



# PREDICTION OF LAYER-INVERSION BEHAVIOR OF BINARY-SOLID LIQUID FLUIDIZED BEDS: COMPARISON OF MODELS

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## ABSTRACT

*Proper characterization of the hydrodynamics of binary-solid liquid-fluidized beds is an important first step in its effective utilization. Of particular importance in this connection is to be able to predict the unique hydrodynamic phenomenon of the layer-inversion, which is associated with the change of the stratification pattern of the two solid species in the fluidized bed brought about due to a change either in the liquid velocity or the bed composition. Past few years have witnessed the development of several models for the prediction of the layer-inversion phenomenon. It is, therefore, important to analyze their predictions in the light of the growing wealth of literature data. Such an exercise is important to examine the underlying assumptions and propose modifications to improve their predictive capability. Predictions of well-known layer-inversion models are compared with the experimental data reported in the literature including our own which comprises of solid species of widely different size, and therefore, provide an important test of predictive capability of models.*

**Keywords:** *binary-solid, liquid fluidized bed, layer-inversion, segregation velocity.*

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## 1. INTRODUCTION

A liquid fluidized bed containing two different kinds of solid species often shows a variation of the concentration of individual species along the length of the bed. The difference in the physical properties of solids understandably has an important bearing on their stratification pattern. The greater the difference in their physical properties, stronger will be segregation tendencies of the binary-solid fluidized bed. An interesting situation arises when the larger solid species is lighter while the smaller one is denser. In this case, a layer mainly consisting of smaller solids constitute the lower layer whereas larger ones are predominantly present in the upper region of the fluidized bed at low liquid velocities. This pattern of solid stratification progressively changes as the liquid velocity is slowly increased marked with a downward migration of larger particles till a uniform concentration of both solid species prevails throughout the fluidized bed. Any further increase of liquid velocity thereafter leads to a new stratification pattern in which larger solids are mainly present in the lower region while smaller ones are present in the upper bed region. This change of stratification pattern of the two solid species is commonly referred to as the layer-inversion phenomenon.

Potential applications of the binary-solid liquid fluidized beds have been proposed for simultaneously carrying out reaction as well as separation in various chemical, petrochemical and biochemical reaction systems. Such a configuration involves the presence of two solid particle species differing in the size as well as the density where the larger of the two solids constitutes the reactive resident phase of the fluidized bed while the smaller one can be used to selectively adsorb the product.

Several explanations have been put forward in the literature for the prediction of the layer-inversion phenomenon. For example, [Van Duijn and Rietema 1982], [Moritomi et al. 1982], [Epstein and LeClair 1985], [Moritomi et al. 1986], [Gibilaro et al. 1986], [Jean and Fan 1986], [Funamizu and Takakua 1995, 1996], [Asif 1998a, 1998b], [Epstein and Pruden 1999] to mention a few. A discussion on some of these can be found in the review article of [Di Felice 1995]. According to Epstein and [LeClair 1985], the layer-inversion takes place whenever the two layers, each assumed to consist of single solid species only, have the same bulk density. This could not, however, explain the reported dependence of the layer-inversion behavior on the bulk composition of the binary-solid fluidized bed. Another notable approach, which is also capable of describing the composition dependence of the layer-inversion phenomenon, appears to be the complete-segregation model of [Gibilaro et al. 1986]. Its predictions have been shown to be in good agreement with the available experimental data by [Di Felice et al. 1987, 1988]. On the other hand, a similar approach of [Asif 1998a] suggested the direct use of the [Richardson-Zaki 1954] correlation in conjunction with the mean particle properties (i.e. diameter and density) for computing the mean values of the particle terminal velocity and the exponent 'n'. Recently, [Asif, 2001] has observed substantial mixing induced contraction of the fluidized bed containing two solid species with significant difference of the size as well as density. Highest degree of contraction was found to prevail when the solids were completely mixed. He used models for

predicting the porosity of the packing of particle mixtures, henceforth simply referred to as packing models, to describe the overall expansion of the fluidized bed. For the binary solid system with about ten-fold difference in the size in his study, predictions of packing models were found to be superior to ones of the serial model. In another subsequent study, it was observed that packing models also provided a better description of the layer-inversion phenomenon as compared to the serial model and the model based on the averaging of particle properties [Asif, 2002]. Since these approaches are based on some kind of the averaging of either the particle properties or the mono-component bed voidages, and therefore, can be classified as averaging approaches.

