

# Fast Adaptive Filter based PN Code Acquisition system in Multiuser Environment

Muhammad Hamid

Electrical & Electronics Engineering Department  
Hafr Al-Batin Community College, KFUPM  
Hafr Al-Batin 31991, Saudi Arabia  
mhamid75@yahoo.com

Asrar. U. H. Sheikh

Electrical Engineering Department  
King Fahd University of Petroleum and Minerals  
Dhahran 31261, Saudi Arabia  
asrarhaq@kfupm.edu.sa

**Abstract**—In CDMA systems, the capacity of acquisition systems is essentially limited by the number of simultaneous users that can achieve and maintain code synchronization rather than by the number of users that can maintain a certain BER performance. This paper presents modified PN code acquisition hybrid scheme employing adaptive filter and it has been shown that for the same SNR conditions, the modified system takes less time for code acquisition in Multiuser environment compared to other schemes as well as it can achieve the BER based capacity.

## I. Introduction

Code synchronization is a very important and essential part of any spread spectrum system in order to remove the spreading effect induced by the transmitter and to exploit the processing gain of the spread signal. Code synchronization must be maintained for the duration of communication. Code synchronization is usually developed over two steps, namely acquisition and tracking ([1] [2] [3] [4]). Acquisition is the process by which the two codes are coarsely aligned such that the phase offset between them is within a chip duration. Tracking is used to minimize the delay offset within a certain fraction of a chip and to maintain the time alignment of the two codes.

Two principal methods are used to acquire the code phase. In serial search method, a locally generated PN code is correlated with the incoming sequence and the correlator output is compared to a threshold and either the position or maximum correlation or the first correlation that exceeded a preset threshold is selected [5]. This technique has poor performance especially if the uncertainty region is large. Serial search has light complexity requirements. In parallel, all chip phases are tested simultaneously, using either convolvers [6], or matched filters [7]. Parallel acquisition is fast but the system can be prohibitively expensive if spreading sequence lengths of more than a few hundred chips are used. A hybrid acquisition system is a compromise between acquisition speed with hardware complexity. This is achieved by dividing the uncertainty region into disjoint intervals of more than one cell. The system starts searching the intervals sequentially while the cells within each interval are tested concurrently.

In CDMA systems, the Multiple Access Interference (MAI) can be dominant and the acquisition performance of both the simple correlator and the matched filter schemes is significantly degraded. Hence the capacity of such systems

is limited by the number of simultaneous users that can achieve and maintain code synchronization rather than by the number of users that can maintain a certain BER performance as explained in [8]. So using a more efficient code acquisition scheme can improve the capacity of such systems. This degradation is compounded when a near far problem exists. Therefore improving the acquisition system performance in CDMA systems by using more advanced near far resistant acquisition schemes would improve the actual capacity of the system to achieve the BER-based capacity.

A Near far resistant PN code Acquisition system using MMSE equalizer was proposed in [9]. It was shown that the system can achieve the reliable acquisition in the presence of the strong MAI, but the adaptation time was very long. Moreover the complexity of the system is very high since the equalizer length should be the same as the code length.

Then a new hybrid scheme for Code Acquisition in MAI in [10] using adaptive filters was proposed which significantly outperforms the conventional matched filter based acquisition schemes. It improves the acquisition based capacity because it takes into consideration the presence of the Multiuser Interference while finding the optimum tap weight setting.

In this paper, we introduce the modified hybrid adaptive filter (AF) based Code Acquisition scheme proposed in [11] in Multiuser environment and it has been shown that for the same SNR conditions, the modified system takes less time for code acquisition in MAI environment than that explained in [10] as well as it can achieve the BER based capacity. Section II presents the system description while acquisition performance is given in Section III, Section IV presents the Analytical and simulation results and the paper ends with conclusions in Section V.

