RESIDUAL VIBRATION RESPONSE SPECTRUM FOR A SERVOMOTOR DRIVEN FLEXIBLE BEAM

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ABSTRACT

In this paper residual vibration response spectra for a servomotor driven flexible beam is discussed. The dynamic model consists of flexible beam and servomotor. Beam is modeled as fixed-free Euler-Bernoulli beam. The rotational trajectory is assumed as triangular velocity (bang-bang) trajectory. The equations governing the vibration of the rotating flexible beam is obtained. Mode summation technique is used to solve the equation. Only first mode is considered. Simulation results show that the maximum amplitude of the residual vibration changes with the change of the frequency ratio. Here the frequency ratio is defined as the first natural frequency of the flexible beam divided by the rotational frequency of the servomotor. When frequency ratios are even numbers like 2, 4, 6 ... the maximum residual vibration amplitudes are minimized. When frequency ratios are odd numbers like 3, 5, 7, ... then the maximum residual vibration amplitudes are maximized. The simulation results are also verified with the experiments.

Keywords: *Vibration, Flexible beam, Residual vibration, Response Spectra*

الملخص

في هذه الورقة تمت مناقشة طيف الاداء المتبقي الاهتزازي لعتبة مرنة تدار بمحرك مؤآزر. النموذج الديناميكي يتكون من عتبة مرنة عولجت كأنها عتبة اويلر-برنولي. العتبة تدار بالمحرك المؤآزر. المسار الدائري يفترض أنه مسار سرعات مثلث الشكل (بانق بانق). المعادلات الممثلة لاهتزاز العتبة المرنة الدوارة تم اشتقاقها. لايجاد الحل تم افتراض طريقة جمع الاطوار المتعامدة. المعادلات تم حلها عدديا. نتائج النمذجة بينت أن أقصى سعة للاهتزاز المتبقي تختلف بتغير نسبة التردد. نسبة التردد تم تعريفها هنا بأنها الهزة الطبيعية الأولى للعتبة المرنة مقسومة على السرعة الدائرية للمحرك المؤآزر. عندما تكون نسبة التردد عدد زوجي٢،٤،٦, وهكذا فإن أعلى سعة للاهتزاز المتبقي يكون أقل مايمكن. وعندما تكون نسبة التردد عددا فرديا ٣،٥،٧, وهكذا فإن أعلى سعة للاهتزاز المتبقي تكون أعلى ما يمكن. نتائج النمذجة تم التحقق منها بالتجربة.

1. INTRODUCTION

The insatiable demand for high performance robotic systems quantified by a high speed of operation, high end-position accuracy and lower energy consumption has triggered a vigorous research trust in various multi-disciplinary areas, such as design and control of lightweight flexible robot arm. The flexible manipulators, although having some inherent advantageous over conventional rigid robots, have posed more stringent requirements on the control system design, such as accurate end point sensing and fast suppression of transient vibration during rapid arm movements. [Bhat, at al, 1991] studied point to point position control of a flexible rotating beam, used Laplace domain techniques and did some experiments. [White and Heppler, 1996] worked on rotating Timoshenko beam equations for pinned-free and clamped-free cases. [Zhu and Mote, 1997] studied dynamic modeling and optimal control of a rotating Euler-Bernoulli beam. [Luo and Guo, 1997] investigated rotating Euler-Bernoulli beam with shear force feedback control. Main concern was the stability analysis of the closed loop equation. [Ankarali and Diken, 1997] studied a single link manipulator arm residual vibration and obtained response spectra. In their study, rotating fixed-free elastic beam is considered. Mode summation technique is used. For the rotation of the beam, cycloidal rise function is assumed. Analytical solution is obtained. Results show that at frequency ratios 2, 3, 4, … residual vibration amplitudes becomes zero. If the flexible beam is rotated with the time that corresponds to one of the frequency ratios mentioned, residual vibration control goal can be achieved.

