Application of Tagged User Approach in the Analysis of Finite-Users Finite-Buffer CSMA over fading channels

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***Abstract*— Tagged user analysis (TUA) is a generic approximate method of analyzing multiaccess protocols that decouples channel contention behavior from user queueing behavior. This technique allows the use of classical queueing theory results to be directly applicable to the analysis of buffered random access methods. In this paper, we analyze a finite-users finite-buffer CSMA system operating over a Rayleigh channel. We also consider some simulation examples which validate our analysis. It is seen that though TUA is an approximate technique, it estimates the system performance quite well.**

I. Introduction

Performance analysis of finite-users finite-buffer multiaccess systems has traditionally been a difficult problem; the exact mathematical analysis using markov models is too complex to be useful [1], [2]. Though each user is in itself a queueing system, queueing analysis cannot be applied to the problem due to dependance amongst users arising from the shared channel [3]. There have been a few attempts to approximately solve the problem exploiting the redundant information in the system [4]–[6]. Common assumptions employed are channel symmetry and existence of an equilibrium state for all users [3], [7], [8]. Unfortunately, most of the available techniques are either too involved or limited in scope, since they do not decouple the multiaccess behavior from queueing behavior. Tagged user analysis (TUA) is a unified technique that can be applied for the analysis of any random multiaccess system [9], [10]; the technique assumes symmetry and equilibrium conditions and uses queueing analysis to solve the system. The foregoing assumptions allow de-coupling of channel contention behavior and user queueing behavior, enabling one to solve the system for any queue discipline without complicating the analysis; in fact the whole analysis simplifies to the analysis of channel contention amongst the transmitting users. Until now, it has been applied only to systems operating in an ideal channel without capture [10]. In this paper we analyze a finite-users finite-buffer CSMA system operating in a Rayleigh channel with capture.

The paper is organized as follows. Section II formulates the system model followed by the TUA analysis in section III. In section IV, we consider the effects of fading channel on the protocol performance. Evaluation of different performance measures are discussed in section V. Finally, simulation results in section VI show the validity of the analysis.

II. System Model

We consider a homogenous centralized slotted non-persistent CSMA system with finite user population N and a finite buffer size L for each user, operating in a slowly fading uncorrelated Rayleigh channel. Packet generation for each user is assumed to occur at the end of a slot with probability λ. We assume an FCFS queue discipline with packet blocking and late arrival model with delayed replacement. Employing a DFT principle, a busy user senses the channel at the start of each slot with probability p: if the channel is sensed idle, it starts its transmission which takes T slots to complete, otherwise it backs off and senses the channel again in the next slot with the same probability p. After the packet is transmitted successfully, the user receives the acknowledgement immediately and deletes the packet from its head of queue. No acknowledgement from the receiver at the end of transmission implies unsuccessful attempt and the user retransmits in the next idle slot.

Assuming a sufficiently high signal to noise ratio, and hence ignoring the thermal noise effects, a transmission from a particular user is successful (it captures the channel) [11], if

$$P_t > z_0 P_i \quad \text{during } \tau, \quad 1 \le i \le N-1, \tag{1}$$

where P_t and $P_i = \sum_{k=1}^{i} P_k$ denote the received power of the concerned user and that of interference, respectively; z_0 is the receiver capture ratio and τ is the slot duration. A slow fading process is assumed such that the channel remains constant during the whole transmission period.
For simplicity we assume that acknowledgements are received error-free and channel sensing is instantaneous and unaffected by the fading process.

III. Tagged User Analysis

For a homogeneous system, using channel symmetry assumption, all users would be operating at an equilibrium state with statistically equivalent behavior. This allows us to concentrate on one representative user—the

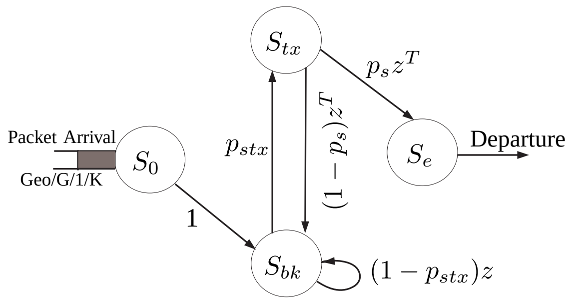

Fig. 1. State flow diagram of tagged user.

tagged user—and analyze its behavior. Once we analyze the tagged user, we have actually solved our multiaccess problem.