## II. Acquisition System Description

The received baseband signal is given by

$$r(t) = \sum_{k=1}^K x_k(t - \tau_k) + n(t) \quad (1)$$

where  $K$  is the number of users,  $n(t)$  is AWGN,  $\tau_k$  is the delay of the  $k$ th user, and  $x_k(t)$  is the  $k$ th user DS/SS signal given by

$$x_k(t) = \sum_{n=-\infty}^{\infty} d_k(t - nT_b) \sum_{j=0}^{L-1} c_k(j) \prod_{T_c} (t - jT_c) \quad (2)$$

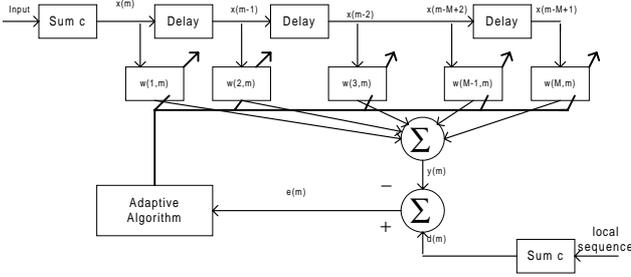


Fig. 1. Schematic of an Adaptive Filter for the proposed Code Acquisition System

where  $\{d_k\}$  is the  $k$ th user information sequence with  $d_k(m) = \pm 1$ ,  $\{c_k\}$  is the  $k$ th user maximal length spreading sequence,  $L$  is the spreading code length and  $\prod_{T_c}$  is a rectangular shaping pulse with unit amplitude and duration  $T_c$ .

The received signal is sampled at the chip rate to produce the sequence

$$r(i) = \sum_{k=1}^K \sum_{n=-\infty}^{\infty} d_k(t - nT_b) \sum_{j=0}^{L-1} c_k(j) \prod(i - j - \gamma_k) + n(i) \quad (3)$$

where we have assumed that the delays are integer multiples of the chip duration, i.e.  $\tau_k \in \{0, 1, 2, \dots, L-1\}$ . This assumption (chip synchronous) is merely to simplify the analysis and does not affect the results significantly. Notice that the dependence on the chip duration has been dropped for notational convenience.

The structure of the proposed adaptive filter based code acquisition system is shown in Fig 1. The working of the acquisition system has been explained in detail in [11]. Our objective is to estimate the delay of the desired user,  $\tau_1$ , from the composite received signal,  $\{r(i)\}$ .

The first task of the system is to find the neighborhood in which the correct phase delay of the desired user is located, before it zeroes it on the phase itself.

The desired sequence is formed by adding  $c$ , called a cell, of the spreading code of the desired user at a certain phase offset into the adaptive filter. Thus

$$u(m) = \sum_{i=1}^c c_1(mc + i) \quad (4)$$

The integer  $c$  is chosen so that  $cM = L$ , the length of the spreading sequence. Note that  $c$  can be chosen so that  $cM \approx L$  if  $cM \neq L$ . The same number,  $c$ , of the received samples are  $\mathbf{r}_m$  also added to form the input to the Adaptive Filter.

$$r(m) = \sum_{i=1}^c a(mc + k + i) + n(mc + k + i) \quad (5)$$

The adaptive filter is clocked at  $cT_c$ , so that the output of

the adaptive filter is given by the convolution sum

$$y(m) = \sum_{j=0}^{M-1} \mathbf{w}_j(m) \mathbf{x}(m-j) = \mathbf{w}^T(m) \mathbf{x}(m) \quad (6)$$

where  $\mathbf{w}(m) = [w_0(m) \ w_1(m) \ \dots \ w_{M-1}(m)]^T$  is the tap weight vector of the AF at  $mcT_c$ , and  $\mathbf{r}(m) = [r(m) \ r(m-1) \ \dots \ r(m-M+1)]^T$  consists of the present and the past  $M-1$  samples of the sums of the filter input vector.

