Reduced Spatial Multiplexing and Diversity Gains due to Rician Fading and Antenna Correlation in MIMO Channels-A Review

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***Abstract*- There continues to be substantial interest in wireless communication systems that employ multiple transmit and receive antennas, due to their promise for dramatically increasing the performance and capacity to 20-40 bit/s/Hz when the system design is optimal. However, performance can deviate significantly from the ideal performance due to Rician fading and antenna correlation. In this paper we review the main developments related to the effect of non-ideal propagation conditions on the spatial multiplexing and diversity gains of MIMO systems. Various correlation and fading channel models are reviewed, for small and large number of antennas, various conditions on signal power and various performance criteria.**

***Index Terms*-Correlation, Diversity methods, MIMO systems, Multiplexing, Rician channels**

I. Introduction

Designing very high-speed links that offer good range capability on the wireless channel is a hard problem for several reasons. The wireless channel is a harsh time-varying propagation environment. A signal transmitted on a wireless channel is subject to interference, propagation path loss, delay spread, doppler spread, shadowing and fading. While it is possible to increase data rates by increasing the transmission bandwidth or using higher transmit power, both spectrum and transmit power are very constrained in a wireless system. The bandwidth, or spectrum, is prohibitively expensive. Increasing transmit power adds interference to other systems and also reduces the battery life-time of mobile transmitters.

The solution that has emerged over the past seven years is to exploit space, i.e., use multiple transmit and receive antennas. Pioneering work by Foschini and Gans [1], and Telatar [2] ignited much interest in this area by predicting remarkable spectral efficiencies for wireless systems with multiple antennas when the channel exhibits rich scattering and its variations can be accurately tracked. Multiple-input multiple-output (MIMO) systems can provide maximum capacity and/or diversity gains over channels with independent Rayleigh fading. When the fading is correlated or not Rayleigh MIMO gains are considerably reduced. Section II summarizes the main capacity results for ideal and actual channels. Section III provides similar results for MIMO diversity gain.

II. Capacity of MIMO Channels

Since the work of Foschini and Gans [1], and Teletar [2], there has been intense research activity in the area of MIMO systems. Here we will list some of the important capacity and diversity results obtained by the researchers in the area of MIMO systems.

A. *Idealized Channels*

Foschini and Gans analyzed the information-theoretic capacity of a multiple-antenna point-to-point wireless system in a narrow-band slowly Rayleigh-fading environment. They assume independent and identically distributed (i.i.d.) fading at different antenna elements, and assume that the transmitter does not know the channel while the receiver is able to track the channel perfectly. With T transmit and R receive antennas, the system is described by the matrix equation

$$\mathbf{y} = \sqrt{\frac{E_s}{T}}\mathbf{Hs} + \mathbf{n} \tag{1}$$

where E_s is the total energy available at the transmitter, $\mathbf{y}$ is the R x 1 vector of signals received on the R antennas, $\mathbf{s}$ is the T x 1 vector of signals transmitted on the T transmit antennas, $\mathbf{n}$ is the R x 1 noise vector consisting of independent complex Gaussian distributed elements with zero mean and variance σ^2, and $\mathbf{H}$ is the R x T channel matrix with components modeled as i.i.d. zero mean circularly symmetric complex Gaussian random variables (ZMCSCG) with unit variance. The capacity for this system is shown to be

$$C = E_{\mathbf{H}}\left\{\log_2 \det\left(\mathbf{I}_m + \frac{\rho}{T}\mathbf{HH}^H\right)\right\} \tag{2}$$

with

$$\rho = \frac{E_s}{\sigma^2}. \tag{3}$$

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Where $E_{\mathbf{H}}\{.\}$ denote the expectation over **H**, $m = min(T, R)$, and the operator $\mathbf{H}^H$ indicates the hermitian of the matrix **H**. The above equation can be decomposed using singular value decomposition (SVD) as

$$C = E_{\mathbf{H}}\left\{\sum_{i=1}^{k}\log_2\left(1+\frac{\rho}{T}\lambda_i\right)\right\}. \qquad (4)$$

Where k, $(k \le m)$ is the rank of **H**, and λ_i (i = 1, 2, ….., k) denotes the positive eigenvalues of $\mathbf{HH}^H$. Foschini and Gans show that the capacity C grows linearly with $min(T, R)$ for a given fixed transmitter power and bandwidth. In other words, without increasing the transmit power or bandwidth the capacity of the wireless channel can be increased by simply increasing the number of transmit and receive antennas. This is an enormous improvement compared to a logarithmic increase in more traditional systems utilizing receive diversity or no diversity.

