A Comparative study between Space-time Block codes (STBC) and Collaborative codes (CC)

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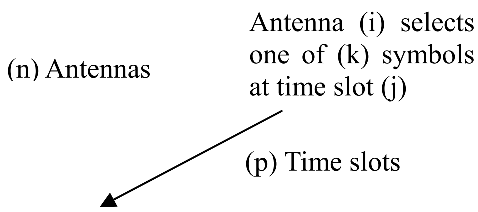
Abstract - The objective of this paper is to compare the CC with STBC, to find out the similarities and differences. It discusses the advantages of both schemes and find out how to utilize the benefits of both to improve system performance. Both schemes have distinct signal combination and hence the performance of STBC can be improved by employing error detection based on table lookup.

Key words: Collaborative codes, multiple access, STBC, diversity

I Introduction

Space-time codes (STC), and its variant space time block codes (STBC) is a new diversity techniques that is bandwidth and power efficient method of communication over fading channels that realizes the benefit of multiple transmit antennas. It achieves bandwidth efficiency through the use of an efficient way of combining forward error correction (FEC) and diversity transmission to overcome the impairments of wireless channels. STBC, which is a subset of STC, was first introduced in 1998 [1] is simpler than STC and lend itself for practical implementation. Due to its simplicity, STBC have been recommended for use in 3rd generation mobile systems (UMTS) [2].

A space-time block code is defined by its transmission matrix G – whose dimension is p x k. p represents the number of time slots used for transmission, while k is the number of symbols utilized by the code. At any point in time, the encoder transmits n symbols (using n antennas) – selected from one of the available k symbols (or their functions). At the first time slot t_1, k symbols (k bits in the binary case) arrive at the encoder, which selects one of the available k signals (s_1, .. s_k) to transmit them. These signal are transmitted using n antennas (where n is independent of k and p, n > k offers diversity). For the remaining time slots (t_2, .. t_p), there is no data from the source, but the n antennas transmit signals that are derived from (s_1, .. s_k). A diagram illustrating the relationship between k, p and n is given below.



$$\begin{pmatrix} a_{11} & . & . & a_{1n} \\ . & & & . \\ . & a_{ij} & & . \\ a_{p1} & . & . & a_{pn} \end{pmatrix}$$

Since k symbols are transmitted in p times, then the rate R, is given by:

$$R = k / p \quad \text{symbols / time slot}$$

An implementation of STBC system is shown in Fig. 1, with 2 transmit and 1 receive antennas.

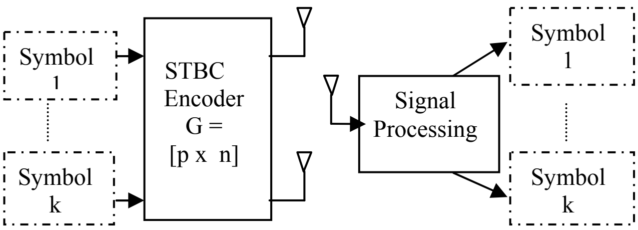


Fig. 1: STBC communication system for single user

Collaborative Codes[1] (CC) provides higher bits/channel use, than TDM or FDM. It allows more users to access the channel at same time in the normal case. CC allows multi users to access the channel at same time where their transmitted signals add up. The receiver will be able to decode this combined message into its constituent code words and deliver them to their respected users respectively [3]. In Fig. 2, a single user is allowed to achieve this higher rate by splitting its data path – this is done just to compare with STBC.

[1] Sometimes referred to as t-user code.

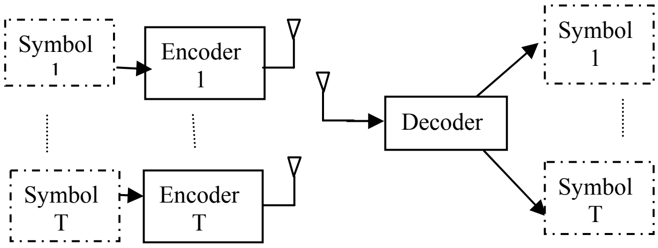


Figure 2: CC communication system for single user

The objective of this paper is to compare the CC with STBC, to find out the similarities and differences. It will discuss the advantages of both schemes and find out how to utilize the benefits of both to improve system performance. To enable comparison both schemes utilizes binary real signals with BPSK modulation.