In this study a dynamic model of a rotating fixed-free Euler-Bernoulli beam is developed. For the rotational trajectory, triangular velocity (bang-bang) function is assumed. Equations are solved numerically to find the residual vibration of the tip of the beam. A residual vibration spectrum is obtained. Results show that for frequency ratios 4, 6, 8 … residual vibration amplitudes are minimized but they are not zero. At frequency ratios 5, 7, 9 … residual vibration amplitudes are maximum but each time the maximum amplitude is lower than the previous one. These results are also verified experimentally.

2. ANALYSIS

Figure 1 shows the rotating flexible beam. OXY is the fixed frame, Oxy is the rotating frame, θ is the rotation angle which, represents the servomotor rotation. y(x,t) is the displacement of the mass *dm* with respect to the rotating frame Oxy. To develop the equation of the motion of the fixed-free Euler beam, if the equilibrium of the small mass *dm* is considered as the equilibrium of the beam element, the following differential equation can be obtained [Thomson, 1981],

$$\frac{d^2}{dx^2}\left(EI\frac{d^2 y}{dx^2}\right) = p(x) \tag{1}$$

Top

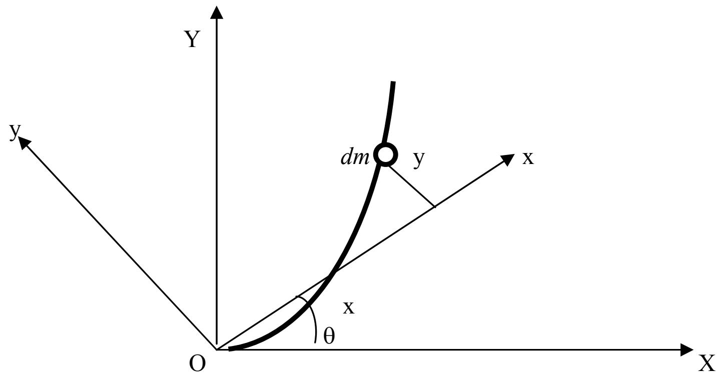

Figure 1. Rotating flexible beam

Here *EI* is the rigidity of the beam element. To find the inertial loading *p(x),* the acceleration of the mass *dm* with respect to the fixed frame should be obtained. The position vector of the mass *dm* with respect to the fixed frame OXY is

$$\vec{r} = (x\cos\theta - y\sin\theta)\vec{I} + (x\sin\theta + y\cos\theta)\vec{J} \tag{2}$$

if equation (2) is differentiated twice with respect to time, acceleration of the mass *dm* will be obtained as,

$$\begin{aligned}\ddot{\vec{r}} &= [(\ddot{x} - y\ddot{\theta} - 2\dot{y}\dot{\theta} - x\dot{\theta}^2)\cos\theta - (\ddot{y} + x\ddot{\theta} + 2\dot{x}\dot{\theta} - y\dot{\theta}^2)\sin\theta]\vec{I} + \\ &+ [(\ddot{x} - y\ddot{\theta} - 2\dot{y}\dot{\theta} - x\dot{\theta}^2)\sin\theta + (\ddot{y} + x\ddot{\theta} + 2\dot{x}\dot{\theta} - y\dot{\theta}^2)\cos\theta]\vec{J}\end{aligned} \tag{3}$$

If the following transformation is used

$$\begin{bmatrix}\vec{I}\\ \vec{J}\end{bmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\vec{i}\\ \vec{j}\end{bmatrix} \tag{4}$$

The acceleration vector of the mass *dm* in rotating coordinate is

$$\ddot{\vec{r}} = (\ddot{x} - y\ddot{\theta} - 2\dot{y}\dot{\theta} - x\dot{\theta}^2)\vec{i} + (\ddot{y} + x\ddot{\theta} + 2\dot{x}\dot{\theta} - y\dot{\theta}^2)\vec{j} \tag{5}$$

The acceleration component in $\vec{j}$ direction has nonlinear terms which will be ignored in this analysis, then the inertial loading caused by the rotation will be,

$$p(x) = -m(\ddot{y} + x\ddot{\theta}) \tag{6}$$

Here *m* is the mass per unit length of the beam. If equation (6) is substituted into equation (1) the following partial differential equation with the forcing function on the right side will be obtained.

