DETERMINING CRITERIA WEIGHTS AS A FUNCTION OF THEIR RANKS IN MULTIPLE-CRITERIA DECISION MAKING[1]

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ABSTRACT

The problem of assigning weights from ordinal ranks appears in many contexts in multi-criteria decision making. In this paper, we present an empirical methodology for converting an ordinal ranking of a number of criteria into numerical weights. Based on this methodology, the weight for each criterion is expressed as a simple mathematical function of its rank and the total number of criteria. The proposed methodology is empirically developed, evaluated, and validated based on a set of experiments involving university students and faculty members. The proposed method is compared with well-known methods in the literature and has shown superiority in assigning criteria weights from ordinal ranks.

INTRODUCTION

In multiple criteria decision making, several methods are used to determine the relative criteria weights. These methods depend on the input of the decision maker(s), i.e., the approach used to compare the different criteria. In many situations, the only input provided by each decision maker is a list (in the order of priority) the factors they consider most relevant for evaluating and comparing the applicable alternatives. As an example the factors used to assign an overall audit score for a quality or maintenance system are ranked by the auditing team and a weight for each factor must be obtained from the given ranks. In goal programming, if the different objectives are ranked, their relative weights can be determined and used to combine multiple objectives into a single objective.

This paper presents an empirically developed methodology to convert criteria ranks into relative criteria weights, using real-life experiments that involve surveys of university students and faculty members. In these experiments., participants were first asked to list the relevant factors in the order of importance, and then asked to give a weight for each factor based on its importance in their point of view. In other words they had to provide a weight for each factor that matches the given rank (the highest weight must be given to the first-ranked factor). Using regression and statistical analysis, a methodology. That best fits the experimental data and minimizes the errors is recommended for general use in assigning weights. In order to validate the proposed methodology, a second set of experiments involving another sample of students and a different set of criteria was subsequently conducted.

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In the following section, relevant literature is reviewed. Subsequently, the problem is defined and experimental design is described. Next, the methodology for conversing ranks into weights is presented. Finally, validation of the methodology is discussed, and then conclusions are drawn.

LITERATURE SURVEY

Lootsma (1999) provides a comprehensive overview of multi-criteria decision making (MCDM) approaches based on ratio and difference judgment. Traditional methods for determining criteria weights include the tradeoff method and the pricing-out method (Keeney and Raiffa, 1976), the ratio method and the swing method (Von Winterfeldt and Edwards, 1986), conjoint methods (Green and Srinivasan, 1990), and the Analytic Hierarchy Process (AHP) (Saaty, 1994). Borcherding et al. (1991) compare the tradeoff, pricing out, ratio, and swing methods. More recent methods include habitual domains (Tzeng et al. 1998), multiobjective linear programming (Costa and Climaco, 1999), and linear programming (Mousseau et al., 2000).

Tzeng et al. (1998) classify weighting methods into objective or subjective, according to whether weights are indirectly computed from outcomes or directly obtained from decision makers. Weber and Borcherding (1993) classify weight-determining procedures as statistical or algebraic, holistic or decomposed, and direct or indirect. Choo et al. (1999) offer several interpretations of weights in different MCDM models. Zopounidis and Doumpos (2002) provide a detailed survey of multicriteria classification and ranking methods. Al-Kloub et al. (1997) describe an MCDM methodology for determining weights and ranks, and apply it to rank water projects in Jordan. Lahdelma et al. (2002) use Stochastic Multicriteria Acceptability Analysis (SMAA) with ordinal criteria to convert criterion-wise rankings of alternatives into cardinal information used for selecting a waste treatment facility location.

Specific functions for assigning weights w_r to n criteria with ranks $r = 1, \ldots, n$, have been suggested by several authors. Stillwell et al. (1981) propose two functions: inverse weights, and linear weights. Barron (1992) proposes centroid weights. Lootsma (1999) and Lootsma and Bots (1999) suggest geometric weights. Barron and Barrett (1996) support using centroid weights, while Gonzalez-Pachon and Romero (2001) favor a linear order under a context of complete ordinal ranking.

PROBLEM DEFINITION AND EXPERIMENTS

This paper considers a deterministic group MCDM problem with m alternatives and n decision criteria. Weights reflect the relative importance of each decision criterion, and are usually normalized by making their sum equal to 1 ($\sum_{j=1}^{n} w_j = 1$). Given the specific performance value a_{jk} of each alternative k ($k = 1, 2, \ldots, m$) in terms of each criterion j ($j = 1, 2, \ldots, n$), the overall performance of each alternative k can be calculated as follows:

$$P_k = \sum_{j=1}^{n} w_j a_{jk}, \qquad k = 1, 2, \ldots, m \qquad (1)$$

We assume that input is obtained as a list of n prioritized (ranked) criteria, where each criterion j has a rank r_j, ($r_j = 1, ..., n$). We assume that rank is inversely related to weight (rank 1 denotes the highest weight, while rank n denotes the lowest weight). Our objective is to convert the list of ranks ($r_1, \ldots, r_n$) into numerical weights $w_1, \ldots, w_n$ for the n criteria.

