Concatenated Coding Schemes with Multiple Transmit and Receive Antennas

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Abstract— In this paper, we investigate the error performance of a serially concatenated system using a nonrecursive convolutional code as the outer code and a recursive QPSK space-time trellis code as the inner code on quasi-static and rapid fading channels. At the receiver, we consider iterative decoding based on the maximum a posteriori (MAP) algorithm. The performance is evaluated by means of computer simulations and it is observed that when the recursive versions of some of the previously known space-time trellis codes are used as the inner codes, the Euclidean distance criterion based space-time codes lead to better error performance for both quasi-static and rapid fading channels. We also investigate the effects of various system parameters on the performance. We obtain the error performance for the large number of transmit and/or receive antennas and observe that, generally, as the number of antennas increases the error performance improves.

I. Introduction

Space-time coding [1] combines the benefits of forward error correction coding and diversity transmission to overcome the impairments of wireless channels. The design criteria for quasi-static Rayleigh fading channels which are rank and determinant of a matrix depending on the space-time code structure were derived in [1]. A number of trellis codes that provide maximum diversity and good coding advantage have also been presented in [1]. Later, some new codes (e.g. [2]) which perform better error performance have been introduced. In [3], a new set of design criteria which depends on the diversity order of the system was proposed. In particular the authors proved that when a reasonably large diversity order is provided the error performance is dominated by the minimum squared Euclidean distance of the space-time code. This criterion is called Euclidean distance (ED) criterion.

Turbo codes proposed by Berrou *et al.* [4] represent a recent breakthrough in coding theory. They were originally introduced as binary error-correcting codes built from the parallel concatenation of two recursive systematic convolutional codes exploiting iterative decoding algorithm. It was shown that turbo codes can perform close to the Shannon limit in AWGN channels. Beyond the form of parallel concatenation, different forms of concatenation such as serial concatenation [5] have been studied. In recent years, several schemes that combine space-time and parallel or serial concatenated codes (e.g. [6]- [9]) were proposed and it was shown that these schemes performs much better than conventional space-time codes of similar complexity.

In this paper, we investigate, by computer simulations, performance of a serially concatenated system with a nonrecursive convolutional outer code and a recursive QPSK space-time trellis inner code on quasi-static and rapid fading channels. Throughout the paper, we consider the case of coherent demodulation with ideal channel state information where the fading coefficients are perfectly known to the receiver. We also assume that the fading effect on the phase of the received signal is perfectly compensated.

II. System Model

We consider a serially concatenated multiple input multiple output (MIMO) communication system that employs n_T antennas at the transmitter and n_R antennas at the receiver. Block diagram of the system is depicted in Fig. 1. In this system, a block of K independent data bits are encoded by a rate-m/l convolutional outer encoder whose output is a block of $N = K(l/m)$ coded bits. After the multiplexing, the binary sequence is interleaved by using a random interleaver (Π) with length N and the interleaved bit sequence $\mathbf{x} = (x_1, x_2, \ldots, x_N)$ is demultiplexed into two (even and odd) bit streams and then both streams are input to the inner encoder which is essentially a space-time trellis encoder. The inner encoder has a rate of $2/(2n_T)$ to encode incoming bit pairs to $2n_T$ output bits. The $2n_T$ bits of the output of the space-time encoder are mapped onto n_T QPSK symbols. In vector representation, the QPSK symbols can be shown as $0 \longleftrightarrow (\sqrt{E_s},0)$, $1 \longleftrightarrow (0,\sqrt{E_s})$, $2 \longleftrightarrow (-\sqrt{E_s},0)$, $3 \longleftrightarrow (0,-\sqrt{E_s})$ where E_s is the average energy per symbol. At time n, a QPSK symbol s_n^i is transmitted throught the ith transmit antenna, $i = 1, 2, \ldots, n_T$. All symbols are transmitted simultaneously, each from a different transmit antenna, and all symbols have the same transmission interval. This system achieves a bandwidth efficiency of $2m/l$ bits/sec/Hz.