The paper starts with presenting the Alamouti STBC code, which is equivalent to the G2 [2 x2] orthogonal design [4], and continues with the G3 and G4 codes. The CC follows the same track by starting with the 2-user code followed by the 4-user code.

II Space Time Block Codes (STBC)

A Alamouti (2 x 2) code

This code utilizes 2 transmit antennas and a single receive antenna. The coding scheme is given in table 1.

Table 1: Alamouti code

	t	t + Δt
Ant. 1	$\mathbf{s_0}$	$\mathbf{-s_1^*}$
Ant. 2	$\mathbf{s_1}$	$\mathbf{s_0^*}$

If the signals uses are binary, then the resulting code is the [2 x 2] orthogonal design [4].

$$\begin{bmatrix} x1 & x2 \\ -x2 & x1 \end{bmatrix}$$

This code matrix is equivalent to table 1, but the antennas are represented by columns and time slots by rows.

The 2 binary data {0,1}, signals from both antennas are sent using the following convention:

Table 2: Symbols sent at time (t)

Data	Ant. 1 (Sends s_0)	Ant. 2 (Sends s_1)
0	-	+
1	+	-

The signals sent at time (t + Δt) are functions of signals sent a time (t) and there is no data coming in during this time slot.

N.B.

- Other data representations may be possible.
- The data sent from the two antennas can be coming from two different users, or from a single data source, through a serial-to-parallel converter.

In BPSK modulation a ‘0’ is sent as $-\cos \omega_c t$ and a ‘1’ is sent as $+\cos \omega_c t$, presented simply as (-) or (+) respectively, i.e. constellation points (–1 and +1) and the resulting codebook is given in Table 3.

Table 3: Alamouti code

Data		Code words (t, t+ Δt)		Channel Signals	
Antenna 0	Antenna 1	Ant. 0	Ant.1	r_0	r_1
0	0	- -	+ -	0	-2
0	1	- -	- +	-2	0
1	0	+ +	+ -	+2	0
1	1	+ +	- +	0	+2

Note that all the channel signals are distinct (just as in CC)!

The decoding procedure of Alamouti STBC is summarized as follows:

- Estimates of signals ($s_0^{\sim}$ and $s_1^{\sim}$) are found, based on which, the data sent by both antennas can be found using table 4 (which is deduced from table 2).

Table 4: Decoding strategy

$s_0^{\sim}$	-	+
Data sent by Ant. 1	0	1

$s_1^{\sim}$	-	+
Data sent by Ant. 2	1	0

B G4 (4x4) STBC

The 4x4 STBC is defined as follows [3]:

$$\begin{array}{c} \\ t1 \\ t2 \\ t3 \\ t4 \end{array} \begin{array}{c} \begin{array}{cccc} A\#1 & A\#2 & A\#3 & A\#4 \end{array} \\ \begin{Bmatrix} x1 & x2 & x3 & x4 \\ -x2 & x1 & -x4 & x3 \\ -x3 & x4 & x1 & -x2 \\ -x4 & -x3 & x2 & x1 \end{Bmatrix} \end{array}$$

Using baseband signaling, the signals (at $t = t_1$) are selected according to Table5. Signals transmitted at t_2, t_3, t_4 are functions of signals transmitted at time = t_1 as given G4 STBC transmission matrix above.

Table 5: Signal set for G4 – binary.

Data Sent at time = t_1	Signals Selected			
	x_1	x_2	x_3	x_4
0	0	0	0	0
1	1	1	1	1

Using BPSK modulation, we have:

Table 6: Signal set for G4 - BPSK

Data Sent at time = t_1	Signals Selected			
	x_1	x_2	x_3	x_4
0	-	-	-	-
1	+	+	+	+

Table 7: Codebook for G4 - BPSK

S. No.	Data	Code words (t_1, t_2, t_3, t_4)				Σ (Channel signals)
		Ant. #1	Ant. #2	Ant. #3	Ant. #4	
1	0000	- + + +	- - - +	- + - -	- - + -	-4 0 0 0
2	0001	- + + -	- - + +	- - - -	+ - + -	-2 -2 2 -2
3	0010	- + - +	- - - -	+ + - -	- + + -	-2 2 –2 -2
4	0011	- + - -	- - + -	+ - - -	+ + + -	0 0 0 –4
5	0100	- - + +	+ - - +	- + - +	- - - -	-2 –2 –2 2
6	0101	- - + -	+ - + +	- - - +	+ - - -	0 –4 0 0
7	0110	- - - +	+ - - -	+ + -+	- + - -	0 0 –4 0
8	0111	- - - -	+ - + -	+ - - +	+ + - -	2 –2 –2 –2
9	1000	+ + + +	- + - +	- + + -	- - + +	-2 2 2 2
10	1001	+ + + -	- + + +	- - + -	+ - ++	0 0 4 0
11	1010	+ + - +	- + - -	+ + + -	- + + +	0 4 0 0
12	1011	+ + - +	- + + -	+ - + -	+ + + +	2 2 2 –2
13	1100	+ - + -	+ + - +	- + + +	- - - +	0 0 0 4
14	1101	+ - + -	+ + + +	- - + +	+ - - +	2 –2 2 2
15	1110	+ - - +	+ + - -	+ + + +	- + - +	2 2 –2 2
16	1111	+ - - +	+ + + -	+ - + +	+ + - +	4 0 0 0

C The G3 STBC code:

The transmission matrix is given by:

$$\begin{array}{c} \\ t1 \\ t2 \\ t3 \\ t4 \end{array} \begin{array}{ccc} A\#1 & A\#2 & A\#3 \\ \hline x1 & x2 & x3 \\ -x2 & x1 & -x4 \\ -x3 & x4 & x1 \\ -x4 & -x3 & x2 \end{array}$$

This is a subset of G4 (the fourth column is deleted). The BPSK scheme is given in Table 8.

Table 8: Codebook for G3 - BPSK

S. No.	Data	Code words (t_1, t_2, t_3, t_4)			Σ (Channel signals)
		Ant. #1	Ant. #2	Ant. #3	(t_1, t_2, t_3, t_4)
1	0000	- + + +	- - - +	- + - -	-3 +1 –1 +1
2	0001	- + + -	- - + +	- - - -	-3 –1 +1 -1
3	0010	- + - +	- - - -	+ + - -	-1 +1 –3 -1
4	0011	- + - -	- - + -	+ - - -	-1 –1 –1 -3
5	0100	- - + +	+ - - +	- + - +	-1 –1 –1 +3
6	0101	- - + -	+ - + +	- - - +	-1 –3 –1 +1
7	0110	- - - +	+ - - -	+ + -+	+1 +1 –3 +1
8	0111	- - - -	+ - + -	+ - - +	+1 –3 –1 –1
9	1000	+ + + +	- + - +	- + + -	-1 +3 +1 +1
10	1001	+ + + -	- + + +	- - + -	-1 +1 +3 -1
11	1010	+ + - +	- + - -	+ + + -	+1 +3 –1 –1
12	1011	+ + - +	- + + -	+ - + -	+1 +1 +1 –1
13	1100	+ - + -	+ + - +	- + + +	+1 +1 +1 +1
14	1101	+ - + -	+ + + +	- - + +	+1 –1 +3 +1
15	1110	+ - - +	+ + - -	+ + + +	+3 +1 +1 +1
16	1111	+ - - +	+ + + -	+ - + +	+3 –1 +1 +1

III Collaborative Codes

A. 2-User Collaborative Codes

A simple code is the 2-use code is given in table 9. The constituent codes are given by:

C1 = {0,1} = {00,11}
C2 = {0,1} = {10,01}

The 2-user code based on the BPSK modulation scheme is given in table 9.

Table 9: 2-user code, BPSK modulated

Users' digits	Users' code words		BPSK signals		Channel output
00	00	01	- -	- +	-2, 0
01	00	10	- -	+ -	0, -2
10	11	01	+ +	- +	0, +2
11	11	10	+ +	+ -	+2, 0

Joint decoding is then carried out using the above table, where for each channel output, we find the transmitted pair of symbols (digits). Note the similarity with G2 STBC!

