A New Hybrid Local Search Successive Interference Cancellation Detector

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***Abstract*— In this paper, a new hybrid multiuser detection scheme is proposed. The structure cascades a linear successive interference cancellation detector and a 1-opt local search algorithm. The proposed multiuser detector is compared to other multiuser detectors in terms of BER and complexity. Simulation results illustrate the superiority of the proposed scheme compared to actual ones existing in literature.**

***Index terms* — SIC, Hybrid detectors, local search, multiuser detection, CDMA.**

I. INTRODUCTION

Multistage detectors have received significant attention in recent years due to their relative low computational complexity [1]. At each stage the estimated interference from other users is subtracted before estimating the desired user's data bit. Depending on the interference cancellation procedure implemented at each stage, two types of multistage detectors are pointed out: successive interference cancellation and parallel interference cancellation detectors. The successive interference cancellation is one of the simplest multi-user detectors. It requires only marginal additional computational complexity over the conventional matched filter detector [2]-[3]. Hybrid schemes combining both approaches are considered in [4]-[5]. Depending on the decision functions used to estimate the interference at each stage, multistage detectors could be divided into two main categories: linear and non-linear. Linear successive interference cancellation (SIC) and linear parallel interference cancellation PIC detectors are shown to be equivalent to Gauss-Seidel and Jacobi iterative methods used in matrix inversion, respectively [2].

Another way to look at the multiuser detection is to consider it as an optimization problem. Doing so, results in a non-deterministic polynomial time (NP)-hard problem. Many approaches are used to get exact and approximate solutions for the multiuser detection problem. An approximate solution is found either by relaxation or by heuristic search methods, while the branch and bound algorithm is used to provide an exact solution [6]. Many heuristic search methods, relaxation methods and combination of the two have been used to solve the multiuser detection problem [7] and [9]. Local search algorithm is one of the simplest heuristics that enjoys low computational complexity *O(K)* [10]. It has been used alone, in combination with relaxation methods and in combination with other heuristic algorithms such genetic algorithms to solve the multiuser detection problem [7] and [9]

In this paper, a new hybrid multiuser detection scheme is proposed. It cascades a linear successive interference cancellation (SIC) detector with a 1-opt local search algorithm. The linear SIC detector provides an initial solution to the 1-opt local search algorithm. The main advantage of the proposed detector is that it maintains the total complexity of the hybrid detector $O(K)$ rather than $O(K^3)$ for the detector proposed in [8].

The rest of the paper is organized as follows: the system model is descripted in section 2, the linear SIC detector and local search algorithm are described in sections 3 and 4, respectively. The proposed linear hybrid detector is discussed in section 5. Finally, simulation results and conclusion are discussed in sections 6 and 7.

II. SYSTEM MODEL

Throughout this paper, an uplink channel where *K* users are transmitting simultaneously over a synchronous additive white Gaussian noise (*AWGN*) channel using Binary-Phase Shift Keying (*BPSK*) is considered. Each user is characterized by its own spreading code of length *N*-chips. The received signal is expressed in vector form as:

$$\mathbf{r} = \mathbf{SAb} + \mathbf{n} \tag{1}$$

where $\mathbf{S}$ is a $N \times K$ matrix of the spreading sequences, $\mathbf{A}$ is a $K \times K$ matrix of the received amplitudes, $\mathbf{b}$ is a K-length vector of binary transmitted symbols, and $\mathbf{n}$ is a K-length vector of independently, identically distributed additive white Gaussian noise with zero-mean and variance equals $N_0/2$.

III. THE LINEAR SUCCESSIVE INTERFERENCE CANCELLATION MULTIUSER DETECTOR

The linear SIC consists of successive interference cancellation units (SICU) arranged in a multistage structure as illustrated in Figure 1. The basic successive interference cancellation unit is shown in Figure 2. The residual signal $\mathbf{e}(m,k)+\mathbf{I}(m,k)$ at the input of the m^{th}-stage, k^{th}-user SICU is first despreaded to estimate the decision variable $y(m,k)$ of the k^{th}-user at the m^{th}-stage, that is, $y(m,k)=\mathbf{s}_k^T\left(\mathbf{e}(m,k)+\mathbf{I}(m,k)\right)$.

The multiple access interference (MAI) termed $\mathbf{I}(m,k)$ due the k^{th}-user at the m^{th}-stage is obtained by spreading the residual decision variable $y(m,k)$ that is $\mathbf{I}(m,k)=\mathbf{s}_k y(m,k)$, which in turn is subtracted out from the residual signal $\mathbf{e}(m,k)+\mathbf{I}(m,k)$ to get a cleaned version of the residual signal $\mathbf{e}(m,k+1)$.

This process is repeated in a multistage structure as it is shown in Figure 1.