The methodology suggested in this paper is empirically developed based on a set of experiments. We would like to test whether rank-weight relationships change with different sets of criteria or different decision makers. Therefore, the experiment involves two groups of participants and two sets of criteria. The main objective of the experiments is to develop a general methodology to convert ordinal data into relative weights for any set of criteria. The experiment consisted of a survey distributed among a sample of 111 students and 23 faculty members at KFUPM, in which the participants were asked to answer two questions:

> **Question 1.** List the factors that hinder students learning and retaining course materials.
> **Question 2.** List the factors that affect the evaluation of course instructors.

For each list, the participants were asked to arrange the factors in order of priority (most important to least important). After they listed them, they were required to give weights to all factors in each prioritized list, starting with a weight of 100% for the most important (first) factor. The factors most frequently listed by the participants in response to the two questions are shown in Table 1.

Table 1. Criteria cited most by participants in the survey and corresponding percentage of citing participants for the two questions

Question 1. Learning hindrances	%	Question 2. Instructor evaluation	%
Homework cheating	30	Fluency in English language	28
Excessive students social activity	24	Judgment in dealing with students	24
Frequent examinations	23	Experience in teaching	22
Tough examinations	20	Good handling of student problems	17
Focus on theories rather than applications	18	Assuming responsibility	16
Student lack of preparation for the lecture	16	Honesty in work	14
Poor attention and note taking in lectures	15	Honesty in work	12
Choosing the wrong roommate	14	Giving students bonus via extra work	10
Memorizing instead of understanding	14	Solving homework problems in class	10
Losing points for absences	14	Instructor ethics	8
Insufficient computer programming skill	14		
Students carelessness and delaying tasks	14		

DETERMINING RANK-WEIGHT RELATIONSHIPS

The methodology for assigning criteria weights based on criteria ranks provided by the decision maker(s) is empirically developed on the basis of the data obtained from the experiments. The process of analyzing the data in order to develop the methodology involved the following steps.

1. First, we separated the data obtained from the survey into four categories based on the two sets of criteria (two questions) and the two sets of decision makers (students and faculty). We then separated each category into distinct groups according to the number of criteria, n, given by each participant in the survey.
2. For each of the four categories (question 1, question 2, students, and faculty) and each value of n, we calculated the average weight for each rank.

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3. For each value of n, we applied statistical tests to determine whether the differences between the two sets of criteria (two questions) or between the two sets of decision makers (students and faculty) are significant. Since these differences were found statistically insignificant, all inputs from the four categories were combined for each value of n, producing the average weights for each rank shown in Table 2.

4. Five different models were applied to estimate the weight of each rank for each value of n. All these models assume that there is a consistent relationship between rank and average weight, which is independent of the problem context. After plotting the data in Table 2, we proposed the first model given below. Our model (M1) assumes that the relationship between the weight and rank is linear, in which the slope is itself a function of the number of criteria n. All models were adjusted for our data by making $w_1 = 100$. The five models used to calculate the weight of each rank are:

M1 Proposed model: Linear weights with variable slope: $w_r = 100 - s_n(r - 1)$, where w_r is the weight, r is the rank, and s_n is the absolute value of the slope calculated by least squares regression when the number of criteria is equal to n, as shown in Table 2.

M2 Stillwell et al. (1981): Linear weights with fixed slope: $w_r = 100(n + 1 - r)/n$,

M3 Stillwell et al. (1981): Inverse or reciprocal weights: $w_r = 100/r$,

M4 Barron (1992): Centroid weights: $w_r = \dfrac{100\sum_{i=r}^{n} 1/i}{\sum_{i=1}^{n} 1/i}$.

M5 Lootsma (1999): Geometric weights: $w_r = \dfrac{100}{\left(\sqrt{2}\right)^{r-1}}$.

5. In order to compare how closely each model approximates actual weights, the mean absolute percentage errors (MAPE) were calculated for all five models and all values of n. The results shown in Table 3 clearly show that Model M1 consistently outperforms all other models. Thus, Model M1 is chosen to represent the relationship between the rank and average weight.

