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Tracking of Mobile Fading Channels by Predictive Type RLS Algorithm

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Abstract— **In this paper, we propose an improved version of recursive least square (RLS) channel estimator by incorporating a single step prediction in the conventional RLS method. One step prediction of channel estimates is obtained by using degree-1 least square (LS) fading memory prediction. The predicted value will be used in the subsequent estimate of the channel by RLS algorithm. The overall effect of the proposed algorithm is to estimate the channel impulse response based on RLS adaptation while taking into consideration the speed of variation of channel impulse response as well. It is observed by computer simulations that for moderately fast mobile fading channels (doppler frequency equals 100 Hz), the proposed predictive-RLS algorithm provides 18 dB better channel estimates, in terms of mean square deviation (MSD), over the conventional RLS. Similarly, for fast fading channels having doppler frequency of 222Hz, the improvement is about 13dB. We also investigated the effect of signal to noise ratio on the performance of the two algorithms.**

Keywords— **mobile fading channel, channel estimation, channel tracking, adaptive filters, RLS, least square fading memory prediction**

I. Introduction

Many of the present day receivers for TDMA or CDMA based mobile communication systems require the estimates of the channel impulse response [1]. In order to achieve high performance through these receivers, it is deem necessary to improve the quality of the channel estimates. For instance, in TDMA systems like GSM (Global Mobile System) where maximum likelihood receiver is employed to combat intersymbol interference, a high quality channel estimates are required for satisfactory performance [2]. Similarly, essentially good quality channel estimates are required in CDMA systems that employ Rake receivers.

Different estimators for fading channel estimation have been proposed in the literature. Of these, the most promising technique is the one that employs adaptive filter for channel estimation. The weights of the adaptive filter can be adjusted based on some error minimization criterion like least mean square (LMS) or RLS algorithm [3]. It is well known that the convergence rate of RLS algorithm is faster than that of the LMS algorithm and is not sensitive to the eigenvalue spread of the input vector correlation matrix [4]. When RLS algorithm is used to estimate time-varying channels like mobile fading channels, the convergence behaviour of RLS adaptive filter is a transient phenomena, whereas its tracking behaviour is a steady state phenomenon. So, a good convergence behaviour does not translate into a good tracking behaviour [5].

The purpose of this paper is to improve the tracking performance of RLS algorithm for fading channel estimation by incorporating degree-1 least square (LS) fading memory prediction. The use of LS fading memory prediction to predict the value of a variable parameter was first introduced by Morrison [6] and then Clark [7] used this approach with LMS to estimate HF channels. In this paper, we extend this scheme and applied it to RLS algorithm to obtain time varying estimates of the mobile fading channel impulse response. The details of the proposed algorithm will be described in Section III. Simulations are carried out to compare the performance of the predictive-RLS algorithm with the conventional RLS. The results are obtained in the form of mean square deviation (MSD) for moderately fast and fast varying vehicular channels having doppler frequencies of 100 Hz and 222 Hz, respectively.

The paper is organized as follows: Section II describes the channel model used in this work. Section III gives a review of RLS adaptive channel estimation. Section IV presents the mathematical formulation of the proposed algorithm. Detail simulation results are presented in Section V; comparing the proposed predictive-RLS and the conventional RLS algorithm.

II. Multipath Mobile Fading Channel Model

The multipath mobile fading channel is assumed to be a three path fading channel corrupted by additive white Gaussian noise(AWGN). The fading in each path of the channel follows Rayleigh distribution and has power spectral density as given by Jakes [8].

$$S(f) = \frac{\sigma^2}{\pi f_m \sqrt{1-(f/f_m)^2}}$$

where $f_m = v f_c/c$ is the Doppler frequency that depends on the speed of the vehicle v and the carrier frequency f_c. The relative strength of the paths is assumed to have exponential power delay profile. For simulation purpose, the complete channel is modeled as finite impulse response (FIR) filter with delay between successive filter taps assumed to be symbol period. The vector representing filter taps or coefficients is

$\mathbf{h}(k) = [\mathbf{h}_1(k)\ \mathbf{h}_2(k)\ \mathbf{h}_3(k)]$, where $\mathbf{h}_i(.)$ are time varying and generated as complex Gaussian according to the Jakes model for fading channel simulator [8]. We will evaluate the performance of the proposed algorithm for fast varying channels, like vehicular channels, with maximum Doppler frequencies of 100 Hz and 222 Hz.