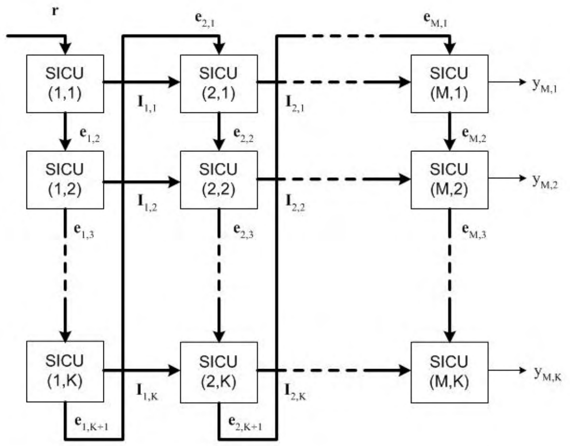

Fig 1: Multistage structure of the linear SIC detector.

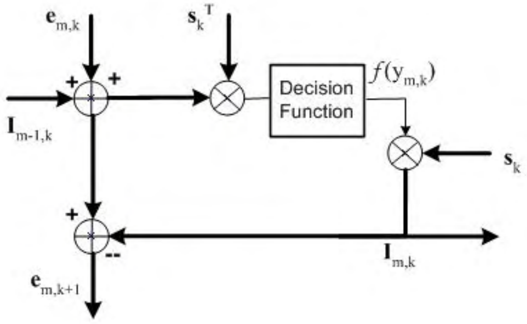

Fig 2: k^{th} user, m^{th} stage successive interference cancellation unit (SICU).

It easy to show that the general expression for the residual signal vector and the decision variable of the k^{th} SICU at the m^{th} stage, respectively, could be obtained as:

$$\mathbf{e}(m,k)=\mathbf{\Phi}_{k-1}(\mathbf{\Phi}_K)^{m-1}\mathbf{r} \tag{2}$$

and:

$$y(m,k)=\mathbf{s}_k^T\mathbf{\Phi}_{k-1}\sum_{i=0}^{m-1}(\mathbf{\Phi}_K)^i\mathbf{r}=\ \mathbf{g}^T(m,k)\mathbf{r} \tag{3}$$

where:

$$\mathbf{\Phi}_k=\coprod_{j=k}^{1}(\mathbf{I}-\mathbf{s}_j\mathbf{s}_j^T) \tag{4}$$

Collecting the decision variables of all users in one matrix we get:

$$\mathbf{y}(m)=\mathbf{G}^T(m)\mathbf{r} \tag{5}$$

where

$\mathbf{G}\ (m)=\left[\mathbf{g}\ (m,1)\ \mathbf{g}\ (m,2)\cdots\mathbf{g}\ (m,k)\cdots\mathbf{g}\ (m,K)\right]$

Hence, the linear SIC could be described as matrix filtering of the received chip-matched signal vector.

It could be shown easily as in [3], that when the number of stages M tends to infinity, the performance of linear SIC detector tends to that of the decorrelator detector, that is,

$$\mathbf{G}\ (\infty)\rightarrow\left(\mathbf{S}^T\mathbf{S}\right)^{-1}\mathbf{S}^T=(\mathbf{R})^{-1}\mathbf{S}^T \tag{6}$$

Where **R** is the cross-correlation matrix of the spreading codes.

IV. THE LOCAL SEARCH ALGORITHM

The multiuser detection problem is equivalent to the optimization of the following objective function [6]-[8]:

$$\begin{aligned}\phi_{ML}&=\arg\min_{\mathbf{b}\in\{-1,1\}^K}((\mathbf{Ab})^T\mathbf{R}(\mathbf{Ab})-2\mathbf{y}^T\mathbf{Ab})\\&=\arg\min_{\mathbf{b}\in\{-1,1\}^K}(\Omega(\mathbf{b}))\end{aligned} \tag{7}$$

To apply the algorithm to local-search we shall first convert the optimization problem to a binary 0-1 quadratic one. That is:

$$\begin{aligned}\phi_{ML}&=\arg\min_{\mathbf{b}\in\{-1,1\}^K}((\mathbf{Ab})^T\mathbf{R}(\mathbf{Ab})-2\mathbf{y}^T\mathbf{Ab})\\\sim\phi_{ML(0-1)}&=\arg\min_{\mathbf{x}\in\{0,1\}^K}(\frac{1}{2}\mathbf{x}^T\mathbf{Qx}+\mathbf{c}^T\mathbf{x})\end{aligned} \tag{8}$$

Let:

$$\mathbf{H}=\mathbf{A}^T\mathbf{RA},\quad \mathbf{x}=\frac{1}{2}(\mathbf{b}+\mathbf{e})\ \text{ where }\mathbf{e}=[1\ldots1]' \tag{9}$$