In TUA, the tagged user is modelled as a G/G/1/K queueing system consisting of a queue and a *virtual server* which is operating at its equilibrium state. A packet is said to be in the virtual server if the packet is ready for transmission (service) or being transmitted (served). Since the arrival process and queue discipline are already known, we only need *virtual packet service time distribution* (VPST), which is influenced by the interference from other busy users, to apply the standard queueing analysis and obtain all relevant performance measures of the tagged user (equivalently the system) [12]. It has been shown that VPST is a function of the busy probabilities of all users [9]. On the other hand, from queueing theory we know that the busy probability of each queueing system is a function of its service time distribution. In general, these functional relationships are non-linear, however they are sufficient and enable us to determine VPST and user busy probabilities which lead to the solution of the system.

The key step in TUA is to find the distribution of VPST for the tagged user (termed as channel contention analysis). Once this is obtained, the queueing system of the tagged user is identified and the corresponding results from the queueing theory can be applied to solve for the system performance measures (termed as queueing analysis).

A. Contention Analysis

To determine the distribution of VPST for the tagged user, we use a state flow graph technique [13], [14], as illustrated in Fig. 1, where S_0, S_{tx}, S_{bk}, and S_e are packet-ready state, transmission-start state, backoff state, and departure state of the tagged user, respectively. At the start of a new slot when a new packet arrives, or when there is already a packet waiting for transmission in the queue, the user enters busy mode and the state changes from S_0 to S_{bk}. At S_{bk}, the user will issue a transmission with probability p_{stx} or defer for one more slot with probability $1-p_{stx}$, where p_{stx} is given by

$$p_{stx} = p\, p_I, \tag{2}$$

and p_I is the probability of channel being idle. Denoting by p_s, the probability that a packet of the tagged user will be transmitted successfully (as per (1)), the transfer function from S_{tx} to S_e is given by $p_s z^T$.

From Fig. 1, (virtual) packet service time is the time duration from the user's entering S_0 to exiting S_e. Hence, the probability generating function $B(z)$ of the packet service time is the transfer function from S_0 to S_e,

$$B(z) = \frac{p_{stx}\, p_s z^T}{1-(1-p_{stx})z - p_{stx}(1-p_s)z^T}. \tag{3}$$

From the state flow graph of tagged user in Fig. 1, at state S_{tx} when the tagged user is about to start its transmission, it must be busy and would also have sensed the channel idle. Since at that time all other busy users in the system would also sense the channel idle we have

$$p_s = \sum_{i=0}^{N-1} \binom{N-1}{i} (pp_b)^i (1-pp_b)^{N-1-i} p_{s|i}, \tag{4}$$

where we have used the statistical equilibrium assumption, and p_b and $p_{s|i}$ denote steady-state user busy probability, and the conditional success probability that the tagged user transmission is successful given i other simultaneous transmissions, respectively.

From (2)–(4), we observe that $B(z)$ is a function of two unknowns p_b and p_I. To apply our TUA model, we have to find a relationship between these two, since queueing theory can provide us only with a functional relationship between $B(z)$ and p_b. An approximate relationship can be found using classical analysis of CSMA [15]–[17]. When the system is in equilibrium state, the time durations in the system would consist of sequence of "cycles"; each cycle consists of an idle duration followed by a busy duration which corresponds either to successful transmission or collision. Given the tagged user is busy, the mean length of the idle duration in each cycle is

$$E(I) = \frac{(1-p)(1-p\,p_b)^{N-1}}{1-(1-p)(1-p\,p_b)^{N-1}}. \tag{5}$$

Denoting by q_u the probability that the busy duration in a cycle corresponds to a successful packet transmission from the tagged user, we obtain

$$q_u = \frac{p\,p_s}{1-(1-p)(1-p\,p_b)^{N-1}}. \tag{6}$$

Denoting by p_{ctu} the probability that, in a busy period, un-successful transmission would involve the tagged user, we would get

$$p_{ctu} = \frac{p\,(1-p_s)}{1-(1-p)(1-p\,p_b)^{N-1}}. \tag{7}$$

In each cycle, we note that the tagged user would sense the channel idle in each idle slot and in the first slot of each busy duration, which may correspond later to a successful transmission or not. Hence, in each cycle