III. Adaptive Channel Estimation by RLS

If $\mathbf{u}(k) = [\mathbf{u}_1(k)\ \mathbf{u}_2(k)\ \mathbf{u}_3(k)]$ is the vector of the transmitted sequence (assumed stationary) and $\eta(k)$ is the noise then the received symbol $r(k)$ is given by

$$r(k) = \mathbf{h^H}(\mathbf{k})\mathbf{u}(\mathbf{k}) + \eta(\mathbf{k}) \tag{1}$$

The adaptive filter used to track the fading channel is an FIR filter with time varying tap weights, $\hat{\mathbf{h}}(\mathbf{k})$. Assuming channel matched adaptive filter such that $\hat{\mathbf{h}}(k) = [\hat{\mathbf{h}}_1(k)\ \hat{\mathbf{h}}_2(k)\ \hat{\mathbf{h}}_3(k)]$, the estimated received signal at the output of the adaptive filter can be written as

$$r'(k) = \hat{\mathbf{h}}^{\mathbf{H}}(\mathbf{k})\mathbf{u}(\mathbf{k}) + \eta(\mathbf{k}) \tag{2}$$

The estimation error between the received signal and its estimate is

$$\xi(k) = r(k) - r'(k) \tag{3}$$

This error will be used to update the filter coefficients based on some error minimization criterion. If conventional RLS is used to update the filter coefficients then the weight update equation will be

$$\hat{\mathbf{h}}(\mathbf{k}) = \hat{\mathbf{h}}(\mathbf{k-1}) + \mathbf{g}(\mathbf{k})\xi^*(\mathbf{k}) \tag{4}$$

where $g(k)$ is the Kalman gain and can be updated in the following manner

$$g(k) = \frac{\lambda^{-1}P(k-1)u(k)}{1+\lambda^{-1}u(k)P(k-1)u(k)} \tag{5}$$

and $P(.)$ is the error covariance matrix that can be updated as

$$P(k) = \lambda^{-1}P(k-1) - \lambda^{-1}g(k)u(k)P(k-1) \tag{6}$$

IV. Proposed Predictive-RLS algorithm

To improve the performance of conventional RLS algorithm for tracking time varying channel a prediction scheme is employed in this paper. The algorithm developed with this modification can be termed as *predictive-RLS*. In [7], so called "degree-1 Least Square fading memory prediction" was employed to take *a priori* information about the channel into the estimation scheme. The method of least square fading memory prediction is based on the fact that a better prediction of $\mathbf{h}(k+1)$ from the sequence of vectors $\hat{\mathbf{h}}(\mathbf{k})$, $\hat{\mathbf{h}}(\mathbf{k-1})$, $\cdots$, is obtained by determining the set of $n+1$ polynomials of given degree (0,1 or 2) each of which gives the LS fit to the components in the corresponding locations in the vectors $\hat{\mathbf{h}}(\mathbf{k})$, $\hat{\mathbf{h}}(\mathbf{k-1})$, $\cdots$, and then using the values of the polynomial at time $t = (i+1)T$ to determine the nth component of $\hat{\mathbf{h}}(\mathbf{k+1})$. Thus, it behaves as a coefficient prediction filter. The proposed predictive-RLS can be mathematically formulated by modifying Eq. (4) as:

$$\hat{\mathbf{h}}(\mathbf{k}) = \hat{\mathbf{h}}'(\mathbf{k-1}) + \mathbf{g}(\mathbf{k})\xi^*(\mathbf{k}) \tag{7}$$