An error signal is formed from the AF output and the reference signals such that

$$e(m) = u(m) - y(m) \quad (7)$$

which is used to adjust the filter taps according to the **LMS** algorithm

$$\mathbf{w}(m+1) = \mathbf{w}(m) + \mu e(m) \mathbf{r}(m) \quad (8)$$

where  $\mu$  is the **LMS** algorithm step size which controls the convergence speed and the steady state mean square error. Following convergence of the tap weights, the maximum of the tap weights is identified. As will be seen later, a record of the maximum tap weights will indicate the location of the cell where the chip phase delay can be found.

Once the correct cell is located, the system will move on to acquire the phase delay itself. It does this by changing the clock rate of the adaptive filter to  $T_c$ , and loading the delay line elements with the subsequence that is half  $M$  chips ahead of the cell which has been pinpointed. A fixed number of samples are taken, and the tap where the majority of the peaks occur will indicate the position of the delay phase.

According to [12], the optimum tap weight vector of the FIR adaptive filter is given by the Wiener Hopf Equation

$$\mathbf{w}_{opt} = \mathbf{R}^{-1} \mathbf{p} \quad (9)$$

where  $\mathbf{R} = E\{\mathbf{r}(m) \mathbf{r}^T(m)\}$  is the  $M \times M$  autocorrelation matrix of the AF input vector, and  $\mathbf{p} = E\{\mathbf{r}(m) u(m)\}$  is the  $M \times 1$  cross correlation vector between the AF input and the (desired user) spreading sequence.

The elements of  $\mathbf{R}$  can be shown to be

$$r_{pq} = \begin{cases} c(1 + \sigma^2) & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases} \quad (10)$$

for  $p, q = \{0, 1, 2, \dots, M-1\}$ .

The elements of the cross correlation vector between  $u(m)$  and  $r(m)$  is an  $M \times 1$  vector with elements  $p(q)$  where  $q = \{0, 1, 2, \dots, M-1\}$  and is

$$p(q) = \begin{cases} 0 & \text{if } a(n)'s \text{ do not match at all} \\ v & \text{if } a(n)'s \text{ match in } v \text{ places with} \\ & \text{superiormatch} \\ (c-v) & \text{if } a(n)'s \text{ match in } (c-v) \text{ places} \\ & \text{withinferiormatch} \end{cases} \quad (11)$$

Based on the previous observations, the optimum tap weight vector may be visualized as to have a peak at the tap corresponding to the correct delay offset i.e. corresponding to the cell where the delay offset is located. The decision strategy is to find the set of optimum coefficients  $\mathbf{w}_{opt} = \{w_0 \ w_1 \ \dots \ w_{M-1}\}^T$ , and then select the delay offset that corresponds to the maximum coefficient.

### III. Acquisition Performance

The performance of the proposed AF acquisition system can be measured by the probability of cell acquisition failure,  $P_{wc}$ . This occurs when the maximum of the optimum filter taps does not correspond to the correct delay estimate, i.e.

$$P_{wc} = 1 - P(w_\alpha \geq w_0, w_\alpha \geq w_1, \dots, w_\alpha \geq w_{M-1}) \quad (12)$$

where  $w_\alpha$  is the AF coefficient that corresponds to the cell having correct delay offset and  $P(w_\alpha \geq w_0, w_\alpha \geq w_1, \dots, w_\alpha \geq w_{M-1})$  is the joint probability density function (PDF). The exact PDF is difficult to derive, however a very good Gaussian approximation has been reported in [13]. The tap weight vector of the LMS adaptive filter has been shown to follow a gaussian random distribution with mean vector of

$$\underline{m} = \underline{w}_{opt} \quad (13)$$

and covariance matrix of

$$\mathbf{C} \approx \frac{\mu}{2} J_{min} \mathbf{I} \quad (14)$$

where  $\mathbf{I}$  is an identity matrix,  $\mu$  is the adaptation step size and  $J_{min}$  is the minimum mean square error (MMSE) given by