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the average number of possible slots in which the tagged user would sense the channel idle is $E(I)+1$. However, in *each cycle* the average number of slots in which the tagged user is *able to sense the channel* is given by $E(I)+1+(T-1)(1-q_u-p_{ctu})$. Therefore the probability that the channel is sensed idle by the tagged user given it senses channel is

$$p_I = \frac{E(I)+1}{E(I)+1+(T-1)(1-q_u-p_{ctu})}. \quad (8)$$

From (5)–(8), we observe that $B(z)$, given by (3), is solely a function of user busy probability p_b or quivalently $B(z)$ *is only a function of user idle probability* p_0. Now we need to use the queueing theory to find another functional relationship between p_0 and $B(z)$ to solve our multiaccess problem.

B. Queueing Analysis

From (3), the queueing model of tagged user is identified as a Geo/G/1/K system. Using the queueing analysis, we can obtain another set of equations relating p_0 and $B(z)$. Corresponding results from [12] are presented here.

If we let a_k the probability that k packets arrive at the tagged user buffer during a packet service time, then the corresponding probability generating function is

$$A(z) = \sum_{k=0}^{L-1} a_k z^k = B(1-\lambda+\lambda z), \quad (9)$$

since at most $L-1$ packets arrive in one service time. Denoting by $\pi_i, i=0,1,\ldots,L-1$, the probability that a leaving packet sees *i* packets in queue, we have the following recursive algorithm [12]:

$$\begin{aligned} &\pi_k = \pi_0 \pi'_k, \quad 0 \le k \le L-1\,, \\ &\pi'_0 = 1\,, \\ &\pi'_{k+1} = \frac{1}{a_0}\Big(\pi'_k - \sum_{j=1}^{k} \pi'_j a_{k-j+1} - a_k\Big),\ 0 \le k \le L-2 \end{aligned} \quad (10)$$

where $\pi'_i, i=0,1,\ldots,L-1$ is an intermediate variable. Using (10) and the total probability theorem, we obtain

$$\pi_0 = \left(\sum_{i=0}^{L-1} \pi'_k\right)^{-1}. \quad (11)$$

Letting $p_k, 0 \le k \le L$, the probability that there are k packets present in the system at any slot boundary, we have

$$\begin{aligned} p_k &= \frac{\pi_k}{\pi_0+\rho}, \qquad 0 \le k \le L-1, \\ p_L &= 1 - \frac{1}{\pi_0+\rho}\,, \end{aligned} \quad (12)$$

where we define

$$b = B'(1), \quad (13)$$

$$\rho = \lambda\, b. \quad (14)$$

C. An Iterative Algorithm

Since the equations relating p_0 and $B(z)$ are nonlinear, making it difficult to obtain closed-form solutions, we have to use numerical methods. An iterative algorithm to solve the equations can be developed as follows.

1) Select a very small positive number δ, (say $\delta = 10^{-8}$), and set an initial value for p_0 between zero and one.
2) Calculate $B(z)$ using (2)–(8) and (19).
3) Calculate $p_k\,, 0 \le k \le L$ using (9)–(14) to obtain a new estimate $\hat{p_0}$.
4) Calculate $\epsilon = (\hat{p_0} - p_0)/p_0$.
   a) If $|\epsilon| \le \delta$ then p_0 and $B(z)$ are obtained.
   b) If $\epsilon > \delta$, let $p_0 = \hat{p_0}$ and go to step 2.
   c) Else, let $p_0 = \hat{p_0} + \epsilon$, and go to step 2.