$$EIy^{iv} + m\ddot{y} = -x\ddot{\theta} \tag{7}$$

Here *EI* is considered constant. Assuming the solution as the summation of the orthogonal modes, the displacement *y(x,t)* will be,

$$y(x,t) = \sum_{i=1}^{n} q_i(t)\phi_i(x) \tag{8}$$

Here $q_i(t)$ is the *i*th generalized coordinate and $\phi_i(x)$ is the *i*th mode of the beam. If this solution is put into equation (7) and orthogonality is used, the following equation for the generalized coordinates is obtained as,

$$\ddot{q}_i + \omega_i^2 q_i = -\frac{m}{M_i}\ddot{\theta}\int_0^l x\phi_i(x)dx \tag{9}$$

Here M_i is the generalized mass calculated from,

$$M_i = m\int_0^l \phi_i(x)^2 dx \tag{10}$$

If the following is defined as the mode participation factor,

$$F_i = -m\int_0^l x\phi_i(x)dx \tag{11}$$

equation (9) will be,

$$\ddot{q}_i + \omega_i^2 q_i = \frac{F_i}{M_i}\ddot{\theta}(t) \tag{12}$$

If viscous damping is introduced, the following equation is obtained

$$\ddot{q}_i + 2\zeta\omega_i\dot{q}_i + \omega_i^2 q_i = \frac{F_i}{M_i}\ddot{\theta}(t) \tag{13}$$

Here ζ is the damping ratio, $\ddot{\theta}(t)$ is the angular acceleration of the servomotor.

Top

Figure 2 shows the triangular velocity trajectory function.

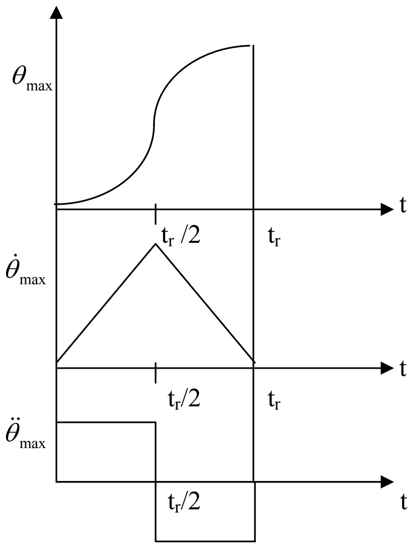

Figure 2. triangular velocity trajectory function.

The acceleration of the servomotor during the travel time t_r is

$$\ddot{\theta}_{max} = \frac{2\theta_{max}}{t_r^2}, \quad 0 < t < \frac{t_r}{2}$$
$$\ddot{\theta}_{max} = -\frac{2\theta_{max}}{t_r^2}, \quad \frac{t_r}{2} < t < t_r \tag{14}$$

To solve the equations, the state space form of equation (13) is obtained and only first mode is considered,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_1^2 & -2\zeta\omega_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ F_i / M_i \end{bmatrix} \ddot{\theta}(t) \tag{15}$$

Simulation results are obtained for 90 degrees of the rotation of the beam. To find the response spectra with respect to the frequency ratio ρ, the rotation time t_r is calculated as,

$$t_r = \frac{\rho}{f_1} \tag{16}$$

Here f_1 is the first natural frequency of the beam

3. SIMULATIONS

Figure 3 shows the vibration amplitudes of the beam when frequency ratio ρ=3. Beam material is chosen as aluminum of the size 500x26x2 mm. First natural frequency of the beam is f_1=8 Hz. According to equation (16) the rotation time is 0.375 second. After the rotation is complete, residual vibration starts as can be seen from Figure 3. The maximum amplitude of the residual vibration is 0.0075 m and occurs at 0.445 second. Figure 4 shows the vibration amplitudes of the beam for ρ=4. Here the rotation time is 0.5 second then residual vibration starts. Maximum amplitude occurs at 0.56 second with an amplitude of 0.0009 m. If the maximum residual vibration amplitudes are plotted against the frequency ratio, the residual vibration response spectra is obtained. Figure 5 shows the residual vibration response spectra. As can be seen from the figure, maximum vibration amplitudes are not decreasing monotonically. When frequency ratios are even numbers like 4, 6, 8, … maximum residual vibration amplitudes are minimized. When frequency ratios are odd numbers, like 5, 7, 9, … then maximum residual vibration amplitudes are maximized. Although several modes of the vibration exist at the same time, only first mode of the beam vibration is considered, because it has highest vibration amplitudes and more significant than the others.