$$J_{min} = \sigma_d^2 - \mathbf{p}^T \mathbf{R}^{-1} \mathbf{p} \quad (15)$$

with  $\sigma_d^2 = c$  is the power of the desired sequence. Equation (14) indicates that the tap weights are uncorrelated. The acquisition failure probability can be written as [14]

$$P_F = 1 - \frac{1}{\sqrt{2\pi\sigma_w^2}} \int_{-\infty}^{+\infty} [1 - Q(\frac{w_c}{\sigma_w})]^{M-2} [1 - Q(\frac{w_c - m_\beta}{\sigma_w})] \exp(-\frac{(w_c - m_c)^2}{2\sigma_w^2}) dw_c \quad (16)$$

where  $Q(x)$  is the Q function and  $m_c$  and  $\sigma_w^2$  are the mean and variance of the correct tap which has the largest match between the incoming sequence and local sequence respectively whereas  $m_\beta$  is the adjacent cell which has an inferior match. We have also assumed that all the coefficients have the same variance  $\sigma_w^2$  obtained from the diagonal elements of Eq 14. This is a reasonable assumption especially when the SNR per chip is small.

### IV. Results

We consider a CDMA system with code length of  $L = 127$  chips. Fourteen cells are used, with each cell containing 9 chips, except for the last that contains 10 chips. For

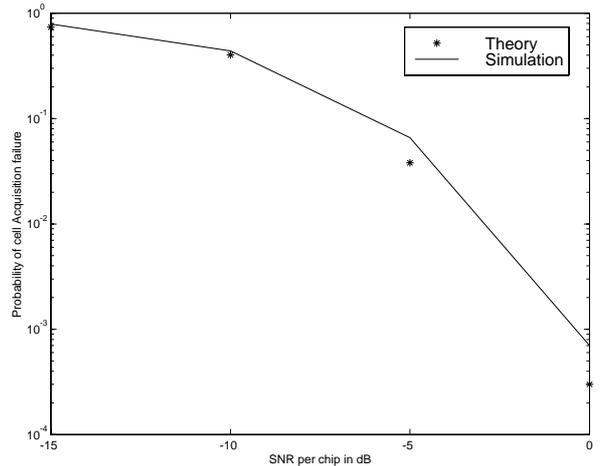


Fig. 2. Theoretical and Simulation results for Probability of Cell Acquisition Failure against  $SNR_c$  for a system using spreading sequence of 127 chips.

the cell identification stage, a value of 0.0001 is used for  $\mu$ . The adaptation period for the filter is selected as multiple of the code length.

Fig 2 shows  $P_F$ , the probability of identifying the cell wrongly for  $SNR_c$  ranging from 0 dB to -15 dB, using just a single sample for a short length spreading sequence of  $L = 127$  in Multiple Access Interference with a maximum of 4 users. The decision criteria is the same as taken in [11] for single user. The adaptation period is taken to be of ten 127 chip sequences. The tap where the peak occur will indicate the position of the cell where the chip phase delay can be found. Here we have one desired user and three interfering users. The interfering users have their data as well while the desired user has been assumed to be without data sequence. The spreading sequence for the users have been generated with a linear shift register of length 7 and with different preferred pair combinations. For the desired user, the delays are generated with uniform distribution over the code length. We can see that both theoretical and simulation results agree quite well.

Fig 3 shows the effect of the **LMS** adaptation period on the acquisition failure probability with different number of users in a Multiple Access Interference environment for  $SNR_c$  of -5 dB. Here we have generated spreading codes for users by using different initial loads of the shift register. The step size parameter is taken to be 0.0001 as in single user environment.