Teletar [2] assumed that the channel state information is available only at the receiver and showed that for i.i.d slowly Rayleigh fading channels with T transmit and R receive antennas,

- Capacity C grows linearly with $min(T, R)$ for a given fixed transmitter power and bandwidth.
- For $T = 1$, Capacity increases logarithmically with the increase in the number of receive antennas R.
- For $R = 1$, Capacity does not increase at all with the increase in the number of transmit antenna T.

However, Teletar also showed that when the channel parameters are known at the transmitter, i.e., if the channel state information (CSI) is available at the transmitter, the capacity given by (4) can be increased by assigning the transmitted power to various antennas according to the "water-filling" algorithm [2].

$$C = E_{\mathbf{H}}\left\{\sum_{i=1}^{k}\log_2(\mu\lambda_i)^{+}\right\} \qquad (5)$$

Where μ is chosen to satisfy:

$$\rho = \sum_{i=1}^{k}\left(\mu - \lambda_i^{-1}\right)^{+} \qquad (6)$$

and "+" denotes taking only those terms which are positive.

B. Channels with Correlated Fading

Shiu et al. [3] investigated the effects of fading correlations in multi-element antenna (MEA) communication systems. They characterized the fading correlation for narrow band Rayleigh fading. They modeled correlated fading using one-ring scattering model as

$$vec(\mathbf{H}) = \mathbf{R}^{1/2}vec(\mathbf{H}_{\omega}). \qquad (7)$$

Where $vec(\mathbf{H})$ is the vector form of the **H** matrix and $\mathbf{R} = \operatorname{cov}(vec(\mathbf{H}))$, and $\mathbf{H}_{\omega}$ represents independent and identically distributed (i.i.d) Rayleigh fading channel. They showed that the effective degree of freedom i.e., the number of independent paths reduces as the correlation increases thus leading to the reduction of the system capacity.

While [3] provides a useful insight on the effect of correlated fading, the results are limited to the case of one end (i.e., either transmitter or receiver) correlation only. The analysis by Gesbert et. al. [4] accounts for both transmitter and receiver correlations using the eigenvalue decomposition technique, and modeling the MIMO correlated fading as

$$\mathbf{H}_{\mathbf{corr}} = \mathbf{R}_{\mathbf{r}}^{1/2}\mathbf{H}_{\omega}\mathbf{R}_{\mathbf{t}}^{1/2} \qquad (8)$$

where $\mathbf{R}_t$ and $\mathbf{R}_r$ are the correlation matrix at the transmitter and at the receiver side, respectively.

Lokya [5] investigated the MIMO channel capacity in correlated channels using the uniform and exponential correlation matrix model. Using uniform correlation matrix model, the correlation matrix **R** is defined as

$$r_{ij} = \begin{cases} r, & i \ne j \\ 1, & i = j \end{cases}, |r| \le 1. \qquad (9)$$

While using exponential correlation matrix model, the correlation matrix **R** is defined as

$$r_{ij} = \begin{cases} r^{\,j-i}, & i \le j \\ r^{*}{}_{ji}, & i > j \end{cases}, |r| \le 1 \qquad (10)$$

where r is the correlation coefficient between any two adjacent antennas, and "*" denotes the complex conjugate. Using the Jensen's inequality and approximations, the capacity of n x n MIMO Rayleigh fading channel in the presence of correlation is shown to be

$$C \approx n \cdot \log_2\left(1+\frac{\rho}{n}\left(1-|r|^2\right)\right) \qquad (11)$$

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The paper compared the two models and showed that the exponential model predicts better MIMO performance, which is obvious. The other findings were as follows

- As the correlation increases, the capacity decreases. In other words, the increase in correlation is equivalent to the decrease in SNR.
- Uniform correlation model predicts the worst-case scenario.
- For the exponential correlation model, correlation coefficient of 0.7 (r = 0.7) is same as 3dB reductions in SNR.
- For the exponential correlation model, the MIMO capacity decrease significantly for $r > 0.6$ which is in accordance with the measurement of MIMO channels.