4. EXPERIMENTS

To verify the simulation results, experiments are also conducted. Figure 6 shows the schematic diagram of the experimental setup, Figure 7 shows the photograph of the experimental setup. Setup consists of servomotor, gearbox, flexible beam, accelerometer, amplifier and computer. Figure 8 shows an example of the experimental result of the beam vibration for ρ=4. Figure 9 shows the simulation and experimental results together. As can be seen from the figure experimental results agree with the simulation results.

Top

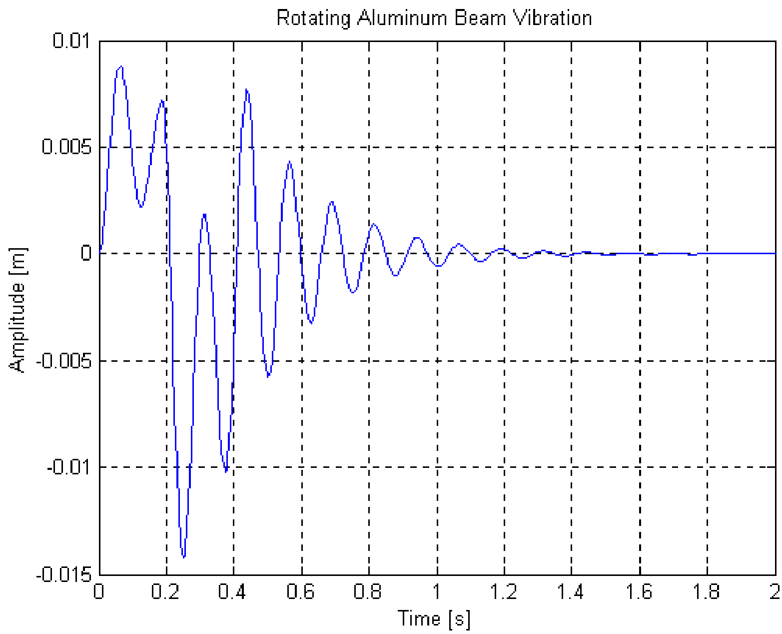

Figure 3. Vibration of the aluminum beam, $\rho = \frac{t_r}{t_1} = 3$.

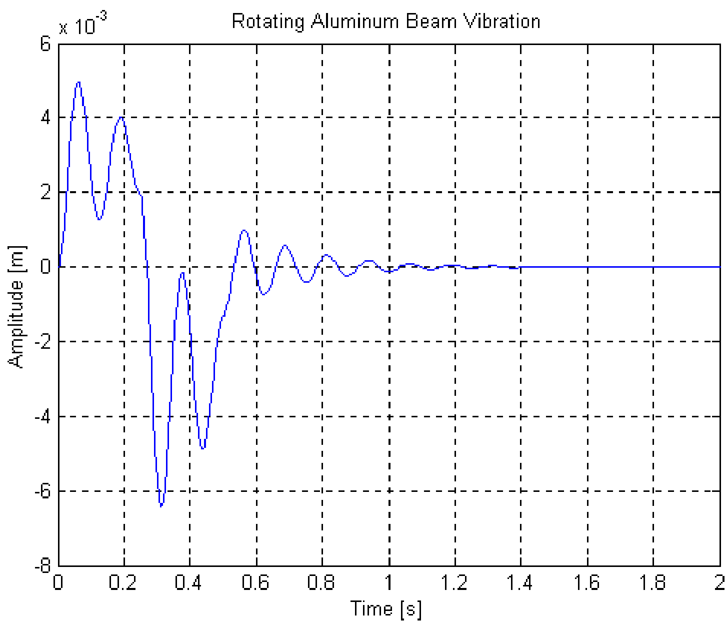

Figure 4. Vibration of the aluminum beam, $\rho = \frac{t_r}{t_1} = 4$.