So the cost function $\Omega(\mathbf{b})$ becomes:

$$\begin{aligned}\Omega(\mathbf{b})&=\mathbf{b}^T\mathbf{Hb}-2\mathbf{y}^T\mathbf{Ab}\\&=(2\mathbf{x}^T-\mathbf{e}^T)\mathbf{H}(2\mathbf{x}-\mathbf{e})-2\mathbf{y}^T\mathbf{A}(2\mathbf{x}-\mathbf{e})\end{aligned}$$

$$\begin{aligned}&=4\mathbf{x}^T\mathbf{Hx}-2\mathbf{x}^T\mathbf{He}-2\mathbf{e}^T\mathbf{Hx}-4\mathbf{y}^T\mathbf{Ax}+(\mathbf{e}^T\mathbf{He}+2\mathbf{y}^T\mathbf{Ae})\\&=\Omega(\mathbf{x})\end{aligned}$$

The last two terms are just additive ones that have no direct influence on the metric parameter. Therefore, we discard them from the above expression:

$$\begin{aligned}\Omega(\mathbf{x}) &= 4\mathbf{x}^T\mathbf{Hx} - 4\mathbf{y}^T\mathbf{Ax} - 2[\mathbf{x}^T\mathbf{He} + (\mathbf{He})^T\mathbf{x}] \\ &= 4\mathbf{x}^T\mathbf{Hx} - 4\mathbf{y}^T\mathbf{Ax} - 4(\mathbf{He})^T\mathbf{x} \\ &= 4(\mathbf{x}^T\mathbf{Hx} - (\mathbf{yA} + (\mathbf{He}))^T\mathbf{x})\end{aligned} \tag{10}$$

Therefore:

$$\begin{aligned}\phi_{ML} &= \min_{\mathbf{b}\in\{-1,1\}^K}\{\Omega(\mathbf{b})\} \\ &= \min_{\mathbf{x}\in\{0,1\}^K}\{4(\mathbf{x}^T\mathbf{Hx} - (\mathbf{yA} + (\mathbf{He}))^T\mathbf{x})\} \\ &= \min_{\mathbf{x}\in\{0,1\}^K}\{\frac{1}{2}\mathbf{x}^T(2\mathbf{H})\mathbf{x} - (\mathbf{yA} + (\mathbf{He}))^T\mathbf{x})\}\end{aligned} \tag{11}$$

If we let: $\mathbf{Q} = 2\mathbf{H}$, and $\mathbf{c} = -(\mathbf{yA} + \mathbf{He})$, (11) becomes:

$\phi_{ML} = \min_{\mathbf{x}\in\{0,1\}^K}\{\frac{1}{2}\mathbf{x}^T\mathbf{Qx} + \mathbf{c}^T\mathbf{x}\}$ which is our target.

Local search (LS) algorithms are improved heuristics that search in the neighborhood of the current solution for a better one until no further improvement can be made, i.e. there is no better solution in the neighborhood in the current solution. The local search algorithm can be characterized by the neighborhoods it considers, in our case only the 1-opt neighborhood is considered. This neighbor can be obtained by flipping a single bit in the solution, i.e., the hamming distance between the current solution and the neighborhood solution is one.

Let $f(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{Qx} + \mathbf{c}^T\mathbf{x}$, and define the gain $g(m) = f(\mathbf{x}_m) - f(\mathbf{x})$, where $\mathbf{x}_m$ means that the m^{th} bit of vector $\mathbf{x}$ which is flipped, that is,: $1 - x_m = \overline{x}_m$. In our implementation, we search for a flip that gives the highest associated gain g ,that is,:

$m^* = \arg\max\{g(m), m = 1,\ldots, K\}$

Moving from one solution to another by flipping one bit only doesn't need the recalculation of the gains but needs an update. The update is calculated by the following formula:

$$g(j) = \begin{cases} g(j) + \Delta g_j(m^*); & j \neq m^* \\ -g(m^*) & ; j = m^* \end{cases} \tag{12}$$

with $\Delta g_j(m^*) = q_{jm}(x_j - \overline{x}_j)(\overline{x}_m - x_m)$

where q_{jm} is j^{th} row m^{th} column of the matrix $\mathbf{Q}$.