Simulation results are presented here for adaptation periods of 6, 8 and 12 code periods respectively for cell identification stage. For the phase identification stage, the adaptation period is taken to be one code length. The capacity of the system is significantly improved from 4 users with 90% success probability at 6 code periods adaptation time to nearly 40 users at 12 code periods. This is because the

longer the adaptation period, the higher the probability that the  $AF$  coefficients converge to the optimum value. But a longer acquisition time is incurred because the  $AF$  should test all cells for a longer period and then phase itself. It has been reported in literature [10] that for 10% failure probability, Matched Filter can support up to 10 users at most which is significantly smaller than that supported by proposed  $AF$  scheme and the one proposed earlier.

The capacity of a  $DS/CDMA$  system based on bit-error-rate (BER) performance can be computed using a Gaussian approximation [15] and the results indicate that a CDMA system with perfect power control and processing gain of 127 can support up to 40 users with BER of  $10^{-3}$  in AWGN. Hence Matched Filter (MF) system has a very limited capacity, while the previously proposed hybrid  $AF$  scheme as well as our modified scheme can achieve the capacity obtained by the BER criterion.

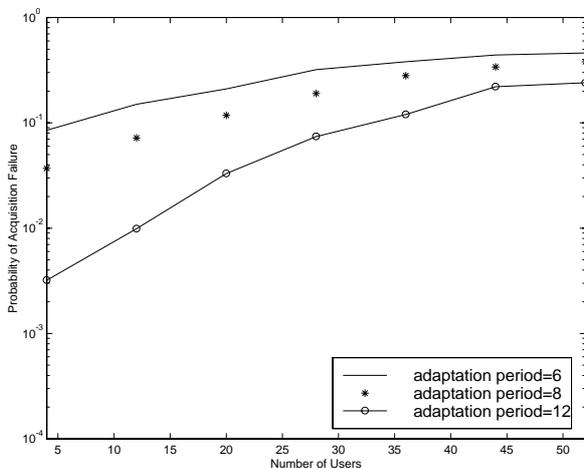


Fig. 3. Probability of Acquisition Failure as a function of Number of users for different values of Adaptation periods with  $SNR_c$  of  $-5$  dB

Fig 4 shows the mean acquisition time in chips needed to acquire the phase of a 127 chip spreading sequence using our proposed system in case of Multiple Access Interference environment with a maximum of 4 users. Also the performance in case of single user (without MAI) is included to assess the degradation due to the Multiple Access Interference. Here for cell acquisition stage, to minimize the probability of false acquisition, we take 11 samples at 127-chip sequence intervals, after a settling down of twelve 127-chip sequences and only if six or more of samples have the peak at the same location will we declare that the cell has been correctly identified [11]. Once one of cells has been correctly identified as having correct phase of desired user, the system will go into the phase acquisition stage by loading the subsequence that starts  $\frac{M}{2}$  from beginning of the identified cell. For phase acquisition stage, 9 samples are taken, at 127-chip intervals, after a settling down time of two chip sequence of 127-chips and if 5 or more samples

have the peak located at the same location will the system declare that the correct phase is acquired. We can see that for low  $SNR_c$ , degradation due to MAI is more than for high  $SNR_c$  case which means that code acquisition system is more affected by MAI for low  $SNR_c$  environment.

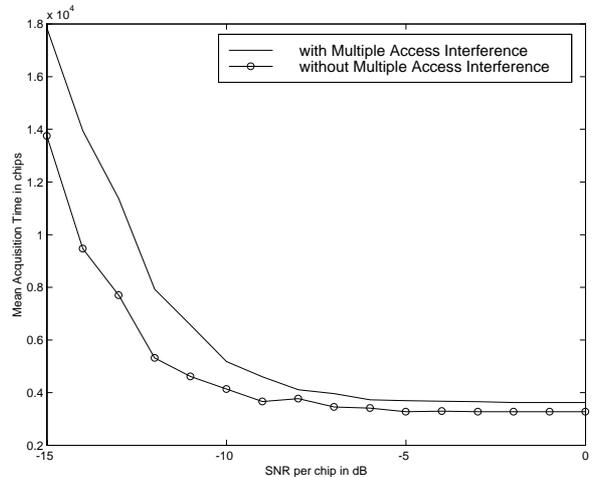


Fig. 4. Mean Time to Acquisition in chip periods against  $SNR_c$  (signal-to-noise-ratio per chip in dB) in the presence of MAI (4 users )