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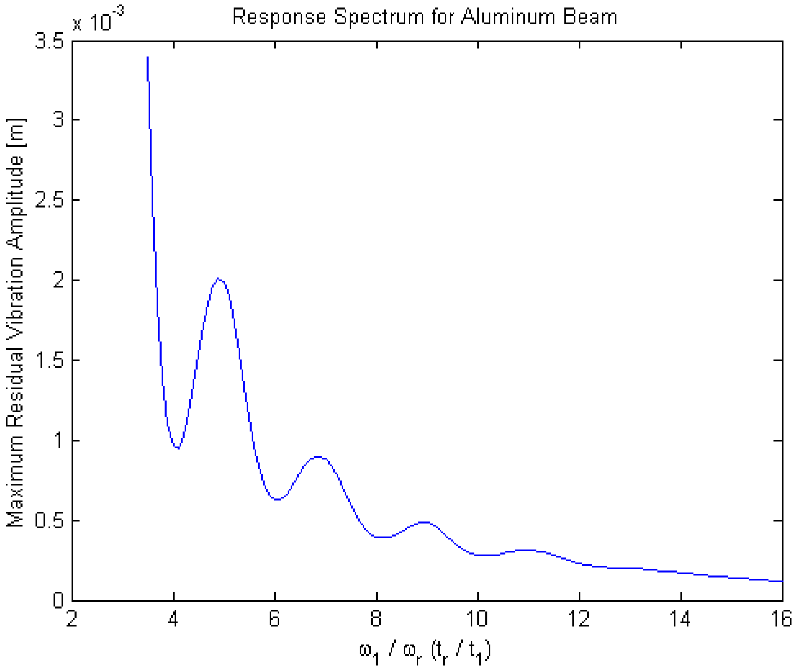

Figure 5. Response spectrum of residual vibration for the aluminum beam.

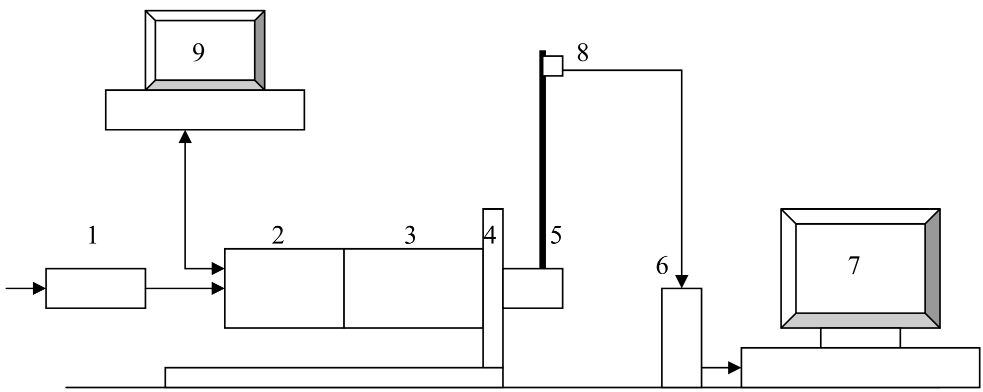

Figure.6 Experimental setup. 1) power supply, 2) servomotor, 3) gear box, 4) stand for motor, 5) flexible beam, 6) amplifier, 7) computer, 8) accelerometer, 9) computer.