Let the sequence $\mathbf{s}=[\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_L]$ be transmitted where $\mathbf{s}_n = [s_n^1, s_n^2, \ldots, s_n^{n_T}]^T$, $n = 1, 2, \ldots, L$. Here T denotes tranpose and L corresponds to the frame length of the transmitted symbol sequence for each antenna. After the demodulation (not shown in the figure), the corresponding $\mathbf{r}=[\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_L]$ sequence is received at the receiver. Here $\mathbf{r}_n = [r_n^1, r_n^2, \ldots, r_n^{n_R}]^T$ and at time n, the received symbol at antenna j, $j = 1, 2, \ldots, n_R$ is given by

$$r_n^j = \sum_{i=1}^{n_T} \rho_n^{i,j} s_n^i + \eta_n^j, \qquad n = 1, 2, \ldots, L \tag{1}$$

where $\rho_n^{i,j}$ is the fading coefficient for the path from trans-

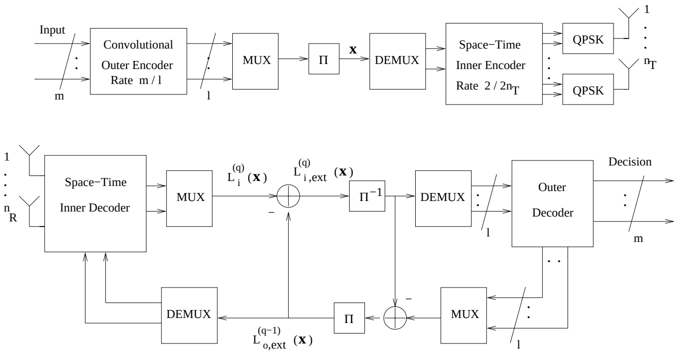

Fig. 1. Block diagram of the serial concatenated QPSK space-time system: Transmitter and Receiver.

mit antenna i to receive antenna j and η_n^j is noise which is modeled as independent samples of a zero mean complex Gaussian random variable with variance $N_0/2$ per dimension. To conform to the definition in [1], we define the signal to noise ratio (SNR) per receive antenna as E/N_0 where $E = n_T E_s$ is the total transmitted energy at each transmission interval.

We consider flat Rayleigh fading and spatially independent channels, i.e. fading is statistically independent from one transmitter-antenna pair to the other. Therefore $\rho_n^{i,j}$ coefficients are modeled as samples of independent zero mean complex Gaussian random variables with variance 0.5 per dimension and satisfy $E\{|\rho_n^{i,j}|^2\}$=1. In this paper, we evaluate quasi-static and rapid Rayleigh fading channel cases. At the quasi-static fading case, the fading coefficients are constant over the entire duration of a frame but change independently from one frame to another. For the rapid fading case, the coefficients change independently from one symbol to another.

As shown in Fig. 1, the receiver uses a message passing decoder, which passes messages (extrinsic log likelihood ratios, LLRs) [4] between the soft output inner decoder (space-time decoder) and an outer decoder in an iterative fashion. Both inner and outer decoders use log-MAP [10] version of the nonbinary maximum a posteriori (MAP) algorithm. Clearly, when $m = 1$, the outer decoder uses the binary log-MAP algorithm. A maximum of Q iterations between the inner and outer decoder are used. During the qth iteration ($q = 1, 2, \ldots, Q$), the inner decoder uses $\mathbf{R}^{(q)} = (\mathbf{r}, L_{o,ext}^{(q-1)}(\mathbf{x}))$ where $L_{o,ext}^{(q-1)}(\mathbf{x})$ is the interleaved extrinsic information (see Fig. 1) obtained from the outer decoder in the $(q-1)$th iteration. The space-time decoder produces LLRs for each bit in the sequence $\mathbf{x}$, given by

$$L_i^{(q)}(x_k) = \log \frac{P(x_k = 0|\mathbf{R}^{(q)})}{P(x_k = 1|\mathbf{R}^{(q)})} \qquad (2)$$

where x_k is the kth element of $\mathbf{x}$, $k = 1, 2, \ldots, N$. The extrinsic information obtained from the inner decoder can be written as $L_{i,ext}^{(q)}(x_k) = L_i^{(q)}(x_k) - L_{o,ext}^{(q-1)}(x_k), \forall k$. This extrinsic information is deinterleaved by using a deinterleaver (Π^{-1}) and, after the demultiplexing, input to the outer decoder. The extrinsic information obtained from the outer decoder is interleaved and input to the space-time decoder.