C. Channels with non-Rayleigh Fading

Farrokh and Foschini [6] modeled the Rician fading channel as

$$\mathbf{H} = a\mathbf{H}^{sp} + b\mathbf{H}^{sc} \quad (12)$$

where the specular and scattered components of **H** are denoted by superscripts, $a > 0$, $b > 0$ and $a^2 + b^2 = 1$. The Rician factor, K is defined as a^2/b^2. Thus, the above **H** matrix can be written as

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}^{sp} + \sqrt{\frac{1}{K+1}}\mathbf{H}^{sc} \quad (13)$$

Khalighi's et al. [7] used a model similar to the above for the Rician fading channel. Using simulation, they showed that for uncorrelated Rician fading and with CSI being available only at the receiver, an increase in Rician factor:

- Reduces the capacity of the MIMO system.
- Increases the capacity of the SIMO system.

D. Channels with Correlation and non-Rayleigh Fading

Channels with non-Rayleigh Fading

Ayadi's et al. [8] derived an upper bound on the average Rician channel capacity for n x n MIMO system with CSI available at the receiver only, and showed that a limit of this capacity is given by the sum of the capacities corresponding to the LOS and Rayleigh components when they are considered separately. The upper bound is given as

$$\overline{C}_{Ric} \le C_{LOS} + C_{Ray} \quad (14)$$

where:

$$C_{LOS} = \log_2\left\{\det\left[\mathbf{I}_n + \left(\frac{\rho}{n}\right)a^2\mathbf{H}_{LOS}{}^H\mathbf{H}_{LOS}\right]\right\} \quad (15)$$

$$C_{Ray} = \log_2\left\{\det\left[\mathbf{I}_n + \left(\frac{\rho}{n}\right)b^2\mathbf{H}_{Ray}{}^H\mathbf{H}_{Ray}\right]\right\} \quad (16)$$

They selected two cases, one when the antenna elements are uncorrelated and the other when they are perfectly correlated. They found that the upper bound is more optimistic to be reached in the correlated antenna elements case than in the fully correlated antenna situation. They also found that for both the cases,

- Average upper bound on the Rician channel capacity is almost reached for small values of SNR.
- For low values of Rician factor, the average upper bound seems to be less tight than the one obtained in the case of high values of Rician factor.

Qaseem and Ali [9] used Monte Carlo simulation to calculate the capacity of MIMO systems, subject to Ricean fading, when the elements of the antenna systems experience correlated fading at the transmit, receive, or both sides. Ergodic and 10% outage capacities are obtained as functions of SNR for various values of the Ricean factor, K, with or without correlation at either the transmitter or at the receiver side or at both sides. The effect of CSI is considered as well. Ergodic and 10% outage capacities shows the same trend; however, ergodic capacity is nearly 3 dB higher than the 10% capacity.

III. DIVERSITY OF MIMO CHANNELS

Alamouti [10] presented a new diversity scheme (Alamouti's scheme). It assumes independent and identically distributed (i.i.d.) fading at different antenna elements, and that the transmitter does not know the channel while the receiver is able to track the channel perfectly. The important results obtained were as follows:

- Using two transmit antenna and one receive antenna, the new scheme provides the same order of diversity as the maximal-ratio receive combining (MRRC) with one transmit antenna and two receive antenna.
- The scheme can be easily generalized to two transmit antennas and R receive antennas to provide a diversity of $2R$.

When compared with MRRC, if the total radiated power is to remains the same, the transmit diversity scheme has a 3-dB disadvantage because of the simultaneous transmission of two distinct symbols from two antennas.