## V. Conclusion

The proposed system is analyzed in case of Multiple Access Interference environment. The capacity of the CDMA system is found to be improved significantly by using acquisition strategies such as the modified hybrid scheme based on adaptive filter structure, proposed in this paper. The proposed system utilizes an LMS-based FIR adaptive filter to estimate the delay offset of the desired user from the tap-weight vector. Probability of cell acquisition failure is derived and simulation and analytical results are compared. It has been shown here that the adaptive filter acquisition system can support the same number of users as that obtained by the BER criterion and thus the degradation suffered by the matched filter based scheme can be avoided. Secondly the degradation due to MAI is less for the low  $SNR_c$  environment using the proposed system. Moreover the performance of the system will be more better for longer spreading sequence in the presence of MAI.

## Acknowledgment

The authors acknowledge the support of KFUPM, for this work.

## REFERENCES

- [1] M. Simon et al. *Spread Spectrum Communications*, volume 3. Computer Science Press, Rockville, Maryland, 1985.

- [2] R. Dixon. *Spread Spectrum Systems*. John Wiley and Sons, New York, 1984.
- [3] G. Sage. Serial synchronization of Pseudo-Noise systems. *IEEE Transaction on Communication*, COM-12:123–127, December 1964.
- [4] J. Homes. *Coherent Spread Spectrum Systems*. Robert Krieger Publishing Company, Malbaro, Florida, 1990.
- [5] M. Simon et al. *Spread Spectrum Communications*, volume 3. Revised Edition, McGraw-Hill, Inc, New York, 1994.
- [6] J. Gevargiz L. Milstein and P. Das. Rapid acquisition for direct sequence spread spectrum communications using parallel SAW convolvers. *IEEE Transaction on Communication*, COM-33(7):593–600, July 1985.
- [7] Y. Su. Rapid Code PN Acquisition algorithms employing PN Matched Filters. *IEEE Transaction on Communication*, COM-36(6):724–733, June 1988.
- [8] U. Madhow and M. Pursley. Acquisition in direct sequence spread spectrum networks. *IEEE Trans. Info. Theory*, 39(3):903–912, May 1992.
- [9] R. Smith and S. Miller. Coarse acquisition performance of the minimum mean squared error receiver. *Proceedings of the IEEE Military Communications Conference; San Diego, California*, pages 1186–1189, 1995.
- [10] M. G. El-Tarhuni and A. U. H. Sheikh. An Adaptive Filtering PN Code Acquisition Scheme with Improved Acquisition based Capacity in DS/CDMA. *The Ninth IEEE international Symposium on indoor and mobile radio communication.*, 3:1486–1490, 1998.
- [11] Muhammad Hamid and A. U. H. Sheikh. Rapid adaptive filter based pn code acquisition scheme in awgn environment. *IEEE Technical Exchange Meeting, King Fahd University of Petroleum and Minerals*, April 2002.
- [12] S. Haykin. *Adaptive Filter Theory*. Prentice-Hall, New Jersey, 2nd edition edition, 1991.
- [13] N. J. Bershad and L. Z. Qu. On the Probablity Density Function of the LMS adaptive filter weights . *IEEE Trans. on Acoustics, Speech, and Signal Processing*, 37:43–56, Jan 1989.
- [14] Muhammad Hamid. *Code Acquisition Technique using Adaptive Filters in Wireless Communication*. Electrical Engineering Department, King Fahd University of Petroleum & Minerals, 2001.
- [15] T. S. Rappaport. *Wireless Communications Principles and Practice*. Prentice Hall Inc., 1996.