III. System Performance

It is well known from [5] that in order to obtain "interleaving gain" and therefore good error performance in a serially concatenated system, the inner encoder must be a recursive encoder. There is no tight constraint on the outer encoder, however, it should be a nonrecursive encoder with large free Hamming distance when the inner encoder is recursive. In [11], the authors proved that these design criteria are valid for serially concatenated space-time systems as well. Therefore, in this paper we use recursive space-time encoders as the inner encoders. We evaluate the error performance of the system considered on the quasi-static and rapid Rayleigh fading channels by computer simulations. In all simulations, unless otherwise stated, the number of iterations is Q=8, each frame consists of L=130 symbols out of each transmit antenna and the interleaver between the outer and inner encoder is S-type random interleaver with length N=260 bits and S=5. We terminate both outer and inner encoders by using appropriate tail bits. We consider the rate-1/2 and 4/5 nonrecursive convolutional outer code cases and plot the frame error rate (FER) curves versus SNR per receive antenna.

A. Rate-1/2 Convolutional Outer Code

When the convolutional outer code rate is 1/2, the bandwidth efficiency of the system becomes 1 bit/sec/Hz with QPSK modulation, ignoring the tail bits used. In this subsection, unless otherwise stated, the input block length is K=130 bits (in order to transmit L=130 QPSK symbols from each antenna) and we use a standart nonrecursive 4-state convolutional outer code with generator polynomial of [5 7], represented in octal, and having a free Hamming distance of 5.

As mentioned in the Introduction, the design criteria for conventional, i.e. without concatenation, space-time codes are maximizing the minimum rank and the minimum determinant of the distance matrices constructed from all possible pairs of distinct transmission sequences for quasi-static fading channel [1]. We called these design criteria the rank and determinant (RD) criteria. In [3], Chen *et al.* showed that, for quasi-static fading channel, these design criteria are no longer valid for the large number of transmit and/or receive antenna (e.g. $n_T n_R >$ 3) case. When large number of transmit and/or receive antennas are used in the system, the MIMO channel converges to an AWGN channel and therefore the error performance is determined by the minimum Euclidean distance between all possible pairs of distinct transmission sequences. In this case one should maximize the minimum Euclidean distance in the code (the ED criterion). For rapid fading channels, the design criteria for conventional space-time codes are maximizing the minimum symbol Hamming distance and minimum product distance [1]. We called this Hamming distance and product distance (HDPD) criteria. In [12] it was shown that the ED criterion is also valid for rapid fading channels when the system has large number of transmit and/or receive antennas.

Here, we first investigate the error performance of the serial concatenated systems employing recursive space-time inner encoders based on the ED criterion, the RD criteria and the HDPD criteria for n_T=2 and n_R=1. Toward this goal, we evaluate several systems and use recursive versions of the following 4-state space-time trellis

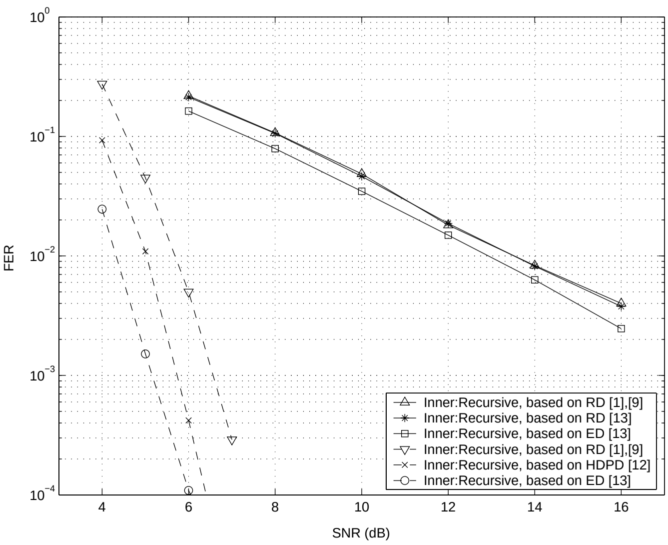

Fig. 2. Error performance of the serially concatenated system with different type of inner encoders. Outer rate-1/2 nonrecursive convolutional encoder, N=260, n_T=2, n_R=1. Solid: Quasi-static fading channel, Dash: Rapid fading channel.