where $\hat{\mathbf{h}}'(\mathbf{k-1})$ can be updated as

$$\begin{aligned} \Delta_k &= \hat{\mathbf{h}}(\mathbf{k}) - \hat{\mathbf{h}}'(\mathbf{k-1}) \\ \hat{\mathbf{h}}''(\mathbf{k}) &= \hat{\mathbf{h}}''(\mathbf{k-1}) + (\mathbf{1}-\theta)^{\mathbf{2}}\mathbf{\Delta_k} \\ \hat{\mathbf{h}}'(\mathbf{k}) &= \hat{\mathbf{h}}'(\mathbf{k-1}) + \hat{\mathbf{h}}''(\mathbf{k}) + (\mathbf{1}-\theta^{\mathbf{2}})\mathbf{\Delta_k} \end{aligned}$$

where θ is the weighting factor $(0 < \theta < 1)$ that controls the forgetting amount of the past in a compromise with an accurate estimate. Δ_k is a measure of tap weight variation. In this scheme θ has to be tuned depending on the signal to noise ratio (SNR) and channel behavior (speed of variation).

V. Simulation Results

We assessed the performance of the predictive-RLS algorithm by computer simulations. The input to the channel and the adaptive filter is BPSK modulated pseudo random sequence +1, -1. The noise is assumed to be AWGN with zero mean and variable variance depending on the signal to noise ratio (SNR).

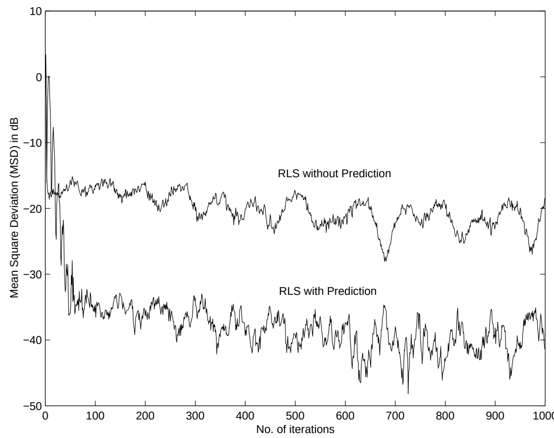

Fig. 1. Tracking performance of channel estimators (Vehicular Channel, f_m = 100Hz, SNR = 40dB)

Figure 1 shows the learning curves for both algorithms obtained by plotting MSD for 1000 iterations in case of moderately fast varying channels (f_m =100Hz). It clearly shows that steady state MSD obtained by predictive-RLS is about 18 dB better than that from conventional RLS. However, the convergence of RLS without prediction is faster than that with prediction.

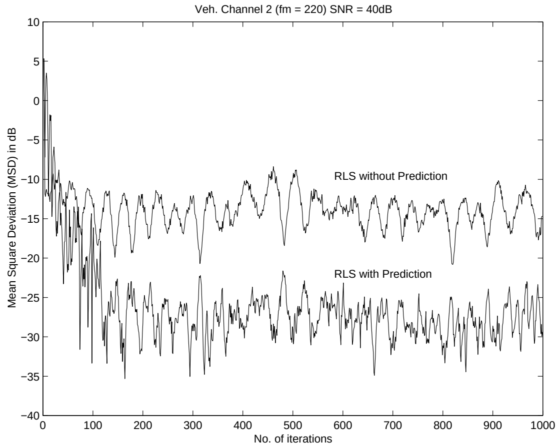

Fig. 2. Tracking performance of channel estimators (Vehicular Channel, f_m = 222Hz, SNR = 40dB)

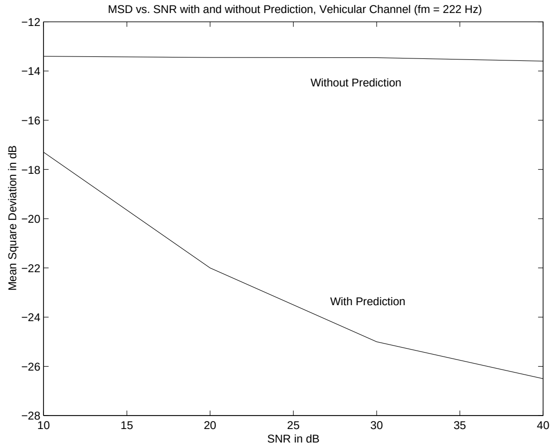

Fig. 4. MSD vs SNR with and without prediction (Vehicular Channel, f_m = 222Hz)

Figure 2 shows similar results for fast vehicular channels with f_m= 222 Hz where an improvement of about 13dB in MSD can be obtained by using the proposed predictive-RLS algorithm.