6. In order to determine the relationship in Model M1 between the slope ($- s_n$) and the number of criteria n, we plotted the values of s_n shown in Table 2 versus n. The plot showed a decreasing nonlinear curve that suggested an inverse model of absolute slope s_n as a function of n. Applying least-squares regression to s_n versus n, we obtained the following model.

$$s_n = 3.19514 + \frac{37.75756}{n}$$

Therefore, for any set of n ranked factors, assuming a weight of 100% for the first-ranked (most important) factor, the percentage weight of a factor ranked as r is given by:

$$w_{r,n} = 100 - s_n(r - 1), \text{ or}$$

$$w_{r,n} = 100 - (3.19514 + \frac{37.75756}{n})(r - 1), \quad 1 \le r \le n \text{ , } r \text{ and } n \text{ are integer} \qquad (2)$$

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Table 2. Actual average weights for each rank r and slope magnitude s_n of the weight-versus-rank line according to the number of criteria n

n / r	2	3	4	5	6	7	8	10	11	12
1	100	100	100	100	100	100	100	100	100	100
2	79.26	81.94	86.42	92.85	93.39	90	93.75	90	99	87.56
3		65.53	72.3	79.35	85.56	90	84.38	88	80	84.94
4			58.55	67.36	71.67	80	76.25	85	80	75.44
5				58	62.5	70	72.63	85	75	71.64
6					54.44	50	63.88	70	75	66.63
7						40	47.63	65	70	63.82
8							38.88	55	60	55.53
9								40	60	51.77
10								30	45	48.93
11									40	41.15
12										35.09
s_n	20.74	17.4	13.81	10.48	9.05	9.01	8.15	6.75	5.72	6.07

Table 3. MAPE values of the 6 models for all values n

n	M1	M2	M3	M4	M5
2	0	0.37	0.37	0.58	0.11
3	0.43	22.59	29.37	38.1	12.47
4	0.1	25.34	38.34	43.67	22.16
5	1.18	28.87	46.51	48.35	33.05
6	1.86	29.86	51.67	50.66	40.67
7	7.45	28.56	54.08	51.06	45.62
8	4.49	27.95	57.32	51.6	50.98
10	8.93	28.53	62.4	52.77	60.27
11	5.21	29.8	65.73	53.88	64.03
12	4.27	24.45	65.41	50.84	65.13
Ave. MAPE	3.39	24.63	47.12	44.31	39.45

VALIDATION

Equation (2) represents our proposed functional relationship between criteria ranks and weights. In order to validate this relationship, a second set of experiments involving a different set of 59 KFUPM students, was conducted. Students were given 3 questions relating to the requirements for the optimal design of a numerical analysis course. The first question listed 7 faculty requirements, the second 6 textbook requirements, and the third 6 grading policy requirements. Students were asked give the proper weight to each requirement (criterion) on a scale from 1 to 9, with 1 = least important and 9 = most important. Although the ranks are not explicitly given, they are implied in the descending order of the weights.

Since both questions 2 and 3 involved 6 criteria (n = 6), their data was combined together. For the two new sets of data, the average weight was determined for each rank, and then normalized to correspond with w_1 = 100. Subsequently, the normalized actual weights were compared with the values suggested by models M1 through M5. The results, summarized in Table 4, confirm the overwhelming superiority of our Model M1 over other models. First, the errors in the weights for Model M1 (MAPE values) are much lower than the four other models. Second, the theoretical slope estimates are quite close to the actual values (relative error = 6.88% and – 4.41% for $n = 7$ and $n = 6$, respectively).

Table 4. Comparing Model M1 with other models using two additional (validation) data sets

Quest.	Rank r	1	2	3	4	5	6	7	MAPE	S_7
	Actual	100	95.18	89.52	81.13	73.17	57.86	44.86		8.036
	M1	100	91.41	82.82	74.23	65.64	57.05	48.47	5.66	8.589
1	M2	100	85.71	71.43	57.14	42.86	28.57	14.29	31.42	
	M3	100	50	33.33	25	20	16.67	14.29	55.92	
	M4	100	61.43	42.15	29.29	19.65	11.94	5.51	56.07	
	M5	100	70.71	50	35.36	25	17.68	12.5	47.67	
Quest.	Rank r	1	2	3	4	5	6		MAPE	S_6
	Actual	100	94.97	89.61	79.89	64.8	36.2			9.926
	M1	100	90.51	81.02	71.54	62.05	52.56		12.36	9.488
2 & 3	M2	100	83.33	66.67	50	33.33	16.67		29.63	
	M3	100	50	33.33	25	20	16.67		50.33	
	M4	100	59.18	38.78	25.17	14.97	6.803		53.5	
	M5	100	70.71	50	35.36	25	17.68		39.68	

CONCLUSIONS

This paper presented a methodology to determine individual criteria weight on the basis of individual ordinal ranking of these criteria. Experiments were conducted to develop the proposed methodology. A model has been proposed in which the weight of each criterion is a function of both its rank and the total number of criteria The linear model defined by equation (2), whose slope depends on the number of criteria, outperforms other functional forms reported in the literature.

The work in this paper can be extended into several directions. One direction is to investigate and assess the performance of the aggregation methods in the literature. Aggregation methods are used for combining different criteria ordinal rankings provided by several decision makers into aggregate numerical criteria weights. An alterative direction is to use the concepts in this paper and propose new aggregation methods.

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