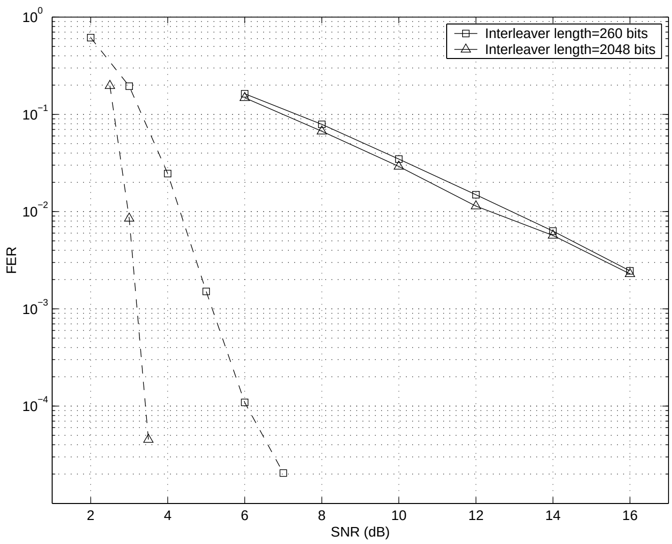

Fig. 3. Error performance of the serially concatenated system with different interleaver lengths. Outer rate-1/2 nonrecursive convolutional encoder, n_T=2, n_R=1, Solid: Quasi-static fading channel, Dash: Rapid fading channel.

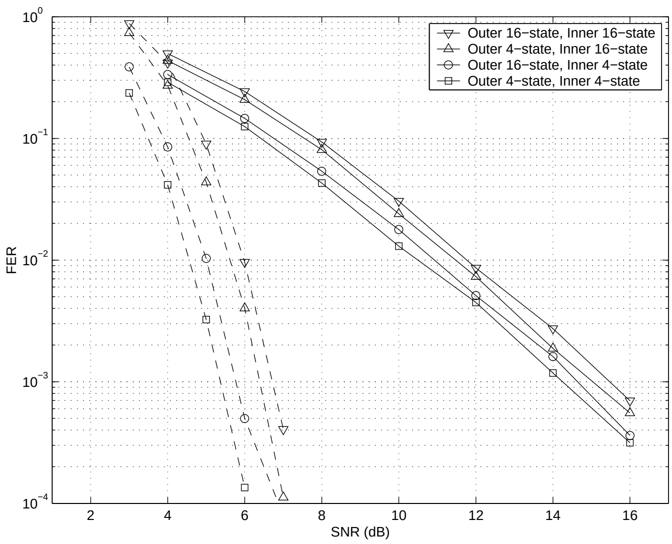

Fig. 4. Error performance of the serially concatenated system with different numbers of inner code state. Outer rate-1/2 nonrecursive convolutional encoder, N=260, n_T=3, n_R=1. Solid: Quasi-static fading channel, Dash: Rapid fading channel.

codes as the inner codes in these systems: The first one is Chen *et al.*'s [3], [13] space-time code based on the ED criterion, the second one is Tarokh *et al.*'s code [1], [9] based on the RD criteria, the third one is again Chen *et al.*'s [13] code based on the RD criteria and the fourth one is Yuan *et al.*'s [12] code based on the HDPD criteria. In order to obtain recursive versions of the codes, we only redefined the input-output transitions of the original codes as done in [9], the output symbol sequence remains the same.