Such averaging approaches, while capable of predicting the layer-inversion phenomenon, fail to describe the local mixing-segregation equilibrium prevailing in the fluidized bed. In an effort to model more realistically the segregation and mixing tendencies of the binary-solid fluidized bed, the pseudo-fluid approach was used to develop the segregation-velocity model [Asif, 1998b]. Using the Richardson and Zaki correlation, the model evaluates the segregation velocity of the larger particle species, which is assumed to be present in the pseudo-fluid consisting of a homogeneous mixture of the liquid and the smaller particle species. A positive value of the segregation velocity predicts the upward movement of the larger species whereas a negative value indicates downward movement of the same. A zero value of the segregation velocity, on the other hand, implies the absence of segregation tendencies, and therefore the two solid species in the fluidized bed will be mixed indicating the onset of the layer-inversion [Asif, 1998b].

In view of the above discussion, it is interesting to compare the predictions of the two approaches, especially in the light of recent data that reports layer-inversion behavior of binary-solid fluidized beds containing two solid species of substantial size difference [Asif, 2002]. It should be noted here that as discussed before although there are several models based on various averaging approaches, it is the one based on the packing model that will be mainly considered for the purpose of comparison here in view of our recent finding that predictions of packing models are superior to other averaging models [Asif, 2002].

Since the main focus of the present study is comparison of two different approaches, only the governing equations describing the segregation velocity model and the packing model will be presented in the following. Other details can be seen in the original works referred to here.

## 2. SEGREGATION-VELOCITY MODEL

The velocity of the larger particles, i.e. species 1,  $U_{p1}$ , in the pseudo-fluid can be evaluated using the [Richardson and Zaki 1954] correlation as follows (Asif, 1998b),

$$U_{p_1} = U_o - (1 - C_1) \overline{U}_{t_1} \overline{\epsilon}^{\overline{n}_1 - 1} \tag{1}$$

where the quantities with an over-bar represent pseudo-fluid properties. The subscript 1 refers to the larger and lighter particle species and the subscript 2 refers to the denser and smaller particle species. The terminal settling velocity of the larger particle in the pseudo-fluid can be evaluated using any standard correlation by simply replacing the density and viscosity of the pure fluid with those of the pseudo-fluid.

The density of the pseudo fluid can be given as

$$\overline{\rho} = \rho_{s_2} \overline{C} + \rho_f (1 - \overline{C}) \tag{2}$$

There are several relationships proposed in the literature for the apparent viscosity of the solid-liquid suspension. For example, the well-known relationship proposed by Happel (1957) is given as

$$\frac{\overline{\mu}}{\mu} = 1 + 5.5 \overline{C} \left[ \frac{4 \overline{C}^{7/3} + 10 - (84/11) \overline{C}^{2/3}}{10(1 - \overline{C}^{10/3}) - 25 \overline{C} (1 - \overline{C}^{4/3})} \right] \tag{3}$$

where

$$\overline{C} = \frac{C_2}{1 - C_1} \tag{4}$$

It is clear from Eq. (1) that the local segregation velocity depends upon the local composition and the local voidage of the bed. However, at the onset of the layer-inversion, when the overall segregation velocity is zero, the bed composition as well as the voidage is uniform throughout the bed. It is, therefore, justified to use the bulk composition and the overall voidage of the bed to compute the occurrence of the zero segregation velocity at the onset of the layer-inversion. Ideally, actual experimental values should be used for the overall bed expansion in Eq. (1). Should these be unavailable, models for predicting the bed voidage can be used. For binaries with large size-difference, the packing model rather than the serial model or the property-averaging model is recommended for the purpose (Asif, 2002). The accuracy of voidage predictions will of course influence the accuracy of the segregation-velocity model. On the other hand, the particle terminal velocity,  $U_t$ , and the index ‘n’ in Eq. (1) are computed using the following correlations of Khan and Richardson (1990).

$$\overline{Re}_{t_1} = \left( 2.33 \overline{Ga}^{0.018} - 1.53 \overline{Ga}^{-0.016} \right)^{13.3} \tag{5}$$

$$\frac{4.8 - \bar{n}_1}{\bar{n}_1 - 2.4} = 0.043 \bar{Ga}^{0.57} \tag{6}$$

where the Galileo number and the terminal Reynolds number are defined as,

$$\bar{Ga} = \left( \frac{(\rho_{s_1} - \bar{\rho}) \bar{\rho} d_1^3 g}{\bar{\mu}^2} \right); \quad \bar{Re}_{t_1} = \left( \frac{\bar{U}_{t_1} \bar{\rho} d_1}{\bar{\mu}} \right) \tag{7}$$

Here  $d_1$  is the diameter and  $\rho_{s_1}$  is the density of the particle species 1, i.e. larger particles.