The 1-opt local search algorithm could be summarized as following:

```
Algorithm LocalSearch_OneOpt( $\mathbf{x} \in \{0,1\}^K$ )
    Step1: calculate gains $g(i)$ for all $i \in \{1,\ldots,K\}$
    Step2: find k with $g(k) = \max_i g(i)$
    Step3: $\overline{x}_k = 1 - x_k$
    Step4: update the gains $g(i)$ using (12)
    Step5: if $g(i) \le 0$ for $\forall i$ ,return $\mathbf{x}$, stop.
           Else go to step2
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V. THE PROPOSED LINEAR HYBRID MULTIUSER DETECTOR

The proposed linear hybrid interference cancellation multiuser detector is illustrated in Figure 3. The received signal $\mathbf{r}$ is passed through M stages of a linear SIC detector.

The vector of the hard decision variables at the output of the linear SIC $\hat{\mathbf{b}}_{SIC}$ is then passed through the 1-opt local search algorithm block to ultimately get the vector of the final hard data estimates $\hat{\mathbf{b}}$.

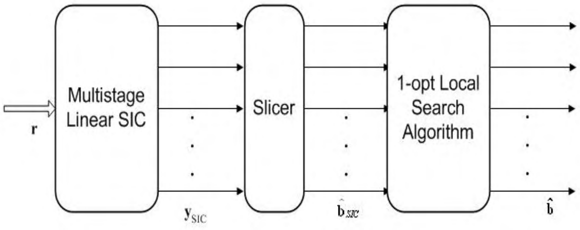

Fig 3: Multistage structure of the linear hybrid multiuser detector.

However, since the initial solution to the 1-opt local search algorithm is not the vector of the hard decisions of matched filter outputs but the hard decisions obtained at the output of the linear SIC detector, the objective function that should be optimized by the 1-opt local search algorithm should be changed to the following:

$$\begin{aligned}\phi_{ML} &= \arg\min_{\mathbf{b}\in\{-1,1\}^K}((\mathbf{Ab})^T\mathbf{R}(\mathbf{Ab}) - 2\mathbf{y}^T\mathbf{Ab}) \\ &= \arg\min_{\mathbf{b}\in\{-1,1\}^K}((\mathbf{Ab})^T\mathbf{R}(\mathbf{Ab}) - 2\left(\mathbf{G}_{SIC}^T(m)\mathbf{r}\right)^T\mathbf{Ab})\end{aligned} \tag{13}$$

Note that if m tends to infinity the performance of the proposed detector approaches that of a 1-opt local search algorithm with the initial solution as the decorrelator's output, that is,:

$$\begin{aligned}\phi_{ML} &= \arg\min_{\mathbf{b}\in\{-1,1\}^K}((\mathbf{Ab})^T\mathbf{R}(\mathbf{Ab}) \\ &\quad - 2\left((\mathbf{R})^{-1}\mathbf{S}^T\mathbf{r}\right)^T\mathbf{Ab}) \\ &= \arg\min_{\mathbf{b}\in\{-1,1\}^K}((\mathbf{Ab})^T\mathbf{R}(\mathbf{Ab}) - 2\mathbf{y}_{DEC}{}^T\mathbf{Ab})\end{aligned} \tag{14}$$

VI. SIMULATION RESULTS

In this section, we compare the performance of the proposed hybrid structure to that of the matched filter, decorrelator detector and finally to that of a 1-opt local search algorithm with an initial solution obtained from the output of the decorrelator detector. The number of users is set to 20. In addition, gold codes of length 32 are

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used, therefore the system could be considered as a heavy loaded system. Results of Figure 4 illustrate the following:

- The local search algorithm initiated by the output of the matched filter performs worse than the decorrelator detector and better than the matched filter detector.
- The local search algorithm performs better than the decorrelator detector for low SNR (less than 5dB).
- The initiation of the local search algorithm by the output of a 5-stage linear SIC detector is enough to achieve the performance of the local search algorithm initiated by the output of the decorrelator detector.
- The total complexity of the proposed detector is still of order K which less than K^3 (the complexity of the decorrelator and that of the local search algorithm initiated by the output of the decorrelator).

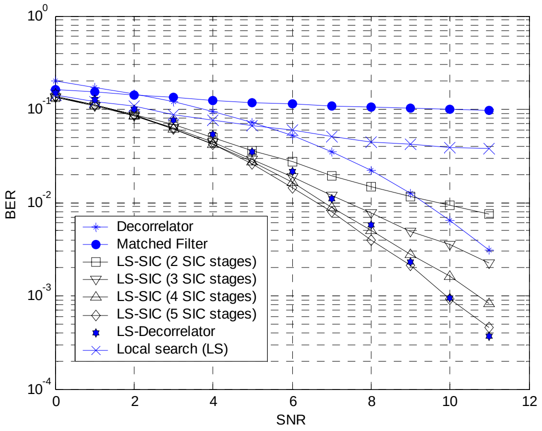

Fig 4: Average BER performance of different multiuser detectors.

VII. CONCLUSION AND SUMMARY

In this work, a new multiuser structure that combines both deterministic and stochastic algorithms is proposed is developed. The new scheme exhibits many desirable features such as: low complexity and excellent performance. Simulation results show that the proposed hybrid detector is an excellent tradeoff between complexity and performance.

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