Fig. 2 compares the frame error performance of the systems employing recursive space-time inner encoders based on the ED criterion and the RD criteria over quasi-static fading channel. Here, we evaluate three systems for quasi-static fading channel. As seen from the figure, even though the serially concatenated system has small number of antennas (n_T=2, n_R=1), the ED criterion for the inner code leads to better error performance. In Fig. 2, we also compare the performance of the systems using recursive space-time inner encoders based on the ED criterion, the RD criteria and the HDPD criteria over rapid fading channel. It appears from the figure, the system using inner code based on the ED criterion achieves better error performance over the rapid fading channel as well.

Fig. 3 demonstrates the effect of the interleaver length of the system for n_T=2 and n_R=1. The inner code is recursive version of Chen *et al.*'s [13] 4-state the ED criterion based space-time code. We evaluate the performance when the system has interleaver lengths of N=260 bits and N=2048 bits. In the case of N=2048 bits, the input block length was K=1024 bits, S parameter of the random interleaver was 10 and each frame consisted of L=1024 symbols out of each transmit antennas. It can be seen from the simulation results that increasing the frame size brings the additional performance gains to the system due to the increased interleaving gain. For example, this gain is 2 dB at FER of 10^{-3} on rapid fading channel. For quasi-static fading channel, however, the SNR gain is quite limited. Obviously, for a larger interleaver length a larger SNR gain is obtained.

Fig. 4 illustrates the effect of number of the states of the outer and/or inner codes. Here the system has n_T=3 transmit and n_R=1 receive antennas. We evaluate the systems using recursive versions of Chen *et al.*'s [13] 4 and 16-state the ED criterion based space-time code as the inner code and 4-state and 16-state nonrecursive convolutional codes as the outer codes. The 4-state outer code has a generator polynomial of [5 7] and free Hamming distance of 5 (the same code used above). The 16-state outer code has a generator polynomial of [23 35] and free Hamming distance of 7. As seen from the figure, the system with 4-state outer and 4-state inner code performs the best performance for both types of channels and increasing the number of states does not improve the performance,

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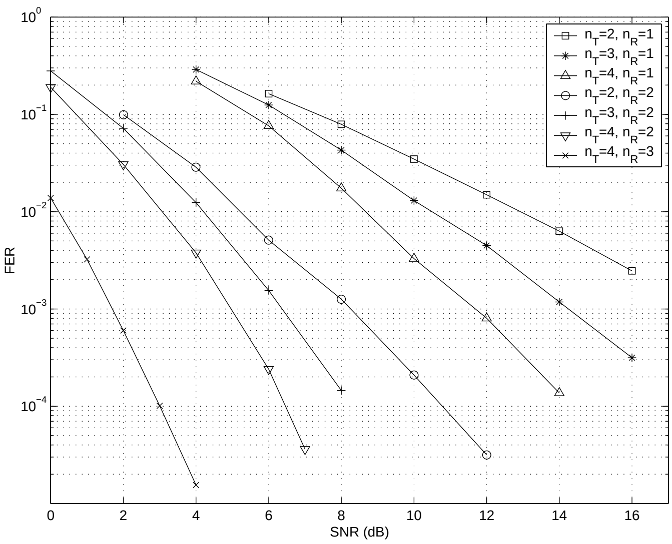

Fig. 5. Error performance of the serially concatenated systems with different number of antennas over quasi-static fading channels. Outer rate-1/2 nonrecursive convolutional encoder, N=260.

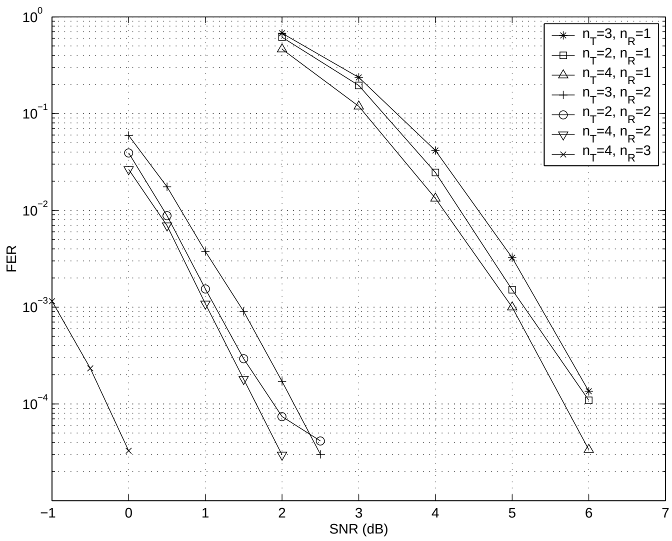

Fig. 6. Error performance of the serially concatenated systems with different number of antennas over rapid fading channels. Outer rate-1/2 nonrecursive convolutional encoder, N=260.

but rather worsens it. This phenomenon is typical in the iterative decoding [9].