### 3. PACKING MODEL

There appears to be a good deal of literature concerning packing models depending upon the perceived mechanism of packing behavior of the particle mixture containing two or more components [Stovall et al., 1986], [Yu and Standish, 1991], [Yu et al., 1996] and [Ouchiyama and Tanaka, 1989]. In the following discussion, however, the model based on the Westman equation (1936) will only be considered. The Westman equation is given as

$$\left( \frac{V - V_1 X_1}{V_2} \right)^2 + 2G \left( \frac{V - V_1 X_1}{V_2} \right) \left( \frac{V - X_1 - V_2 X_2}{V_1 - 1} \right) + \left( \frac{V - X_1 - V_2 X_2}{V_1 - 1} \right)^2 = 1 \tag{8}$$

where  $V$  is the specific volume of the binary-solid fluidized bed, and  $V_1$  and  $V_2$  are specific volumes of mono-component fluidized beds of solid species 1 and 2, respectively. Here,  $G$  depends upon the ratio of the diameter of the two species. It is easy to see that setting  $G=1$  in the above equation yields the serial model  $V = X_1 V_1 + (1 - X_1) V_2$ . Yu et al. (1993) have proposed the following functional form of the parameter  $G$  in the Westman equation

$$\frac{1}{G} = \begin{cases} 1.355 r^{1.566} & (r \leq 0.824) \\ 1 & (r > 0.824) \end{cases} \tag{9}$$

where  $r$  is the size ratio (smaller to larger) of the two solid species. On the other hand, [Finkers and Hoffmann, 1998] have recently suggested another expression for the parameter  $G$  in the Westman equation. Their approach makes use of the structural ratio rather than the diameter ratio, and is equally applicable for both spherical and non-spherical particles. This is given by

$$G = r_{str}^k + (1 - \varepsilon_1^{-k})$$

$$r_{str} = \left[ \frac{((1/\varepsilon_1) - 1) r^3}{1 - \varepsilon_2} \right] \tag{10}$$

where a value of exponent  $k = -0.63$  has been recommended by the authors. Since it has been shown before that both the above two definitions of  $G$  give comparable predictions of the onset of the layer-inversion phenomenon, Eq. (9) is preferred here owing to the simplicity of its expression.

**4. RESULTS AND DISCUSSION**

In this section, the predictive capability of the above-mentioned models will be first examined in the light of new layer-inversion data reported for the PET-sand binary system, which has significant difference in size. The commonly reported data of [Moritomi et al. 1982, 1986] is considered next.

**4.1. Comparison of PET-sand system**

The physical and the mono-component fluidization properties of the PET and sand are shown in Table 1. As can be seen from the table, the lighter PET resins are almost ten times larger than the denser sand while their buoyed density ratio is about 0.24. Parameters of the Richardson-Zaki equation are also reported in the table. The cross-over point of the bulk densities of the two components occurs around 23-mm/s, which is essentially what the serial model would predict as the layer-inversion velocity.

Table 1: Physical properties of the two species and their fluidization properties using water at 20 degrees C (Asif, 2002)

<i>Solids Species</i>	<i>Shape</i>	<i>Size Range</i>	<i>Mean Diameter</i>	<i>Solid Density</i>	$U_t$	$n$
		( $\mu m$ )	( $\mu m$ )	( $kg/m^3$ )		
Sand	nearly spherical	250-300	275	2664	34.4	3.79
PET	cylindrical ( $\psi=0.85$ )		2790	1396	94.3	2.61

The comparison is presented in Fig. 1. The predictions of the segregation velocity model, represented by Eq. (1) are shown along with ones of the serial model, the packing model and the property-averaging model. As pointed out before that for binaries of large size difference, the predictions of both the serial model and the property-averaging model are poor. It is quite clear from Fig. 1 that even the best model of the averaging approach class is not able to describe the inversion behavior as closely as the segregation velocity model presented here. Predictions of Eq. (1) are clearly outstanding for all values except for  $X_1=0.86$  when the binary-solid fluidized bed contains high fraction of larger component. Since the bed expansion here is calculated using Eqs. (8) and (9), discrepancy in its prediction could lead to such discrepancy in the prediction of the layer-inversion velocity. Note that percentage mean deviation in the prediction of Eq. (1) is 1.2% while the same for Eq. (8) is 7.7%.

