
- Has the most impact on the rate of convergence

Comparison of SimE and SA

- In SA a perturbation is a single move
- For SA, the elements to be moved are selected at random
- SA is guided by a parameter called temperature, while for SimE the search is guided by the individual fitness of the solution components

Comparison of SimE and GA

- SimE works on a single solution called population while in GA, the set of solutions comprises the population
- GA relies on genetic reproduction (using crossover, mutation, etc).
- In SimE, an individual is evaluated by estimating the fitness of each of its genes. (Genes with lower fitness have a higher probability of getting altered)

Other facts

- Fairly simple, yet very powerful
- Has been applied to several hard problems (such as VLSI standard cell placement, high level synthesis, etc)
- Parallel implementations have been proposed (for MISD and MIMD)
- Convergence analysis presented by designers of the heuristic and others

Stochastic Evolution

- **StocE**, often confused with Simulated Evolution
- Distinguishing features:
  - » The probability of accepting a bad move increases if no good solutions are found
  - » Like SimE, is based on compound moves (perturb function)
  - » There is a built in mechanism to reward the algorithm whenever a good solution is found

Parameters & Inputs

- An initial solution S_o
- An initial value of control parameter p_o
  - » Gain (m) > RANDINT(-p,0) (accepting both good and poor solutions)
- Stopping criterion parameter called R

Functions

- **PERTURB**: To make a compound move to a new state.
- **UPDATE** function: p = p + incr (p is incremented to allow uphill moves)
- Infeasible solutions are accepted, and then a function **MAKESTATE** is invoked to undo some last k moves.

Comparison of StocE and SA

- In StocE a perturbation is a compound move
- There is no hot and cold regime
- In SA, the acceptance probability keeps decreasing with time (decreasing values of temperature)
- StocE introduces the concept of reward whereby the search algorithm cleverly rewards itself whenever a good move is made

Common features of All heuristics

- All are general iterative heuristics, can be applied to any combinatorial optimization problem
- All are conceptually simple and elegant
- All are based on moves and neighborhood
- All are blind
- All occasionally accept inferior solutions (i.e, have hill-climbing capability)
- All are non-deterministic (except TS which is only to some extent)
- ñAllò(under certain conditions) asymptotically converge to an optimal solution (TS and StocE)

Some Research Areas

- Applications to various hard problems of current technology?
- Hybridization?
  - » How to enhance strengths and compensate for weaknesses of two or more heuristics
  - » Examples: SA/TS, GA/SA, TS/SimE, etc
- Fuzzy logic for multi-objective optimization
- Parallel implementations
- Convergence aspects

Thank you