In Fig. 5, we show the performance when the system has various number of transmit and/or receive antennas over quasi-static fading channels. Here, the inner codes are recursive versions of the 4-state the ED criterion based space-time codes given in [13]. As seen, the SNR value required for a given FER decreases as the number of transmit and/or receive antennas increases. For example, at a FER of 10^{-2}, the system with n_T=2, n_R=2 provides 6.8 dB SNR improvement over the system with n_T=2, n_R=1. For n_R=2, at a FER of 10^{-3}, the performance improvement is 1.8 and 1.3 dB when n_T is increased from 2 to 3 and from 3 to 4, respectively. As expected, the system with n_T=4, n_R=3 achieves the best error performance with respect to the other systems evaluated.

In Fig. 6, the performance is demonstrated for several number of transmit and/or receive antennas over rapid fading channel. In general, the systems exhibit better performance as the number of transmit and/or receive antennas increases. For example, at a FER of 10^{-3}, for n_T=4, the system with n_R=3 yields 2 dB and 6 dB SNR gain over the systems with n_R=2 and n_R=1, respectively. However, for n_R=1 and also for n_R=2, when n_T is increased from 2 to 3, we observe worse error performance. Such kind of behaviour was also observed in [13] for conventional space-time codes over quasi-static fading channels. If we compare Fig. 5 and Fig. 6, we can see that increasing the number of transmit antennas provides larger SNR gain for quasi-static fading channels than that of rapid fading channels.

B. Rate-4/5 Convolutional Outer Code

When the convolutional outer code rate is 4/5, with QPSK modulation, the bandwidth efficiency of the system increases to 1.6 bits/sec/Hz. Here, we use input block length of K=208 bits and interleaver length of N=260 bits in order to transmit L=130 QPSK symbols from each antenna. Unless otherwise stated, we use a nonrecursive 4-state convolutional outer code with a generator polynomial of [67 15 26 52 57] and free Hamming distance of 2. We are aware of the case that such an outer code does not provide an interleaving gain [5] when a maximum likelihood decoder, which is almost impossible to implement in practice for concatenated systems, is employed at the receiver. This is due to the fact that its free Hamming distance is smaller than 3. In that case, to obtain a better error performance, a convolutional code with larger free Hamming distance can be used. However, the error performance behaviour of the iterative decoder for this case is quite different and increasing the free Hamming distance from 2 to 3, results in worse performance. Indeed, we have also simulated the system by using an 8-state rate-4/5 nonrecursive convolutional outer code with generator polynomial of [013 023 056 132 174] and with free Hamming distance of 3 and observed worse error performance with respect to that of 4-state rate-4/5 nonrecursive convolutional outer code. This phenomenon is similar to that in the previous section. Thus, we use a convolutional outer code with free Hamming distance of 2.

For n_T=2 and n_R=1, we have evaluated the performance of the systems with recursive space-time inner codes based on the ED, RD and HDPD criteria and observed that, as in the previous section, the ED criterion for inner code leads to better error performance for both quasi-static and rapid fading channels. The error performance of the system using the 4-state convolutional outer code and recursive space-time inner codes based on the ED criterion is illustrated in Fig. 7 and in Fig. 8 when the system has various number of transmit and/or receive antennas for quasi-static and rapid fading channels, respectively. The performance behavior of the systems is similar to those of the systems with rate-1/2 outer code. Increasing the number of transmit and/or receive antennas, generally, leads to better performance. We also compare our results with Tarokh *et al.*'s [1] and Yuan *et al.*'s [12] conventional 32-state QPSK space-time codes with 2 bits/sec/Hz bandwidth efficiency. Note that Tarokh *et al.*'s code is based on the RD criteria and one of Yuan *et al.*'s code (in Fig. 7) is based on the ED criterion, the