#### 4.2. Comparison with Moritomi et al. (1982, 1986) data

The widely reported data of [Moritomi et al. 1982, 1986] involve the fluidization of 0.775-mm hollow char and 0.163-mm glass beads. The physical properties and Richardson-Zaki correlation parameters of their binary system are presented in Table 2. The qualitative comparison is shown in Fig. 2 and a quantitative comparison is presented in Table 3. Two different curves are shown based on Eq. (1). The difference in these curves stem from the use of different models in computing the value of the void fraction in Eq. (1). It is worthwhile to mention in this connection that the serial model assumes the

Table 2: Physical properties of Moritomi et al. (1982, 1986) system

<i>Solid species</i>	<i>Diameter</i> (mm)	<i>Density</i> (kg/m <sup>3</sup> )	<i>U<sub>t</sub></i> (mm/s)	<i>n</i>
Hollow char (1)	0.775	1380	46.0	3.00
Glass beads (2)	0.163	2450	14.4	3.98

Table 3: Percentage mean deviation in predictions of different models

<i>Model</i>	<i>% Error</i>
Eq. (1) (Packing model)	5
Eq. (1) (Serial model)	17
Eq. (8) with Eq. (9)	10

complete segregation of the two solid species and therefore tends to over-predict the bed expansion. On the other hand, although packing models do recognize the bed contraction brought about by the mixing of the two unequal solid species, yet the extension of their applicability from the binary-solid packings to binary-solid fluidized beds has not been fully established except probably for binaries of large size difference. Moreover, the difference in the predictions of the two models often gets more prominent as the fraction of larger species in the bed increases. Against this backdrop, it becomes clear why the difference in the prediction of the layer-inversion velocity increasingly differs as the amount of larger component in the bed increases. Coming back to the issue of the comparison of the segregation velocity approach and the averaging approach, it can be contended that once the actual experimental value or an appropriate model for describing the bed void fraction is incorporated, the former provides a better prediction of the layer-inversion phenomenon.

#### 5. CONCLUSIONS

An unmistakable trend is obvious from comparison of models presented in Figs (1) and (2). That is, the segregation velocity model can describe the layer-inversion phenomenon more effectively. Some deviations nonetheless appear at higher  $X_1$  for PET-sand system while the same is more apparent for smaller  $X_1$  for char-glass system. This clearly indicates that no error trend in the prediction of segregation velocity model exists unlike its closest competitor in the class of averaging approaches (i.e. packing model), which under-predict the inversion velocity at lower  $X_1$  and over predict the same at higher  $X_1$ . It is, however, important to note that it is a serious limitation of the segregation velocity model that it relies on empirical correlations for the

prediction of the viscosity of the pseudo-fluid and the bed expansion. In fact, it is the prediction of the bed expansion that appears to have an important bearing on the prediction of the segregation velocity model as clearly indicated by Fig. 2.

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### NOMENCLATURE

$C_i$	fractional volumetric concentration of the particle species $i$	
$\bar{C}$	solid concentration in pseudo fluid defined by Eq. (4)	
$G$	Parameter $G$ in Westman equation defined by Eq. (9) and Eq. (10)	
$Ga$	Galileo number defined by Eq. (7)	
$n$	Richardson and Zaki correlation index	
$r$	Size ratio (smaller to larger solid species)	
$Re_t$	Reynolds number based on the particle terminal velocity (Eq. 7)	
$U_o$	liquid superficial velocity	mm.s <sup>-1</sup>
$U_{p1}$	segregation velocity of particle species 1	mm.s <sup>-1</sup>
$U_{ti}$	terminal velocity of the particle species $i$	mm.s <sup>-1</sup>
$V$	Specific volume of binary-solid bed defined as $\left( = \frac{1}{1-\varepsilon} \right)$	
$V_i$	Specific volume of the mono-component bed of species $i$	
$X_i$	fraction of lighter component defined as $X_i = \frac{C_i}{(C_1 + C_2)}$	

### Greek symbols

$\varepsilon$	bed void fraction or fluid fractional volumetric concentration	
$\varepsilon_i$	Mono-component bed void fraction of species $i$	
$\mu$	fluid viscosity	Pa.s
$\bar{\mu}$	pseudo-fluid viscosity defined by Eq. (3)	Pa.s