A finite buffer system may be either locally stable or globally stable; a system is locally stable when it has two (or more) operating points. On the other hand, globally stable system has only one operating point [3]. TUA can be used to learn about the stability of the system. In the case of a locally stable system, different initial values for p_0 will converge to different operating points. While, for the globally stable system, we would obtain the same results for any initial guess.

IV. Channel Considerations

The conditional packet success probability $p_{s|i}$ in (4) depends not only on the state of other users in the system but also on channel and receiver characteristics. For a slowly fading channel using (1)

$$\begin{aligned} p_{s|i} &= \Pr[P_t > z_0 P_i] = 1 - \Pr\big[\frac{P_t}{P_i} < z_0\big] \\ &= 1 - \Pr[Z_i < z_0], \quad i = 1,\ldots,N-1. \end{aligned} \quad (15)$$

where $Z_i \triangleq P_t/P_i$. Using independence of P_t and P_i, and simple transformation of random variables

$$p_{s|i} = \int_{z_0}^{\infty} dz \int_{0}^{\infty} f_{P_t}(zw) f_{P_i}(w) w\, \mathrm{d}w \quad (16)$$

where $f_{P_t}(p_t)$ and $f_{P_i}(p_i)$ are the respective p.d.f.s of the received power of the test packet (tagged user transmission) and that of interference.

A. Slow Rayleigh Fading Case

Assuming all packets are received through uncorrelated Rayleigh channel with equal mean power $\overline{P}_k = P_0, 1 \le k \le N$, the p.d.f. of the power of kth received signal, $f_{P_k}(p_k)$, will be exponentially distributed,

$$f_{P_k}(p_k) = \frac{1}{\overline{P_k}} \exp\Big[-\frac{p_k}{\overline{P}_k}\Big]. \quad (17)$$

For non-coherent demodulation, the p.d.f. of joint interference power of i interferers is found by i-fold convolution of (17) to be the gamma distribution

$$f_{P_i}(p_i) = \frac{1}{P_0} \frac{(p_i/P_0)^{i-1}}{(i-1)!} \exp\Big[-\frac{p_i}{P_0}\Big]. \quad (18)$$

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Substituting (17) and (18) in (16), and carrying out the integration we obtain

$$p_{s|i} = \frac{1}{(z_0+1)^i}, \qquad (19)$$

which is a well-known result [11], [18].

V. Performance Measures

In a multiaccess network, we are interested in parameters such as channel throughput, user queue length, packet response time, and wait time. Since we modelled the system as a queueing system, these performance measures can be easily found using queueing theory [12].

1) Packet Blocking Probability: An arriving packet will be rejected (blocked) if it finds the buffer full, which occurs with probability p_L. Hence, using (12),

$$p_B = p_L = 1 - \frac{1}{\pi_0 + \rho}. \qquad (20)$$

2) Channel Throughput: The user throughput θ is defined as average number of successful transmissions per slot, i.e.,

$$\theta = \lambda(1-p_B)T, \qquad (21)$$

since $\lambda(1-p_B)$ is the average rate of packet acceptance into a user buffer and T is the packet transmission time. Employing channel symmetry, the channel throughput Θ is

$$\Theta = N.\theta. \qquad (22)$$

3) Queue Length: If we let I_q denote the number of packets in a user buffer, then using (12) we have

$$E(I_q) = \sum_{k=0}^{L} k.p_k, \qquad (23)$$

where we have used our assumption that all users are statistically equivalent at equilibrium point.

4) Waiting Time and Response Time: The packet response time t_r is the time that a packet stays in the system from its arrival to departure. Using Little's theorem

$$E(t_r) = \frac{E(I_q)}{\lambda(1-p_B)}. \qquad (24)$$

Similarly, the packet waiting time t_w is the time elapsed from packet arrival to its being ready for transmission, which is

$$E(t_w) = \frac{E(I_q)}{\lambda(1-p_B)} - b. \qquad (25)$$

VI. Numerical Results

We consider two examples of TUA application. In the first, we investigate the performance of a CSMA system in a slow fading Rayleigh channel. Our second example deals with an approximately optimal CSMA system in terms of channel throughput, as described in [8].