B 4-User Code

This code is defined by table 10

Table 10: Codeword allocation for 4-user code

User #	Codeword for	
	'0'	'1'
1	000	001
2	000	010
3	000	101
4	001	100

Since 4 bits are transmitted during 3 time slots, then code rate is given by:

$R_{CC} = 4/3$ symbols / time slot

If BPSK modulation for 4-user code, then a '0' is sent as –cos $\omega_c t$ and a '1' is sent as +cos $\omega_c t$, presented simply as (-) or (+) respectively. The resulting codebook is given in table 11.

Table 11: BPSK transmission for 4-user code

S. No.	User's Data	Code words (t_1, t_2, t_3)				Σ (Channel signals) (t_1, t_2, t_3)
		User #1	User #2	User #3	User #4	
1	0000	- - -	- - -	- - -	- - +	-4 –4 -2
2	0001	- - -	- - -	- - -	+ - -	-2 –4 -4
3	0010	- - -	- - -	+ - +	- - +	-2 –4 0
4	0011	- - -	- - -	+ - +	+ - -	0 –4 -2
5	0100	- - -	- + -	- - -	- - +	-4 –2 –2
6	0101	- - -	- + -	- - -	+ - -	-2 –2 -4
7	0110	- - -	- + -	+ - +	- - +	-2 –2 0
8	0111	- - -	- + -	+ - +	+ - -	0 –2 -2
9	1000	- - +	- - -	- - -	- - +	-4 –4 0
10	1001	- - +	- - -	- - -	+ - -	-2 –4 -2
11	1010	- - +	- - -	+ - +	- - +	-2 –4 +2
12	1011	- - +	- - -	+ - +	+ - -	0 –4 0
13	1100	- - +	- + -	- - -	- - +	-4 –2 0
14	1101	- - +	- + -	- - -	+ - -	-2 –2 -2
15	1110	- - +	- + -	+ - +	- - +	-2 –2 +2
16	1111	- - +	- + -	+ - +	+ - -	0 –2 0

IV Comparison between STBC and CC

A. General

- In STBC, signal transmitted from an antenna at time (t + Δt) is a function of another signal transmitted from (usually) a different antenna. In CC, a signal transmitted at (t +Δt) is selected from a pair of 'symbols' corresponding to a specific transmitted symbol – e.g. a '0' is sent as (0,1) during time (t, t +Δt).
- STBC decides on each signal separately, so it may be advantageous over CC in cases where only one of the signals was received correctly.
- STBC has a maximum rate of 1.0, while CC has a rate of >= 1.
- CC decoding algorithm is simpler to implement. The only drawback is that it needs more memory to store the lookup table.
- CC has the advantage of Error Detection – in case the combined signal is not one of the entries in our lookup table.

B. 2-user case

- We may consider CC the same as STBC, i.e. we have 2 bits in time, and one receiver. In STBC, we have 2 bits in space (2 antennas), and 1 receiver.
- CC utilizes joint detection of transmitted signal, while STBC is based on each signal separately.
- Received signals in both cases are the same.
- In STBC the receiver has to estimate $s_0^{\sim}$ (and $s_1^{\sim}$), and then calculate the distance between $S_0^{\sim}$ and every possible signal to select the one with minimum distance. For binary signaling scheme, comparison is made between two symbols {+ve or –ve}. For CC case, a comparison is made with four pairs of data, with 3 symbols {-ve, 0, +ve}.
- STBC can be considered equivalent to CC, where each antenna in STBC represents a user in CC.

C. 4-user Case

- Encoding in CC is easier and straightforward, a simple transformation of bits is carried out by each encoder. This might reduce the overall complexity of the system!

V Conclusions

A comparison between STBC and CC was presented in this paper. The main findings are as follows. In STBC all the channel signals are distinct, just as in CC. CC has the advantage of Error Detection – since a combined message, which is not one of the codebook entries, will signal an error. This is in contrast to STBC which bears no such relationship between the various signals. Based on the above, we suggest that STBC can be decoded using a two step procedure. First the transmitted signals are estimated using the standard technique normally implemented by STBCC detectors. Second, a lookup table (just like CC), is used to check if these estimated signals constitute a valid combination. If they are not, then a retransmission is initiated by the receiver, which is in effect introducing error control using automatic repeat request (ARQ).

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