Tarokh et al. [11]–[12] first introduced Space-time codes to provide transmit diversity in wireless fading channel using multiple transmit antenna. They showed that for i.i.d slowly Rayleigh fading channels:

- Space–time block codes (STBCs) constructed from known orthogonal designs achieves full diversity and are easily decodable by maximum likelihood decoding via linear processing at the receiver, but suffers from the lack of coding gain.
- Space–time trellis codes (STTCs) posses both diversity and coding gain, yet is complex to decode and arduous to design.

Tarokh's et al. [13] showed that orthogonal space-time block codes (OSTBCs) decouple the vector detection problem into scalar detection problems, i.e., it converts a MIMO system to a corresponding SISO system, without any degradation in the performance, thereby significantly reducing decoding complexity. Thus, the diversity performance of the scalar codes is the same as that corresponding to vector codes. Applying OSTBC to the vector equation in (1), the corresponding scalar equation is

$$y = \sqrt{\frac{E_s}{T}}\|\mathbf{H}\|_F^{\,2}\, x + n \qquad (17)$$

where y denotes the scalar processed received signal, x is the scalar transmitted signal, n is AWGN, and $\|\mathbf{H}\|_F$ is the *Frobenius norm* of MIMO channel matrix **H**.

Sun and Reed [14] recently presented diversity analysis for MPSK in Rician fading channels. The error-rate performance using maximal-ratio combining (MRC) was expressed as a function of the mean signal-to-noise ratio (SNR), the Rician factor K, and the order D of diversity. The analysis made was not confined to any particular mode of channel diversity i.e., transmit, receive, frequency, space, polarization, and space-time codes. They assumed the fading channel to be identically independent i.e. the channels are uncorrelated, slow and frequency non-selective and perfect channel information is available at the receiver. The probability of symbol error for coherent MPSK over a Rician fading channel with Rician parameter K and diversity D is as follows:

$$P_s(E)=\frac{1}{\pi}\left(\frac{D+K}{\bar{\gamma}_s}\right)^D\cdot\int_{\pi/2}^{\pi/2-\pi/M}\frac{\exp\left(-\dfrac{K\sin^2\frac{\pi}{M}\sec^2\theta}{\frac{D+K}{\bar{\gamma}_s}+\sin^2\frac{\pi}{M}\sec^2\theta}\right)}{\left(\frac{D+K}{\bar{\gamma}_s}+\sin^2\frac{\pi}{M}\sec^2\theta\right)^D}d\theta \qquad (18)$$

where $P_s(E)$ is the probability of symbol error, D is the order of diversity, K is the Rician factor, $\bar{\gamma}_s$ is the average value of the SNR, and M represents M-ary PSK Modulation.

In a companion paper [15], Qaseem and Ali used Monte Carlo simulation to calculate the diversity of MIMO systems, subject to Ricean fading, when the elements of the antenna systems experience correlated fading at the transmit, receive, or both sides

Table I presents the gain in capacity as the fading statistics changes from Rayleigh (K=0) to Rician (K>0), for several vales of the spatial multiplexing gain (*m*). With no diversity (*m*=1), capacity increases as K increases and reaches its maximum value for Gaussian channel. With spatial multiplexing (*m*>1), the capacity reaches its maximum value on rich scattering (Rayleigh) channel, and decreases (negative gain) as the scattering component decreases.

TABLE I
CAPACITY GAINS FOR RICIAN FADING
COMPARED TO (K = 0, R = 0)
SNR = 10 dB, R = 0

m	Capacity Bps / Hz *K*= 0, *r*= 0	*K* = 0	Gain dB		
			K = 2	*K* = 4	*K* =10
1	2.903	0	0.203	0.32	0.449
2	5.546	0	- 0.250	- 0.47	- 0.74
4	10.932	0	- 1.605	- 2.53	- 3.73
6	16.372	0	- 3.220	- 4.92	- 7.08

Table II presents the decrease in capacity (negative gain) for correlated Rayleigh fading Channels. It can be seen that the decrease in capacity, in dB, is more for larger *m* and increased correlation

TABLE II
CAPACITY GAINS FOR CORRELATED CHANNELS
COMPARED TO (K = 0, R = 0)
SNR = 10 dB, $K=0$

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IV. Conclusion

This paper reviewed the main developments related to the effect of non-ideal propagation conditions on the spatial multiplexing and diversity gains of MIMO systems.