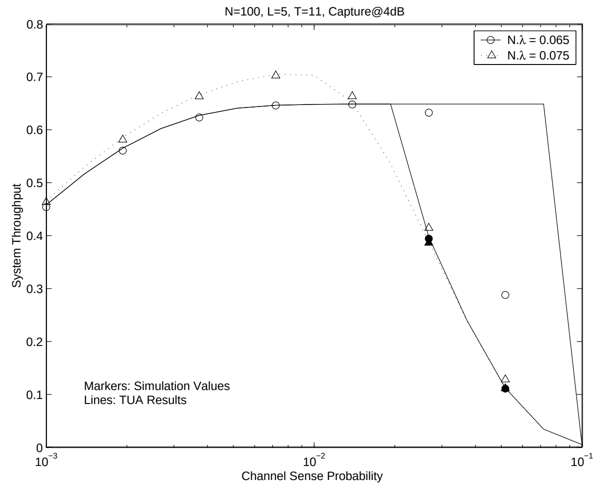

Fig. 2. Channel throughput of a CSMA system operating in a fading channel: $L = 5, N = 100, T = 11$

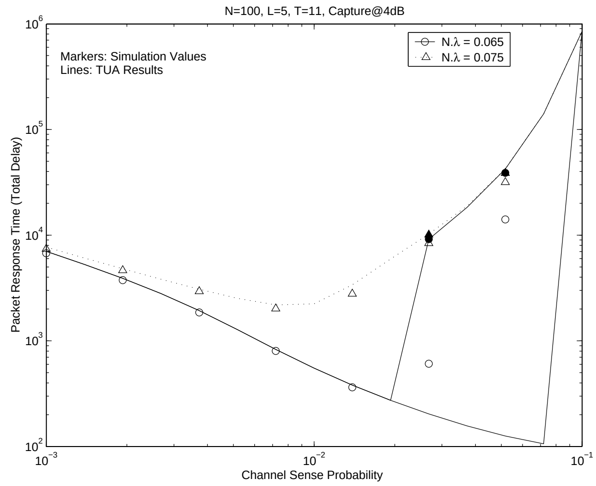

Fig. 3. Packet response time of a CSMA system operating in a fading channel: $L = 5, N = 100, T = 11$

A. CSMA in Rayleigh Fading

The system considered here is with $N = 100, L = 5$ and $T = 11$. We assume a slowly fading Rayleigh channel with receiver signal capture ratio 4dB. Fig. 2–4 show the simulation results for system throughput, packet response time, and blocking probability plotted against TUA analytical results for $N.\lambda = 0.065$ and 0.075. The simulation was run for 200000 slots and the results were averaged over 8 runs. To investigate the system stability, the experiment was carried out with two different initial conditions for the user buffers. The empty symbols were obtained under an empty buffer initial condition while solid symbols denote the results obtained under a full buffer initial condition. It is observed that the system becomes bistable for $N.\lambda = 0.065$ for $p \geq 0.02$. Furthermore, the simulation results do not match well with TUA analysis in this bistable region. This is because that the system may oscillate between the locally stable operating points; though TUA gives the two operating points, it cannot account for

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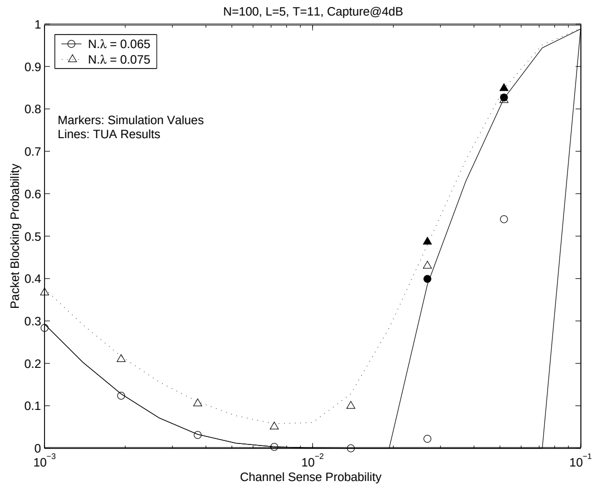

Fig. 4. Packet blocking probability of a CSMA system operating in a fading channel: $L = 5, N = 100, T = 11$

the oscillations among them. Nevertheless, it gives the bounds for system performance.