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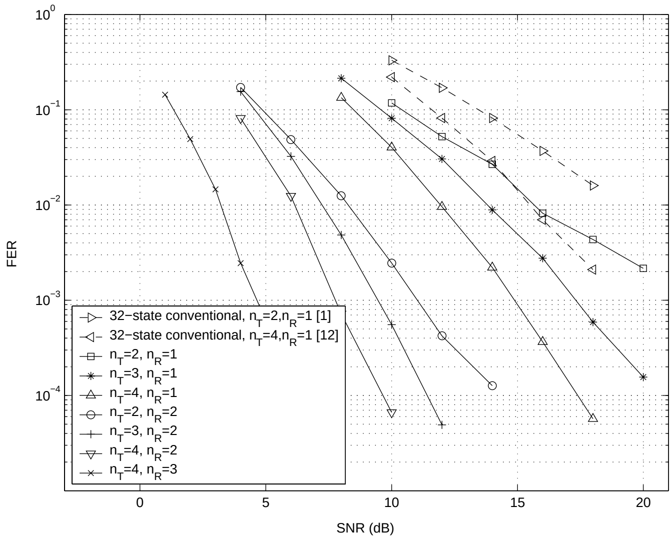

Fig. 7. Error performance of the serially concatenated systems with different number of antennas over quasi-static fading channels. Outer rate-4/5 nonrecursive convolutional encoder, N=260.

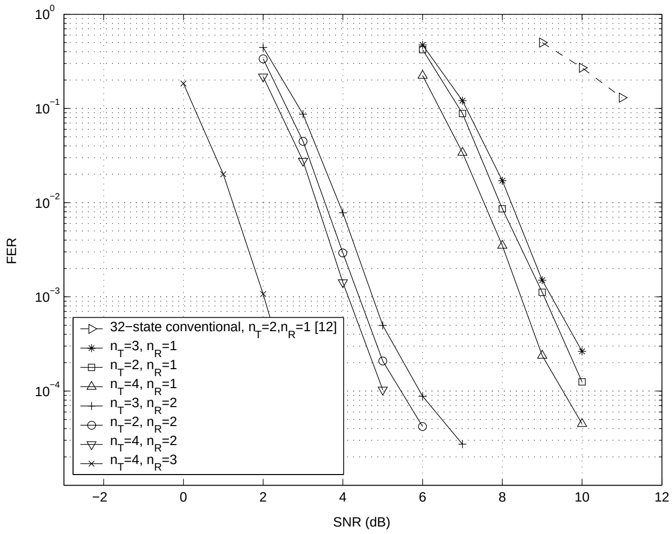

Fig. 8. Error performance of the serially concatenated systems with different number of antennas over rapid fading channels. Outer rate-4/5 nonrecursive convolutional encoder, N=260.

other (in Fig. 8) is based on the HDPD criteria. As seen from the figures, the serial concatenated systems which actually have 0.4 bit/sec/Hz bandwidth efficiency penalty outperform the conventional codes.

IV. Conclusion

We studied a serially concatenated coding scheme with multiple transmit and/or receive antennas. In particular we used a rate-1/2 and a rate-4/5 nonrecursive convolutional code as the outer code and a recursive QPSK space-time trellis code as the inner code. The channel was subject to quasi-static and rapid Rayleigh fading. We compared the performance of the systems using as the inner codes recursive versions of the some of the known space-time codes based on the ED criterion, the RD criteria and the HDPD criteria and observed that the inner codes based on the ED criterion provide better error performance for both quasi-static and rapid fading channels. The effects of interleaver length, number of states of the outer and/or inner codes and number of transmit/receive antennas on the performance were investigated. We showed that the increasing number of states of the outer and/or inner codes from 4 to 16 actually results in worse performance when iterative decoding is used. It was also demonstrated, generally, that the larger number of antennas provides better performance.

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