Various correlation and fading channel models have been presented.

For MIMO systems, as the value of *K* increases, capacity decreases for all values of SNR. For SIMO, MISO, and SISO systems, however, the existence of LOS component enhances the capacity. Correlation is seen to reduce the capacity of the system as expected. However, the correlation at the transmitter or at the receiver has the same effect on the capacity of the system. It is also interesting to note that the water-filling gains i.e., when the CSI is available at the transmitter over equal power i.e., when no CSI is available at the transmitter are significant at low SNR and reduces at high SNR.

The essence of the results is that correlation always reduces capacity, whereas the existence of LOS component can reduce MIMO capacity and enhances capacity for SIMO, MISO, and SISO systems.

m	Capacity bps/Hz K =0 r= 0	Gain dB				
		r= 0	r= 0.3	r = 0.5	r = 0.7	r = 0.9
1	2.903	0	0	0	0	0
2	5.546	0	-0.1	-0.4	-0.9	-1.5
4	10.932	0	-0.4	-1.2	-2.6	-4.9
6	16.372	0	-0.7	-2.1	-4.4	-8.4

References

[1] G. J. Foschini, M.J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, no. 3, pp. 311-335, Mar. 1998.

[2] I. E. Telatar, "Capacity of multi-antenna gaussian channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585-595, 1999.

[3] D. S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multi-element antenna systems," *IEEE Trans. Commun.*, vol. 48, pp. 502-513, Mar. 2000.

[4] D. Gesbert, H. Bolcskei, D. J. Gore, A. J. Paulraj, "Outdoor MIMO wireless channels: Models and Performance prediction," *IEEE Trans. Commun.*, vol. 50, pp. 1926-1934, Dec. 2002.

[5] S. L. Loyka, "Channel capacity of MIMO architecture using the exponential correlation matrix," *IEEE Commun. Lett.*, vol. 5, pp. 369-371, Sept. 2001.

[6] F. R. Farrokhi, G. J. Foschini, A. Lozano and R.A. Valenzuela, "Linkoptimal space-time processing with multiple transmit and receive antennas," *IEEE Commun. Lett.*, vol. 5, no. 3, pp. 85-87, March 2001.

[7] M. A. Khaligi, J.-M. Brossier, G. Jourdain, and K. Raoof, "On capacity of Rician MIMO channels", Globecome, 2001.

[8] J. Ayadi, A. Hutter and J. Farserotu, "On the Multi input multi output capacity of Rician channels", Globecome, 2002.

[9] S. M. Qaseem and Adel A. Ali, "Capacity of Correlated MIMO Ricean Channels" International Conference of Communication Computer and Power - ICCCP'05, February14-16, 2005, Muscat, Oman

[10] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Select. Areas Commun.*, vol. 16, pp. 1451-1458, Oct. 1998.

[11] V. Tarokh, N. Sheshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communications: Performance criterion and code construction," *IEEE J. Select. Areas Commun.*, vol. 17, pp. 451-460, Mar. 1999.

[12] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744-765, May 2003.

[13] ——— "Space-time block codes from orthogonal design," *IEEE Trans. Inform. Theory*, vol. 45, pp. 1456-1467, July 1999.

[14] J. Sun, and I. Reed "Linear diversity analysis for M-PSK in Rician fading channels" *IEEE Trans. Commun.*, vol. 51, pp. 1749-1753, Nov. 2003.

[15] S. M. Qaseem and Adel A. Ali, "Effect of Antenna Correlation and Ricean Fading on Capacity and Diversity Gains of Wireless MIMO Systems", International Symposium on Wireless Systems and Networks ISWSN'05, April 25-27 2005, Kingdom of Bahrain.

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