Top

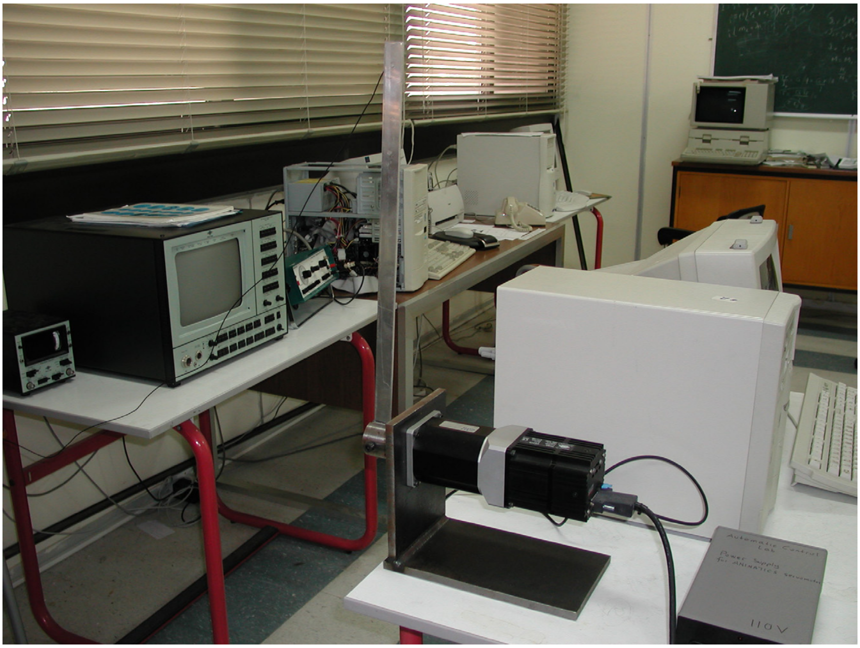

Figure 7. Photograph of the experimental setup.

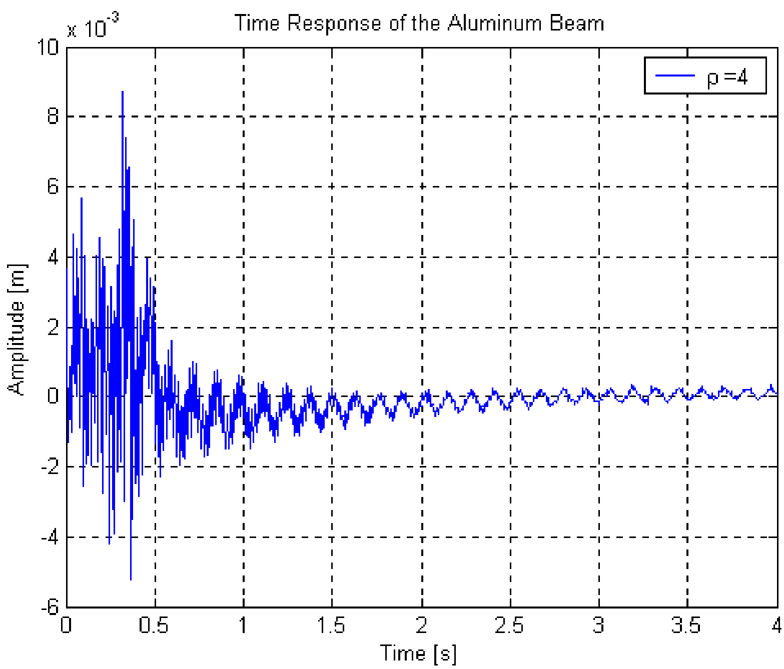

Figure 8. Example of an experimental result.