$\bar{\rho}$	pseudo-fluid density defined by Eq. (2)	kg.m <sup>-3</sup>
$\rho_s$	solid density of the particle	kg.m <sup>-3</sup>
$\rho_f$	fluid density	kg.m <sup>-3</sup>
$\psi$	shape factor	

#### Subscripts

1	larger and lighter particle species
2	smaller and denser particle species
i	solid particles of species i

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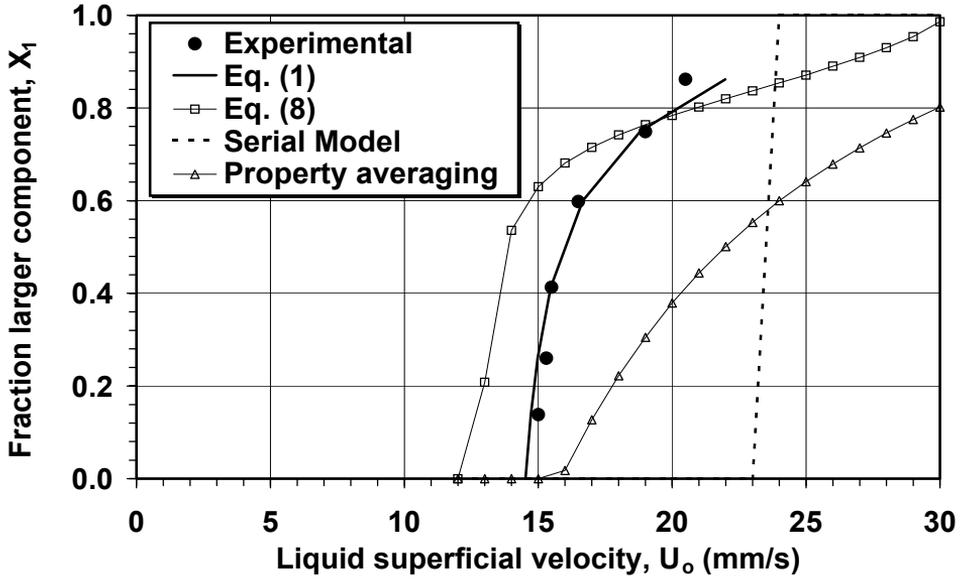


Figure 1: Comparison of predictions of layer-inversion models for the PET-sand system of Table 1

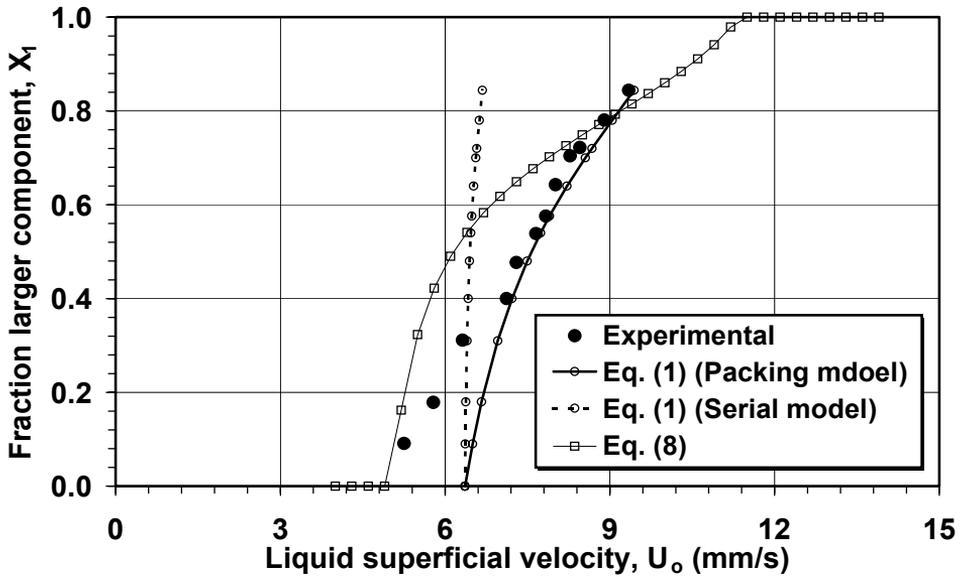


Figure 2: Comparison of predictions of layer-inversion models for the system of Moritomi et al. of Table 2