Figure 3 studies the influence of input SNR on the conventional RLS and predictive-RLS channel estimation schemes. The performance of predictive-RLS gains about 2dB for poor SNR (10 dB) while it gains around 18dB for SNR of 40 dB. As expected, predictive system has superior performance than the conventional RLS. For fast varying vehicular fading channels with f_m= 222 Hz, the plot of MSD vs. SNR is shown in Fig. 4. An interesting observation from Fig. 3 & 4 is that the conventional RLS does not show any significant improvement with the increase in SNR. This can be attributed to the slow tracking properties of the conventional RLS algorithm. The proposed algorithm that introduces prediction in conventional RLS adds a dimension to its tracking capability under fast varying fading channels.

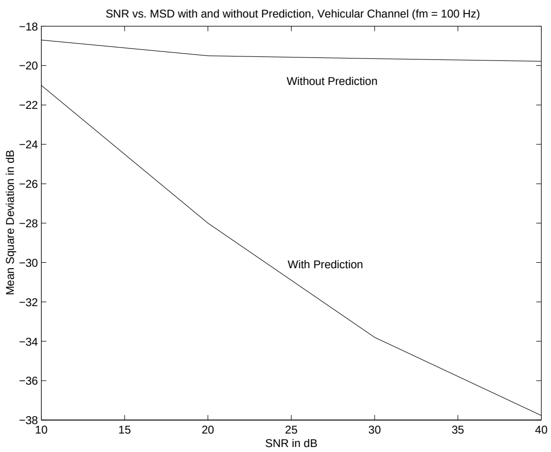

Fig. 3. MSD vs SNR with and without prediction (Vehicular Channel, f_m = 100Hz)

VI. Conclusion

The paper presents a modified version of RLS for mobile fading channel estimation. The proposed algorithm, named as predictive-RLS, uses RLS estimation algorithm to update the predicted value obtained through degree-1 LS fading memory prediction. Simulation results show clear improvement that can be achieved by using predictive-RLS algorithm over the conventional RLS. For moderately fast varying channels, with f_m=100Hz, the gain in steady state MSD is about 18 dB. Similarly, for fast varying channels (f_m=222Hz) the gain is about 13 dB. It has been observed that increasing the value of SNR does not affect the performance of conventional RLS algorithm while the predictive-RLS shows noticeable improvement with the increase in SNR. Also the proposed algorithm does not add any substantial computation complexity.

References

[1] J. G. Proakis, *Digital Communications*. McGraw Hill Inc., 1995.

[2] J. G. Proakis, "Adaptive equalization for TDMA digital mobile radio," *IEEE Trans. Veh. Technol.*, vol. 40, May 1991.

[3] S. Haykin, *Adaptive Filter Theory*. Prentice Hall, 3rd ed., 1996.

[4] E. Eweda, "Comparison of RLS, LMS and sign algorithms for tracking randomly varying time-varying channels," *IEEE Trans. Signal Proc.*, vol. 40, Nov. 1991.

[5] S. Haykin, A. H. Sayed, J. Zeidler, *et al.*, "Adaptive tracking of linear time-variant systems by extended RLS algorithms," *IEEE Trans. Signal Proc.*, vol. 45, pp. 1118–1127, May 1997.

[6] N. Morrison, *Introduction to Sequential Smoothing and Prediction*. Mc Graw Hill, 1969.

[7] A. P. Clark, "Channel estimation for an HF radio link," *Proc. IEE, part F,*, vol. 128, pp. 33–42, Feb 1981.

[8] W. C. Jakes, *Microwave Mobile Communication*. Newyork: Wiley, 1974.