B. A Near Optimal CSMA System

Here we consider a slightly modified form of a system considered in [8] with $N = 10$ and $T = 40$. In each slot, we let $p = 1/n^\circ$, where n° is the number of busy users at the beginning of the slot. It is proved in [8] that for a CSMA/CD system working in an ideal channel, this provides optimal system performance in terms of maximum channel throughout. However, our scenario differs from that of [8], in the sense that not only do we assume a CSMA system but also a slowly fading Rayleigh channel.

To apply TUA to this system, we note that when the tagged user contends for the channel, there are, on average, $n^\circ = 1 + (N-1)p_b$ busy users in the system. Thus

$$p = \frac{1}{1 + (N-1)p_b}. \tag{26}$$

The numerical and simulation results are given in Fig. 5 and 6 for a receiver capture ratio of 4dB. As before the lines denote analysis results using TUA while markers correspond to simulation values.

VII. Conclusions

Tagged user analysis is an approximate technique that can be applied to the analysis of buffered random multiaccess systems. The simplicity and power of TUA lies in its decoupling of channel contention analysis from user queueing analysis. Thus, there is no extra complexity incurred by arbitrary queueing disciplines.

In this paper, we have analyzed CSMA protocol operation in a slowly fading channel using tagged user analysis. It has been shown, through simulation experiments, that the TUA analytical results, though approximate in nature, are quite good in estimating system performance. In fact, in the case of CSMA, the approximation error in TUA not only results from the steady state equilibrium

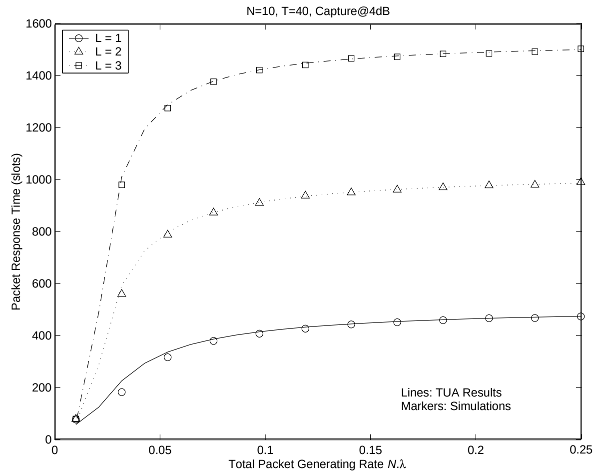

Fig. 5. Packet response time of an approximately optimal CSMA system in a fading channel: $N = 10, T = 40$

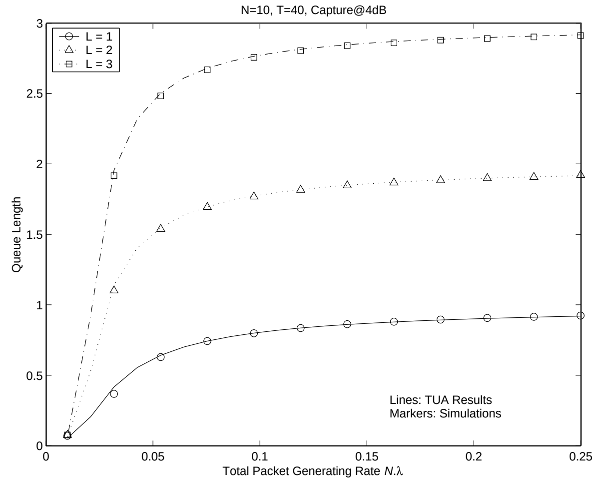

Fig. 6. Average queue length of an approximately optimal CSMA system in a fading channel: $N = 10, T = 40$

assumption but also from the approximate technique used in section III-A to find a relationship between p_b and p_I.

Acknowledgment

The authors acknowledge the support of KFUPM to carry out this work. The work reported in this paper is supported by KACST under project LG 6-8.

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