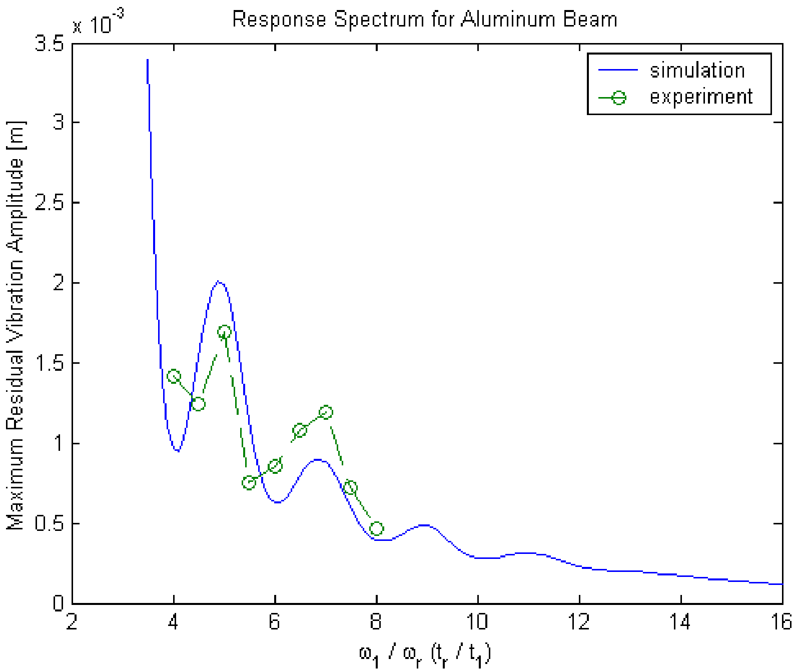

Figure 9. Comparison of simulations and experiments for the beam.

5. CONCLUSIONS

In this study response spectra of the residual vibration of a servomotor driven flexible beam is obtained. Rotational trajectory for the servomotor is assumed as the triangular velocity (bang-bang). Simulation results shows that the maximum residual vibration amplitudes of the flexible beam is sensitive to the frequency ratio. Here the frequency ratio is defined as the first natural frequency of the beam divided by the rotational frequency of the servomotor. For the even number of the frequency ratios, like 4, 6, 8, … the residual vibration amplitudes are minimized. For the odd number of the frequency ratios, like 5, 7, 9, … the residual vibration amplitudes are maximized. If the flexible beam is rotated with even number of frequency ratio the amplitude of the residual vibration is substantially reduced.

ACKNOWLEDGMENT

This work is supported by the Scientific Research Council of the King Abdulaziz University under the grant number 420/102.

Top

REFERENCES

1. Ankarali, A. and Diken, H., 1997, "Vibration Control of an Elastic Manipulator Link," *Journal of Sound and Vibration*, 204, pp 162-170.
2. Bhat, S. P. and Miu, D. K., 1991, "Experiments on point to point position control of a Flexible Beam using Laplace Transform Technique-Part II: closed loop," *ASME Journal of Dynamic Systems, Measurements and Control*, 113, pp. 438-443.
3. Bhat, S. P., Tanaka, M., and Miu, D. K., 1991, "Experiments on point to point position control of a Flexible Beam using Laplace Transform Technique-Part I: open loop," *ASME Journal of Dynamic Systems, Measurements and Control*, 113, pp. 432-437.
4. Choi, S. B., Thomson, B. S. and Gandhi, M. V., 1995, "Control of a Single Link Flexible Manipulator Fabricated from Composite Laminates," *Journal of robotic Systems*, 12, pp 45-54.
5. Hoa, S.V., 1979, "Vibration of a Rotating Beam with Tip Mass," *Journal of sound and Vibration,* 67, pp 369-381.
6. Kojima, H., 1986, "Vibration of a Beam-Mass System Fixed to a Rotating Body," *Journal of Sound and Vibration*, 107, pp 149-154.
7. Low, K. H. and Lau, M.W.S., 1995, "Experimental Investigation of the Boundary Condition of Slewing Beam Using High Speed Camera Systems," *Mechanisms and Machine Theory*, 30, pp 629-643.
8. Luo, Z. H. and Guo, B. Z., 1997, "Shear Force Feedback Control of a Single Link Flexible Robot with a Revolute Joint," *IEEE Transactions on Automatic Control*, 42, 53-65.
9. Mitchell, T.P. and Bruch, J. C., 1989, "Dynamics of a Rotationally Accelerated Beam," *AIAA Journal of Guidance and Control*, 12, pp 935-938.
10. Thomson, T., 1981, Theory of Vibration with applications, 2nd edition, Prentice Hall, Englewood Cliffs.
11. Zhu, W. D. and Mote, C. D., 1997, "Dynamic Modeling and Optimal Control of a Rotating Euler-Bernoulli Beams," *ASME Journal of Dynamic Systems, Measurements and Control*, 119, pp. 802-808.

Top